# OPPONENT'S REPORT
# ON THE DISSERTATION THESIS OF MS. MANVI GROVER

### ALEŠ NEKVINDA

I express a strong endorsement of the Ph.D. thesis submitted by Ms. Manvi Grover at the Faculty of Mathematics and Physics at Charles University.

The thesis has an introduction, a bibliography, and three attachments, each containing one of the following three papers:

- [1] B. Opic and M. Grover, Description of K-spaces by means of J-spaces and the reverse problem in the limiting real interpolation, Math. Nachr., 296 (2023), 4002-4031.
- [2] F. Cobos, L.M. Fernández-Cabrera and M. Grover, Measure of non-compactness and limiting interpolation with slowly varying functions (accepted by Banach Journal of Mathematical Analysis).
- [3] M. Grover, B. Opic, Duality of limiting interpolation spaces (to appear).

The common feature of the papers is their focus on some very specific problems from contemporary theory of interpolation. More precisely, the author together with her co-authors studies mainly interpolation spaces which are obtained via either the $K$-method or the $J$-method of real interpolation. The interpolation functors which are applied here are in a limiting form and their characteristic feature is that they use slowly-varying weight functions. In Paper [1], certain equivalence of the $K$ method and the $J$ method of interpolation is proved. Furthermore, the paper contains a density result. The specific feature of Paper [2] is that it works with compatible couples of quasi-Banach space. In this paper, compactness and measure of non-compactness of operators acting on the spaces obtained via interpolation are studied. The principal result gives estimates for measure of noncompactness. It also describes applications to operators on Lorentz–Karamata spaces, yielding, in particular, a quantitative extension of older compactness results involving operators on Lorentz–Zygmund spaces. Paper [3] gives a characterization of the associate spaces of the interpolation spaces described in Paper [1].

To summarize, the Ph.D. thesis of Ms. Manvi Grover contains new results which are interested both from the theoretical point of view, and also from the point of view of possible applications to concrete types of operators. The results also bring some new open questions which will be worth studying in the future. The thesis is very carefully written, it brings interesting new results, and sufficiently innovative approaches to the questions studied. A good job was obviously done also by the supervisor. I have no hesitation to strongly recommend that Ms. Grover is awarded the Ph.D. degree.

doc. Aleš Nekvinda (FSv ČVUT Praha)