

## Report on the doctoral thesis

*Title:* Weighted inequalities, Limiting real interpolation and Function spaces

*Author:* Manvi Grover

Weighted inequalities, Limiting real interpolation, and Function spaces are very important parts of Mathematical Analysis and results from these fields have many applications in various areas of mathematics.

The doctoral thesis of Manvi Grover consists of the following papers:

- [1] B. Opic and M. Grover, Description of  $K$ -spaces by means of  $J$ -spaces and the reverse problem in the limiting real interpolation, *Math. Nachr.*, 296 (2023), 4002-4031. Doi: 10.1002/mana.202100545
- [2] F. Cobos, L.M. Fernández-Cabrera, and M. Grover, Measure of non-compactness and limiting interpolation with slowly varying functions, pp. 25 (Accepted by *Banach Journal of Mathematical Analysis*).
- [3] M. Grover, B. Opic, Duality of limiting interpolation spaces, pp. 37 (To appear).

In the paper [1] conditions are established under which  $K$ -spaces in the limiting real interpolation involving slowly varying functions can be described by means of  $J$ -spaces and also the reverse problem is solved. To this end, several versions of the fundamental lemma of the real interpolation theory are proved. The results are applied to obtain density theorems for the corresponding limiting interpolation spaces.

In the paper [2] the authors found estimates for the measure of non-compactness of operators interpolated by the limiting real interpolation involving slowly varying functions. The techniques are based on the vector-valued sequence spaces, the definition of limiting interpolation spaces and their description as  $J$ -spaces. The results are applied to establish estimates of the measure of non-compactness of operators acting between Lorentz-Karamata spaces.

Finally, in the paper [3] duals of limiting real interpolation  $K$ - and  $J$ -spaces are determined. To this end, the method used to established duals of classical real interpolation spaces in the book *Interpolation spaces*, by J. Bergh and J. Löfström, was extended to the limiting situation. Together with results from [1], this gives a solution of the given problem.

Note, that the results of the doctoral thesis are new. Some of them extend results which are known for the case when weight functions involved in the definition of the limiting real interpolation spaces are of a logarithmic type. I have lectured about results of paper [1] and some of the results from [3] at a conference in Madrid and my lecture have a very positive acceptance.

Note also that the contribution of Manvi Grover to papers [1] and [2] is about 1/2, while her contribution to the paper [3] is about 1/3.

The doctoral thesis of Manvi Grover satisfies all the required conditions. Thus, I recommend to assign the title Ph.D. to M. Grover.

Bohumír Opic