In this thesis, we present new results related to combinatorial properties of topological spaces given by abstract simplicial complexes, their relations and computational complexity.

First, we generalize a result of Hachimori on relations between shellability and collapsibility which are important combinatorial properties of simplicial complexes.

Next, we study the computational complexity of the PL geometric category of 2-dimensional polyhedra introduced by Borghini which is a combinatorial notion providing a natural upper bound for the Lusternik–Schnirelmann category. For 2-dimensional polyhedra it can be equal to 1, 2 or 3. While it is easy to decide whether the PL geometric category of a 2-dimensional polyhedron is equal 1, we show that it is NP-hard to decide whether this category is at most 2.

Finally, we show that computing the rank of higher homotopy groups of a simply connected topological space is #W[2]-hard using a problem called VEST, given by Anick, as an intermediate problem. We also establish results for the decision version of VEST and for its variants as self-contained problems. For most of them we show W[1]- or W[2]-hardness.