Assessment of the thesis

Combinatorics, group theory, computational complexity & topology

by

Michael Skotnica

The thesis of Michael Skotnica consists of three main topics organized in Chapters 2, 3 and 4. (Chapter 1 serves as an introduction.) Chapters 2 and 3 are based on already finished works. Chapter 4 is based on two preprints but with a reasonable level of confidence I am persuaded that the results in Chapter 4 should be correct.

The contents of Chapters 2 and 3 can be connected via the notion of shellability. The goal in Chapter 2 is to provide a strong connection between shellability and so called collapsibility. In Chapter 3, a connection between shellability and PL geometric category is found which allows to transfer (in a non-trivial way) an earlier result on hardness of recognition of shellable complexes to the setting of PL geometric category. Chapter 4 deals with so called VESTs (vectors evaluated after a sequence of transforms) and the motivation of their study comes from computations of higher homotopy groups. Now, let me explain the contents of individual chapters in more detail.

Chapter 2 (joint work with T. Magnard and myself): Roughly speaking, a simplicial complex is shellable if its inclusion-maximal faces (called facets) can be ordered into a certain nice sequence (each facet meets the previous ones in a controlled way). Shellability is, for example, an important tool for induction, or counting/estimating the number of faces. In 2008, Hachimori proved that for 2-dimensional complexes the existence of a shellable subdivision is equivalent with shellability of the second barycentric subdivision which is in turn equivalent to certain connectivity condition of links. It is not possible to generalize this equivalence to higher dimensions in full strength (there are counterexamples); however, the main aim of Chapter 2 is to generalize an interesting implication that condition on links implies shellability of the second barycentric subdivision. This is done via so called hereditary removal-collapsibility condition. It takes a nontrivial effort to work out all the details.

Chapter 3 (joint work with myself): PL geometric category (not defined here) is a combinatorial variant of a well known Lusternik-Schnirelmann category or a closely related geometric category. It has been introduced by Borghini with a hope that this category may be more accessible than the two aforementioned ones. Borghini showed that for 2-dimensional complexes, this category may be 1, 2 or 3, and he provided a partial characterization of complexes with this category at most 2. The main result of Chapter 3 is to show (at least in algorithmic sense) that one cannot expect anything more than a partial characterization as it is NP-hard to decide whether this category is at most 2 already for 2-complexes. Surprisingly, this is done via relating the problem to shellability and a non-trivial modification of an earlier proof that it is NP-hard to decide whether a given 2-dimensional complex is shellable.

Chapter 4 (based on a preprint solely authored by M. Skotnica and another preprint which is a joint work with C. Brand, V. Korchemna and K. Simonov): As mentioned earlier, here the main object to

study in this chapter is so called VEST. The VESTs are studied here mainly from point of view of parametrized complexity. The author shows that computing so called M-sequence of a VEST is #W[2]-hard in a suitable parametrization. This improves an earlier result of Matoušek showing #W[1]-hardness. This is interesting because the hardness result can be translated to hardness of computing higher homotopy groups (when parametrized by the dimension). I should also point out that this corresponds to the content of the first preprint solely authored by Michael. Then the VEST is studied further in this chapter. Several natural restrictions are put on VEST and either the parameterized complexity is determined exactly or at least a hardness result is provided.

I believe that Michael's contribution to aforementioned works is on average proportional to the number of coauthors though this is probably sometimes more and sometimes less for specific works. In particular, the contents of a part of Chapter 4 is based solely on his work which demonstrates that he is able to work independently. Regarding the progress during the PhD studies, I can confirm that he spent a quite nontrivial effort on a progress during his PhD studies. On the other hand, sometimes I was hoping that this progress could be a bit faster. (But this was also negatively influenced by limited options for personal meetings during the COVID outbreak or by spending quite some time on a probably too hard topic chosen by myself).

Altogether, on the one hand, it would be nice if Michael could get a bit more results (or some a bit more influential one); on the other hand, I am strongly persuaded (with no doubt) that he did more than enough work required for awarding the PhD degree. Thus **I recommend that he should be awarded with the PhD degree**.

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