This thesis is focused on studying the properties of function spaces from three distinct angles: abstract classes of function spaces, one particular class of function spaces, and some specific applications of the function space theory. It contains six papers; two for each of the above mentioned topics.

The first paper studies the properties of quasi-Banach function spaces. We prove several results that provide useful tools for working with concrete examples of spaces belonging to this class.

The second paper studies the so-called Wiener–Luxemburg amalgam spaces, an abstract framework that allows for constructing a space where the conditions on local and global behaviour of its functions is prescribed separately. We describe the fundamental properties of such constructed spaces and develop tools for working with them.

The third paper provides a thorough and comprehensive treatment of Lorentz–Karamata spaces. We consider a wide variety of topics (e.g. normability, absolute continuity of the (quasi)norm, associate spaces) and investigate each of them extensively.

The fourth paper is focused on proving the result, that for every slowly varying function b there exists an equivalent (hence also slowly varying) function c which has continuous classical derivatives of all orders.

The fifth paper is dedicated to proving a so-called reduction principle, which states that operators of a certain form are bounded if and only if their restriction to the cone of non-increasing functions is bounded.

In the sixth paper we develop a new strongly non-linear version of the Gagliardo–Nirenberg inequality which we then use to obtain a priori estimates for several examples of non-linear PDEs. The paper also contains several variants of the main result and other applications.