Point processes are stochastic models widely used in biology, forestry, or astronomy. In this thesis, we are going to deal mainly with anisotropic cluster point processes. We present a new method for estimating parameters of such models. The basis of this method is the use of Bayesian statistics combined with Markov Chain Monte Carlo algorithms, which are a useful way to estimate parameters which are difficult or impossible to estimate using traditional methods. We describe the method in detail and present several examples of its application to simulated and real-life datasets and discuss the difficulties associated with it. Finally, we prove theoretical results about the convergence of the corresponding Markov chain under specific assumptions on the model and discuss the difficulties we encounter when examining these properties.