Question 1. We briefly describe options of parameter settings in the simulation study. We start with a market valuation of interest rate derivatives (caplets, caps and swaptions). The reason, why we use the shifted Black-Scholes model with the shift 3 %, is that quoted volatilities are given from prices by this model. To stay consistent and get the right and senseful prices, we use the shifted Black-Scholes model.

The next choice is a parameter space setting for the calibration procedure. We try to calibrate the Hull-White model to be useful and interpretable. Interest rate movements need to be senseful. Hence we do not reflect very small and relatively high values of the parameters. Specific values corresponds with the methodics in Česká spořitelna. The small values can lead to a different model. As was shown in the thesis, the caplets calibration indicate the use of the Ho-Lee model with no mean reversion ($\alpha = 0$).

The high values can lead to a worse interpretation (either one of them is high or both). The high volatility cause a wide spread of simulated interest rates. An example in the thesis was the caps calibration with estimated $\sigma = 3{,}17$. There may be a relevant lower choice of an upper bound for *sigma*. The high mean reversion speed rate can oppositely cause a very narrow sperad of interest rates. The high values of both parameters lead to a strange behaviour of the rates with abrupt movements from the mean reversion level and back immediately.

The objective function is taken as a square differences of market and model prices, because it is consistent with objective function used in*curve_fit* Python optimization procedure. We also try to use a square relative differences in the second Python optimization function, *differential_evolution*, but results are nearly the same.

Question 2. The estimated Hull-White model through our procedure can be widely used in risk management. A possibility to simulate future interest rates is very advantageous when a company has liabilities sensitive to the interest rates. The simulations provide an option to create an empirical distribution of rates in a given time, implement scenarios of movements and apply stress tests under extreme movement conditions.

We can introduce one specific example of the use in banking sector. Mortgages are long-term loan contracts (liabilities) which are sensitive to the interest rates not only by a cause that we look at discounted cashflows, but also due to a possibility to repay the loan earlier during the fixed interest rate period. In that case, a bank can borrow a new mortgage with a new interest rate which can be much different to one in the repaid mortgage. With the possibility of interest rate simulations by the Hull-White model, we can calculate the safety margin which can be added to the interest rate for the new fixed period.

The use in investment strategies is mainly in a pricing of complex and excotic derivatives with interest rate as an underlying asset. Monte Carlo simulations based on the estimated Hull-White model enable us to price probably any interest rate derivative. Even the mortgage can be viewed as the exotic derivative. Hence the pricing can be also used in risk management.

Question 3. The extension of the calibration procedure on more complex financial derivatives and hybrid products may be possible, but it brings a lot of challenges. A crucial condition is an analytical form of a price implied by the Hull-White model based on calibrated parameters. If the financial derivative has this pricing formula, the extension is

clear. The problem is that they usually do not have easily derived analytical expression, hence is very difficult to use our proposed calibration procedures. This complex products can have specific settings in cashflows, expiry and maturity and it may be impossible to propose analytical form of the price. This can be challenging.

Different way how to get the price, which can be used in the square differences of the model and market prices for a minimization procedure, are simulations of the Hull-White model on a grid of feasible values of parameters. For each pair of values, it is necessary to perform sufficient number of simulations to establish the price for this pair of parameter values. We need this procedure for at least 10 different products to reasonably calibrate two-parameter model. It can be challengenig in time requirements, software and programming limits.

The last but not least, we can think about some approximative procedure like in the swaption calibration when we approximate the Hull-White model with the different one and compare the model and the market by different characteristics like implied volatility. It can be difficult to find a potentially suitable model and even harder to prove, that this approximation is correct, and derive a theoretical value of the chosen characteristic.