Errata to the master thesis

The assertion of Theorem 12 and following proof was not correct. This finding was given by review performed by doc. Kulich. In this errata the correction is published and the differences are emphasized by red colour.

Theorem (Martingale Representation of $\widehat{CIF}_j$)**.** *Assuming the conditions of Theorem 9 and $S(t) > 0$, the following holds:*

$$\sqrt{n}\left(\widehat{CIF}_j(t) - CIF_j(t)\right) = \sqrt{n}\left(\int_0^t \widehat{KM}(u-)d\widehat{\Lambda}_j^{CS}(u) - \int_0^t S(u-)d\Lambda_j^{CS}(u)\right)$$

$$= \sqrt{n}\int_0^t \frac{S(u-)\mathbf{1}\{\overline{Y}(u) > 0\}d\overline{M}_j(u)}{\overline{Y}(u)} - \sqrt{n}CIF_j(t)\int_0^t \frac{\mathbf{1}\{\overline{Y}(u) > 0\}d\overline{M}(u)}{\overline{Y}(u)}$$

$$+ \sqrt{n}\int_0^t \frac{CIF_j(u)\mathbf{1}\{\overline{Y}(u) > 0\}d\overline{M}(u)}{\overline{Y}(u)} + o_p(1)$$

The expression $o_p(1)$ means that this part of the formula converges in probability to zero as n tends to infinity. Furthermore, the convergence is uniform.

Proof. The proof is correct until the following expression. One step of the proof was incorrect because the sign was omitted. This fact is emphasized by changing the colour.

Based on the result from Lemma 5 we have $S(u)d\Lambda_j^{CS}(u) = dCIF_j(u)$. Then we use Lemma 10. We obtain double integral and again we rewrite it to more convenient expression

$$\sqrt{n}\int_0^t - \int_0^u \left(\frac{\widehat{KM}(s-)}{S(s)}\right) d\left(\widehat{\Lambda} - \Lambda\right)(s)dCIF_j(u) + o_p(1) =$$

$$= \sqrt{n}\int_0^t - \int_0^u \left(\frac{\widehat{KM}(s-) - S(s)}{S(s)} + 1\right) d\left(\widehat{\Lambda} - \Lambda\right)(s)dCIF_j(u) + o_p(1) =$$

$$= \int_0^t o_p(1)dCIF_j(u) - \sqrt{n}\int_0^t \left(\widehat{\Lambda}(u) - \Lambda(u)\right) dCIF_j(u) + o_p(1).$$

The last formula consists of the final desired result and integral negligible in probability, again the negligibility does not depend on t. It can be shown again by uniform consistency of a Kaplan-Meier estimate and by similar usage of the central limit theorem for the sum of martingale differences as was provided before. By the last discussion was proven that $\sqrt{n}\left(\widehat{CIF}_j(t) - CIF_j(t)\right)$ is equal to

$$\sqrt{n}\int_0^t S(u-)d\left(\widehat{\Lambda}_j^{CS} - \Lambda_j^{CS}\right)(u) - \sqrt{n}\int_0^t \left(\widehat{\Lambda}(u) - \Lambda(u)\right) dCIF_j(u) + o_p(1) \quad (1)$$

As the next step, we use integrating by parts for Lebesgue-Stieltjes on the second part of the equation. We got

$$\sqrt{n}\int_0^t S(u-)d\left(\widehat{\Lambda}_j^{CS} - \Lambda_j^{CS}\right)(u) - \sqrt{n}\left(\widehat{\Lambda}(t) - \Lambda(t)\right)CIF_j(t)$$
$$-\sqrt{n}\int_0^t CIF_j(u)d\left(\widehat{\Lambda} - \Lambda\right)(u) + o_p(1).$$

To finalise the proof we need to plug in formulas for

$$\sqrt{n}\left(\widehat{\Lambda}(t)-\Lambda(t)\right) \text{ and } \sqrt{n}\left(\widehat{\Lambda}_j(t)-\ \Lambda_j(t)\right),$$

which are provided in Theorem 9. We receive the final formula which is an assertion of the theorem. Note that the term $o_p(1)$ tends to zero uniformly. We obtained final expression

$$\begin{aligned}&\sqrt{n}\int_0^t \frac{S(u-)\mathbf{1}\{\overline{Y}(u)>0\}d\overline{M}_j(u)}{\overline{Y}(u)}\\ &-\sqrt{n}CIF_j(t)\int_0^t \frac{\mathbf{1}\{\overline{Y}(u)>0\}d\overline{M}(u)}{\overline{Y}(u)}\\ &+\sqrt{n}\int_0^t \frac{CIF_j(u)\mathbf{1}\{\overline{Y}(u)>0\}d\overline{M}(u)}{\overline{Y}(u)}+o_p(1).\end{aligned}$$

□

Remark. As a special result from Theorem we could obtain an asymptotic representation for an event of interest ($j=1$). If we merge all competing events into one, there is a possibility for just two values of j, equal to either one or two. In this specific setting of problem, it could be easily proved that $\sqrt{n}\left(\widehat{CIF}_j(t)-CIF_j(t)\right)$ is equal to

$$\begin{aligned}\sqrt{n}\Big[\int_0^t \frac{(1-CIF_2(u))\,\mathbf{1}\{\overline{Y}(u)>0\}d\overline{M}_1(u)}{\overline{Y}(u)}+\int_0^t \frac{CIF_1(u)\mathbf{1}\{\overline{Y}(u)>0\}d\overline{M}_2(u)}{\overline{Y}^2(u)}\\ -CIF_1(t)\int_0^t \frac{\mathbf{1}\{\overline{Y}(u)>0\}(d\overline{M}_2(u)+d\overline{M}_1(u))}{\overline{Y}(u)}\Big]+o_p(1).\end{aligned}$$