Supervisor's Report

Title: **Detection and Correction of Silent Errors in Pipelined Krylov Subspace Methods**
Author: **Jakub Hercík**

Summary of the work's content

This thesis revolves around the topic of silent errors (i.e., bit flips) in variants of Krylov subspace methods for solving large-scale sparse linear systems. In contrast to other types of errors that one might encounter when executing an algorithm on a computer (e.g., node failures), silent errors may be particularly hazardous as there is often no indication that something has gone wrong until the algorithm fails to converge to a reasonably accurate solution. Silent errors are expected to become more common as the scale of computer hardware increases.

One approach to dealing with silent errors is Algorithm-based Fault Tolerance, which is based on using information about the algorithm to monitor certain quantities and detect when errors have occurred. This is the approach studied in this thesis. Previous work has developed silent error detection approaches for other Krylov subspace methods such as the basic conjugate gradient method, and this is shown to be effective.

This thesis develops such a method for a "pipelined predict-and-recompute" variant of the conjugate gradient method. Pipelined Krylov subspace methods are variants designed for high-performance environments which introduce additional recurrences and alter how, e.g., inner products are computed in order to enable overlapping of costly operations and "hide" data movement costs. The "predict-and-recompute" variant fixes stability problems in the basic pipelined variant by using "predictors" for certain computed quantities (computed via recurrence) and then later computing the "true" values of these quantities; this serves to prevent the accumulation of rounding errors.

In his thesis, Jakub Hercík has developed, analyzed, and tested a detection method for silent errors in the pipelined predict-and-recompute conjugate gradient method (Pipe-PR-CG). The approach is based on rigorous floating point rounding error analysis, which bounds the size of the differences between certain computed quantities. By monitoring when the actual difference exceeds the theoretical bound, one can then detect that a bit flip must have occurred. There are four different detection criteria which, when combined, are able to detect errors in all the scalar and vector quantities computed during the algorithm with the exception of the approximation solution to the linear system, for which this is not possible in this approach.

After providing a thorough background on finite precision arithmetic, silent errors, and relevant variants of the conjugate gradient method, Jakub Hercík first presents a number of experiments that demonstrate the sensitivity of the convergence in the Pipe-PR-CG method to bit flips, testing various flip indices, in various quantities, and in various parts of the iterative process. The next chapter gives the finite precision analysis which provides theoretical bounds on the "gaps" between quantities and uses these to develop detection criteria.

This is followed by an extensive set of experiments which evaluate the reliability of the set of detection criteria in terms of false positives and false negatives on a set of real-world sparse matrices. A method for correcting the algorithm after the detection of an error is provided, as well as a clever method of adaptively refining a threshold parameter which reduces the number of false positives.

Evaluation

- **Thesis topic:** The thesis topic is adequate for the Master's thesis level and the student has fulfilled the outlined requirements.
- **Author contribution:** Jakub Hercík has provided the first method for the detection and correction of silent errors in a popular class of Krylov subspace methods designed for high-performance

computing. In doing this, he has also proved new theoretical bounds between predicted and recomputed quantities within the Pipe-PR-CG method. This work is a valuable contribution to the field of algorithm-based fault tolerance for high-performance numerical solvers. He has also contributed a useful set of Python codes which can be used to reproduce the experimental results.

In completing this thesis, Jakub Hercík has demonstrated a high-level of independence, self-determination, and scientific maturity on par with that of a quality PhD student.

- **Mathematics level:** In my opinion, the level of mathematics is on par with what is typically required for a Master's thesis. The work represents the successful combination of finite precision arithmetic, floating point error analysis, and matrix computations, with the included consideration of high-performance parallel computing.
- **Literature and sources:** The sources used are appropriate and correctly cited, and include a good mix of textbooks, classical articles, and recent research articles.
- **English and writing:** The quality of English language and grammar is very good.

Conclusion

I fully recommend accepting the presented thesis as a Master's Thesis.

in Prague, 29.05.2024

Dr. Erin Claire Carson, Ph.D.