Ball-Evans aproximation problem is a highly studied one in Geometric function theory. Approximation of homeomorphisms by diffeomorphisms would have many implications e.g. in the theory of regularity of minimizers or in Finite element method. Recently, this problem was solved in two dimensions. However, in 3D, which is the physically most interesting case, it remains open.
In this thesis, we study the following two problems. Firstly, for some $1 \leq p < \infty$ and a given homeomorphism $f \in W^{k,p}\left((a,b)\right)$, can we approximate it by diffeomorphisms in $||\cdot||_{\mathcal{W}^{k,p}((a,b))}$ with an arbitrarily small error? Secondly, for some $1 \leq p, q < \infty$ and a given homeomorphism $f \in W^{1,p}\left((a,b)\right)$ such that also $f^{-1} \in W^{1,q}\left((c,d)\right)$, can we find a sequence of diffeomorphisms $\{f_n\}_{n=1}^{\infty}$ such that

$$||f_n - f||_{\mathcal{W}^{1,p}((a,b))} \xrightarrow[n\to\infty]{} 0 \quad \text{and} \quad \left|\left|f_n^{-1} - f^{-1}\right|\right|_{\mathcal{W}^{1,q}((c,d))} \xrightarrow[n\to\infty]{} 0?$$

We show positive results for both problems. The latter one is not known in higher dimensions.