Review by the opponent of the bachelor thesis

Thesis title: Permutohedral varieties as Chow quotients
Thesis author: Magdaléna Mišinová

Summary of the thesis content

Permutohedron is a polytope those vertex coordinates are the permutations of the first n natural numbers. Permutohedral variety is a toric variety corresponding to the permutohedron. Permutohedral variety plays a key role in recent breakthroughs in matroid theory by June Huh and his coauthors. This makes the understanding permutohedral variety an important task in combinatorial algebraic geometry. The main topic of the thesis is to give a new perspective on permutohedral variety via representing it as a Chow quotient of the product of projective lines under the diagonal action of $\mathbb{C}^*$. Although this construction is probably known to the experts, up to my knowledge, it is not available in the literature.

Overall evaluation of the thesis

Thesis topic. To obtain the main goal of the thesis one has to gain confidence with toric varieties, Chow varieties (including Grassmanians), and Chow quotients. I find the difficulty of the topic to be appropriate for bachelor thesis.

Author's contribution. Author's contribution is twofold. First the author gives a brief introduction to the theories of toric varieties, Chow varieties and Chow quotients. Then the author construct a map between the Permutohedral variety and the Chow quotient of product of projective lines. The constructed map is expected to be an isomorphism of algebraic varieties, however it is only proven in dimensions up to two.

Mathematical level. All the new results of the thesis are clearly stated and have rigorous proofs. However, in the introductory parts, there are some minor imprecisions/typos. Overall I find mathematical level of the text to be very good.

Work with sources. Work with sources is appropriate, there are no copied or word-to-word translated passages.

Comments and questions

1. In the paper *Kapranov, M.M., and Sturmfels, B. "Quotients of toric varieties." Mathematische Annalen (1991)* authors study general Chow quotients of toric varieties. Perhaps results of this paper could be useful to give proof that permutohedral variety is a Chow quotient of product of projective lines in all dimensions.

2. I believe Definition 6 of the flat morphism is not necessary for the text as the author really uses Definition 7 instead and most terms of Definition 6 are not explained anyway.

3. There is some number of typos in the text starting from second line of introduction (projectivisation of $\mathbb{C}^n$ should be $\mathbb{P}^{n-1}$) and ending with references (there is no journal name for reference [3]).

Conclusion

I consider the thesis to be very good and I recommend that it be accepted as a bachelor's thesis.

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