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Assessment of the doctoral thesis presented by Petr Kotlařík

Petr Kotlařík presented a doctoral dissertation entitled "Gravitational sources in the vicinity of black holes." The thesis is 140 pages long. It is organized in 4 chapters, an introduction and conclusion sections, as well as two appendices. Main results of the thesis are collected in 5 papers:

1. Chen, C.-Y., & Kotlařík, P. 2023, Quasinormal Modes of Black Holes Encircled by a Gravitating Thin Disk, Physical Review D, 108, 064052

2. Kofroň, D., & Kotlařík, P. 2022, Debye Superpotential for Charged Rings or Circular Currents around Kerr Black Holes, Physical Review D, 106, 104022

3. Kofroň, D., Kotlařík, P., & Semerák, O. 2023, Relativistic Disks by Appell-ring Convolutions, arXiv, doi: 10.48550/arXiv. 2310.16669

4. Kotlařík, P., & Kofroň, D. 2022, Black Hole Encircled by a Thin Disk: Fully Relativistic Solution, The Astrophysical Journal, 941, 25

5. Kotlařík, P., Kofroň, D., & Semerák, O. 2022, Static Thin Disks with Power- law Density Profiles, The Astrophysical Journal, 931, 161

Four of these papers have been published in the Astrophysical Journal and in Physical Review. The remaining paper is, at the moment, available as a preprint. It should be emphasized that both the Astrophysical Journal and the Physical Review belong to the best and most prestigious journals publishing scientific results in General Relativity. Moreover, all papers have been written in small groups of either two or three authors.

Papers 3, 4, and 5 are devoted to a study of static and axially symmetric solutions of the Einstein equations, representing razor thin disks or disks around a static (Schwarzschild type) black hole. Such solutions can be understood as an approximation valid in cases in which the rotation is slow and dragging effects do not play a significant role (there is also an interpretation in terms of two counter-rotating streams of matter understood

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as a source of the gravitational field). On the other hand, they allow to investigate the effects caused by the self-gravity of matter, in particular around black holes. The main observation employed in these papers is that in Weyl coordinates one of the metric functions describing a static and axially symmetric spacetime satisfies a standard Poisson equation (or Laplace equation in the vacuum region). Hence, one can employ the knowledge of axially symmetric solutions of the Laplace equation to generate solutions of the Einstein equations. The remaining metric function has to be found separately, which makes the whole study a challenging one. The authors succeeded in reproducing known solutions, but also developed new techniques and obtained new exact solutions.

In paper 1, written with Chen, Petr Kotlařík studied quasi normal modes (QNM) of a massless scalar field propagating on a static and axially symmetric background of a black hole surrounded by a self-gravitating disk. It is an interesting work, going beyond the usual assumption of spherical symmetry. On the other hand, the dynamics of the massless scalar field serves as a toy model of the quasinormal modes of the gravitational waves. The authors demonstrate the influence of the disk on the obtained QNM frequencies, which is a valuable contribution to our understanding of quasinormal modes in the general-relativistic context.

In paper 2 (written with David Kofroň), the authors investigate stationary electromagnetic fields on the Kerr background. More specifically, they find stationary and axially symmetric electromagnetic fields corresponding to circular loops (either a charged rings or current loops) using the so-called Debye superpotentials. This is an exciting paper, as it gives an opportunity to provide expressions for the electromagnetic field on axially symmetric spacetimes, corresponding to stationary and axially symmetric currents, e.g., associated with accretion disks. I believe that this technique could offer a new perspective in the study of black hole magnetospheres, alternative to working, e.g., with the Grad-Shafranov equation. Another possibility to develop this approach further – the one that the authors actually suggest – is to deal with perturbations of stationary spacetimes, corresponding to rotating sources. This paper uses extensively the Newman-Penrose and the Geroch-Held-Penrose formalisms.

Personally, I am truly impressed by the results obtained in Petr Kotlařík's thesis. All investigated subjects require mathematical maturity and an expert knowledge in General Relativity. The authors work comfortably with special functions and complicated series expansions, being able to provide closed-form solutions, where others would probably give up. The systems investigated in this paper may seem idealized, but they are never devoid of a clear physical context. Moreover, the authors stay firmly within the well-established General Relativity, providing new results that were difficult to obtain, but which will most probably resist temporary fashions in modern physics.

Let me now allow myself to state some more remarks and questions. I was happy to see the effect noticed in paper 5 which the authors describe by stating that ``…several occasions can be seen where the disk is so heavy that it is not possible to orbit freely below its maximum (even a particle at rest is attracted outward to the disk, despite the opposite effect of the black hole).'' I have noticed a similar effect in my own work on heavy hydrodynamical disks, published with Wojciech Dyba and Wojciech Kulczycki: "Self-gravitating perfect-fluid tori around black holes: Bifurcations, ergoregions, and geometrical properties," Phys. Rev. D 101, 044036 (2020). In this case some regions of the spacetime occupied by the disk also do not allow for circular geodesics.

On page 54 the authors define the circumferential radius (in the usual way). Since the authors consider massive disks (in comparison to the mass of the central black holes), I was wondering if this radius remains monotonic within the disc. It is known, see, e.g., our paper Phys. Rev. D 101, 044036 (2020), or H. Labranche, D. Petroff, and M. Ansorg, Gen. Relativ. Gravit. 39, 129 (2007), that the circumferential radius may exhibit a local maximum within the disk. In other words, the circle with the largest circumference embedded within the disk region does not have to be the outermost one. Such an effect would also signal a regime of a strong gravitational field in the vicinity of the disk.

In Poland, it is customary to list in the report some mistakes or errors spotted in the dissertation. I this case, I admit that they are almost negligible. The thesis is very well written, and even typos are hard to find.

On page 36, I suppose there is a mistake in the first formula on the page (above Eq. (52)) – I think that the expression in the parenthesis should be a binomial coefficient instead of a fraction.

On page 77 the author states: 'Because the whole system is "open" (asymptotically flat), the gravitational waves exponentially decay. Such a behaviour is described by quasi-normal modes – a sum of exponentially damped oscillations.' This (as an implication) is not quite true. There are many "open" systems in physics, where no quasinormal modes can be observed. Apart from a possibility to disperse energy, the dynamics of the field (usually on a given background) must allow for quasinormal modes.

The graphs in the paper "Debye superpotential for charged rings or circular currents around Kerr black holes" lack the description (labels) of the axes (cf. Figs. 1, 2, 3, 4), which makes them difficult to read.

Very little is written in paper 1 on the actual method of computing QNMs and their frequencies (choosing the boundary conditions, etc.). In my opinion, a very convenient framework of dealing with QNMs is to work in a suitable

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hyperboloidal slicing (see, e.g., M. Ansorg and R. Panosso Macedo, Phys. Rev. D 93, 124016 (2016) for QNMs on the Schwarzschild spacetime, or my old work: P. Bizoń, P. Mach, Trans. Amer. Math. Soc. 369, 2029 (2017) for an example with the Yang-Mills field), but I believe, that it might be too complicated for the problem at hand.

The thesis presented by Petr Kotlařík is one of the best, if not simply the best, among several doctoral dissertations which I was reviewing in the past few years. Given the quality of the results and their mathematical rigor, I would suggest that this thesis for a distinction, if this custom exists in Prague. I would also like to congratulate both Petr Kotlařík and his supervisor – prof. Oldřich Semerák, as well as David Kofroň – the co-author of most of papers included in this thesis for their work.

Summing up, I believe that the doctoral dissertation presented by Petr Kotlařík constitutes a valuable original contribution to General Relativity and to theoretical physics in general. I am also convinced that it meets all the criteria prescribed by the law for a doctoral dissertation. In particular, it proves the author ability for creative scientific work. Therefore, I request that this dissertation should be admitted to a defense.