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Bremen, June 6, 2024

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Report on the PhD thesis of Petr Kotlařík

Dear Members of the Committee,

It is my pleasure to provide the following report on the doctoral thesis presented by Petr Kotlařík. Please note that the candidate is not personally known to me; my review is based solely on the submitted thesis and publicly available data on his publications.

The doctoral thesis by Petr Kotlařík, entitled "Gravitational Sources in the Vicinity of Black Holes," focuses on the theoretical physics of gravitational field equations, particularly solutions involving black holes. The thesis includes five original research articles, four of which have already been published or accepted in the most respected professional journals of the field.

The past decade has seen significant breakthroughs in relativistic astrophysics, including the first observations of gravitational waves and images of supermassive black holes. According to the no-hair theorem in General Relativity, black holes are described by the Kerr-Newman solution to the Einstein field equations. Astrophysically, it is expected that the electric (and magnetic monopole) charge is irrelevant, leading to the Kerr spacetime. However, the no-hair theorem assumes that the compact objects are isolated—a strong idealization, as black holes are typically surrounded by accretion discs and strong magnetic fields, often related to powerful jets. The primary focus of the presented thesis is the model of black holes in conjunction with disc or ring sources of gravitational or electromagnetic fields in their vicinity, representing the influence of the accretion disc. Given recent observational developments, which hold promise for precise testing of the black hole parameters, this topic is extremely timely and relevant from both theoretical and observational perspectives.

The research questions outlined above are analyzed in the thesis with depth and high-impact results. The thesis begins with a well-written general introduction to the topic, followed by a concise yet thorough treatment of basic assumptions and relevant literature in the first chapter, demonstrating Mr. Kotlařík's comprehensive understanding of the subject.

The second chapter then focuses on the construction of static solutions to Einstein's equations representing discs, and the superposition of these disc solutions with Schwarzschild black holes. While the construction of closed form disc solutions is already challenging, the main difficulty is in the superposition, which is linear in the Newtonian-like part but highly nontrivial in the remaining metric function. Mr Kotlařík not only presents several new solutions to the Einstein equation representing black hole – disc systems but also developes a new general method to generate such solutions. These results are extremely impressive and could alone suffice for a doctoral thesis. As a question to the PhD candidate, I am interested in the comparison of the radial velocity profiles he derived for the Schwarzschild black hole – disc systems with the usual Keplerian profiles assumed in Shakura-Sunyaev thin disc, and what are the conclusions of such a comparison for the properties of the system. An interesting point to follow up on the results presented in this chapter would be to compare them to local solutions of black holes surrounded by mass distributions, as e.g. recently derived by Faraji (Universe 8, 2022), for example by comparing the horizon geometry. Additionally, it would be interesting to explore if specific disc density distributions in the newly derived solutions could mimic astrophysically relevant accretion discs, such as in Cygnus X-1 or M87*.

Chapter three applies the previously obtained results to the area of quasi-normal modes. The impact of a particular black hole – disc model is considered in a perturbative way up to first order in the mass ratio. Interestingly, evidence is found for a universal relation between the disc parameters in their influence on the modes, and even hints towards a general universal influence of external masses. The obtained results are again very impressive and impactful. An intriguing point would be to discuss the disc's influence concerning observational uncertainties in the black hole's mass parameter.

In chapter four, the thesis addresses stationary systems. Given that nonvanishing rotation of the black hole or disc significantly complicates the problem, exact analytical solutions to Einstein's equations for black hole-disc systems are not feasible, necessitating perturbative methods. The Newman-Penrose formalism (and the related GHP formalism) is appropriately introduced and utilized. The approach is then used to first tackle the somewhat simpler but still very complicated problem of the electromagnetic field of a rotating ring on the curved background of a Kerr black hole. Impressively, closed form analytic solutions are found for circular sources, possibly located above or below the equatorial plane. This result could also have interesting applications in the context of equilibrium configurations of charged tori, that are known to exist outside of the equatorial plane as well. The chapter closes with a convincing outline how to tackle the more complicated problem of constructing the gravitational perturbation due to a ring source based on the previously developed methods.

The thesis closes with a brief but very useful conclusion and outlook, that nicely illustrated the immeadiate research directions opened up by the presented results.

In summary, the thesis by Mr. Petr Kotlařík addresses very timely and relevant topics from both the theoretical and the observational point of view. The material ranges from mathematical physics to applications in gravitational wave observations. The thesis is well structured, the results are very clearly presented and discussed, and supported by several high quality images. The quantity and quality of the presented results are exceptionally high. This is reflected in Mr. Kotlařík's publications, an important indicator of the research quality: his results have been published in four peer-reviewed articles (one accepted for publication) in the most respected journals in the field, complemented by a conference proceedings article. Another preprint is already on the arXiv, likely to be accepted for publication in a respected journal soon.

With these impressive results, Mr. Kotlařík has clearly demonstrated his outstanding capability for creative scientific work. I strongly recommend the acceptance of the thesis presented by Mr. Petr Kotlařík and the awarding of the PhD title.

Yours faithfully

Eva Hackmann