Fachbereich Physik, Mathematik und Informatik
Institut für Mathematik

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Johannes Gutenberg-Universität Mainz (JGU) D 55099 Mainz
Fachbereich 08, Institut für Mathematik

Fachbereich 08
Physik, Mathematik und Informatik

Institut für Mathematik

Prof. Dr. Maria Lukacova
AG Numerik

Johannes Gutenberg-Universität Mainz (JGU)
Staudingerweg 9
55128 Mainz

Tel. +49 6131 39-22831
Fax +49 6131 39-23331
Mail lukacova@mathematik.uni-mainz.de

Sekretariat:

Brigitte Burkert
Raum: 05-431
Tel. +49 6131 39-22270
Fax +49 6131 39-23331
Mail burkertb@mathematik.uni-mainz.de

www.mathematik.uni-mainz.de/Members/lukacova

prof. Zdeněk Doležal
Vice-Dean of Fac. Math. Phys.
Charles University
Prague

10.05.2023

Referee Report on the Habilitation thesis Mechanics and Thermodynamics of Viscoelastic Fluids by Vít Průša

The present habilitation thesis has a cumulative character and consists of an introductory part and ten attached published papers. The introductory part consists of four chapters explaining the main concepts, such as an evolving natural configuration and asymptotic stability analysis. The publications included in the habilitation thesis were published from 2016 till 2019 in internationally well-recognized journals. All publications are co-authored, the journals, where the papers were published, are focused on fluid dynamics, mechanics and mathematical modelling.

The aim of the thesis is to present the recent candidate's results on the viscoelastic-rate type fluids. Here the main focus is put on thermodynamically consistent modelling. The author (together with his co-authors) overcome the classical modelling approach for visco-elastic fluids and uses rather innovative, thermodynamically consistent approach based on the natural configuration. The latter has been proposed by Rajagopal and Srinivasa (2000). The key idea is based on the fact that the response of a given fluid is decomposed into a dissipative part and an elastic part. Thus, instead of working with the conformation stress tensor, in order to model an elastic response of a fluid, the candidate uses the left Cauchy-Green stress tensor associated to the elastic response.

In the introductory part the Dr. Průša formulates four main questions that are studied and at least partially answered in the habilitation thesis. I particularly appreciate an attempt to rigorously justify a nonlinear response of a jump discontinuity in the strain. It is interesting that in some cases the approximation by smooth functions does not lead to a desired result.

Does this depend on the regularity assumptions of the initial strain, initial data? Would it be possible to work with more general concept of solutions, including defect measures?
The author hypothesizes that in these cases the response is sensitive to microscopic details, which can very well be governed by an underlying measure.

The concept using evolving natural configuration is nowadays quite well accepted in the community. As it is purely phenomenological approach a natural question arises, whether the whole variety of the models derived by the natural configuration approach can be recovered by the hydrodynamic limit of corresponding kinetic/particle models. *Do there exists a general structural approach to derive a link between a model obtained on the base of natural configuration and the corresponding microscopic (kinetic) model?*

I positively value the fact, that using the above mentioned approach of Rajagopal and Srinivasa also the stress diffusion term (in the form of Laplace acting on the elastic stress tensor) can be explained. The derived model heavily depends on the choice of entropy production and energy dissipation (energy storage). A natural question arises: *how is the proposed approach related to the well-accepted GENERIC approach due to Grmela and Öttinger? Are they compatible in the sense that there is a general procedure how to connect one derivation scheme to another? If not, what are the main drawbacks and advantages of the corresponding approaches?*

The results on the asymptotic stability, presented in papers [P1-P4] are very relevant and yield another argument for physical reliability of the studied viscoelastic rate-type models. The author speaks about the Lyapunov functional, which, in fact, is the relative entropy functional, possibly augmented by appropriate Langrange multipliers if some additional constraints need to be taken into account. The present approach has not only application in the asymptotic analysis around a steady state, but can be also used in the context of weak-strong uniqueness or even in order to measure the errors between numerical solutions and an exact strong solutions. All of the calculations presented in the thesis are formal, in the sense that all derivatives, that are needed, are assumed to be available. In this context it will be interesting to rigorously classify (minimal) regularity class of both solutions, so that the corresponding calculus goes through well. *Is it possible to work in this context with weak solutions? Are there any results on the existence of global-in-time weak solutions known for the studied models? What about (local) existence of the strong solution?*

I would like to mention that the relative energy/entropy approach has indeed been recently extended also to open systems for viscous compressible fluids, see the recent book of Feireisl, Novotny (2022) and a paper by Basaric et al. (2022). I believe that not only the Lypanov functional, but also (standard) energy method need to be combined in order to achieve the desired results (weak-strong uniqueness, stability of a particular solution in the larger class of solutions). *Can the candidate comment on the connection of his Lyapunov approach and the ballistic energy approach used in the works of Feireisl for temperature-dependent compressible fluids?*

The candidate succeeded to obtain novel scientific results in highly competitive field of matematical modelling of complex fluids. The thesis is written in a clear understandable way; theoretical results are nicely combined with examples and corresponding pictures. The thesis clearly documents the candidate expertise in the field of fluid dynamics and mathematical modelling. The results presented in the thesis are new and original. The originality of the thesis was verified by the Turnitin system.

I will appreciate if the candidate comments/answers my questions presented above in the slanted style.

In conclusion, I believe that the present thesis fulfils all requirements for a successful habilitation and ***can be accepted for the award of „Docent" title.***

Best regards,

Prof. Dr. Maria Lukacova