UNIVERSIDADE
DE AVEIRO **Carlos A. R. Herdeiro, Research Full Professor**

Carlos A. R. Herdeiro
Investigador Coordenador
Mathematics Department
University of Aveiro
Campus de Santiago
3810-193 Aveiro, Portugal
Phone: +351 963337940
herdeiro@ua.pt
gravitation.web.ua.pt/herdeiro

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Assessment of Barbora Bezdekova's Ph.D. thesis.

This thesis of Barbora Bezděková deals with analytic studies on the propagation and transfer of electromagnetic radiation in refractive and dispersive media in relativistic spacetimes. It is a well written and structured document, based on several original contributions. After a brief Introduction the thesis is divided into 6 chapters.

Chapter 1 introduces the geometrical optics limit. This is done quite formally, which brings an element of rigour but, perhaps, makes the discussion more physically opaque. It is complemented by a review of rays and waves in the Hamiltonian theory, in a Riemannian spacetime, and then focusing on Synge's formalism applicable to refractive, dispersive media. The chapter ends with an informative literature review concerning the usage of Synge's approach, mostly assuming a cold non-magnetized plasma medium surrounding a gravitating object.

Chapter 2 presents original results published in two articles (one in J. Math. Phys. and another in Phys. Rev. D, also covered in Chapter 3). The idea is to apply Synge's formalism to an axially symmetric object surrounded by cold plasma finding conditions such that light propagation is still integrable. This implies a generalized Carter constant associated with a conformal Killing tensor exists, also in the presence of (appropriate) plasmas. Next, the photon region (i.e. the spherical photon orbits) are studied, which have a direct connection to the (edge of the) black hole shadow, which is considered next. The chapter ends by considering a general formula for the deflection angle of light in the considered class of spacetimes.

Chapter 3 provides applications of the general results obtained in the two previous chapters. One application is obtaining the photon regions in explicit examples, including the "hairy" Kerr metric, the Hartle-Thorne metric, the Melvin Universe and the Teo wormhole metric. Another application is comparing the previously obtained formula for the deflection angle with an approximate weak field solution, which is done for the Kerr and Hartle-Thorne metrics, first without plasma and then with plasma. The chapter ends with a discussion on ray tracing to illustrate deflection angles.

Chapter 4 takes one step back and one step forward: the more idealized case of spherical symmetry is considered but now the plasma may be moving and is not necessarily cold. These results are based on a paper accepted in Phys. Rev. D. The first case considered is a radially falling medium. The deflection angle is computed in general and then specialized for particular forms of the refractive index: cold plasma, non-dispersive medium, general non-cold plasma medium. A second case is a rotating medium and again the deflection angle is studied. The final part of the chapter is devoted to a different approach in which a medium is regarded either as a perturbation of vacuum or as perturbation of cold plasma, bringing perturbation schemes as a tool for performing the previous studies of the deflection angle.

Chapter 5 discusses light rays around a compact object from a different perspective, characterizing them by their impact parameters (not their deflection properties). This chapter is based on a paper submitted to J. Math. Phys.). This is handy in particular for describing forbidden spacetime regions for light rays which are studied for the Kerr metric with and without plasma. Chapter 6 deepens some technical aspects on the study of the allowed regions for rays in a plasma discussed in chapter 5.

Overall the thesis represents a solid body of work. Perhaps it could use more physical discussion and insight on the studied cases and how they helped pushed the boundaries of the field. Nonetheless, it is technically non-trivial, well motivated and trying to make discussions on lensing - often quite academic - more adapted to some astrophysical environments, without giving up analytical tools. I think it certainly merits a Ph.D. degree.

Three possible questions:

1) One could expect that in a medium the possibility that different light polarizations propagate differently would arise, like birrefringence seen in some crystals. This does not seem to be the case of the media considered. Are there interesting media around compact objects in which this would be likely to occur?

2) There has been a lot of interest in computing deflection angles using the Gauss-Bonnet theorem, which provides an elegant technique to do it (at least in spherical symmetry). Would this also be an interesting technique in the context of plasmas?

3) There is some mention of ray tracing in Sec. 3.4.. How were the numerical integrations done (which software or numerical scheme)?

Sincerely yours,

Carlos Herdeiro

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