THERMODYNAMICS OF SPACETIME: CORRECTIONS FROM THE QUANTUM REALM

PH.D. THESIS EVALUATION
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1 Introduction

The relationship between gravity and thermodynamics defines a central and transversal paradigm in all current theoretical developments relating to possible generalizations of Einstein's theory of general relativity, and in particular in the attempt to describe gravity at the quantum level. Marek Liška's Thesis work focuses on a precise aspect of this relationship, namely the possibility of expressing the equations of gravitational dynamics, in a semiclassical regime and beyond, in terms of constitutive equations of thermodynamic equilibrium on causal horizons. This possibility, first realized by Ted Jacobson in 1995, suggested that gravity itself, together with Einstein's equations, may "emerge" as the thermodynamic level of a fundamental quantum theory not necessarily of a gravitational nature. Beyond such induced gravity perspective, today the emerging interpretation of gravity at the semiclassical level is supported by most holographic approaches to quantum gravity, with concrete examples in low-dimensional gravity. In this sense, the Marek's Thesis is extremely timely. The original content of the research work done, providing both novel formal tools as well as a series of conceptual advances and insights on open issues, is of absolute relevance to the field.

2 Summary of the Thesis

The Thesis takes up Jacobson's argument on the derivation of Einstein's equations as thermodynamic equilibrium equations of state, starting from its most recent generalisations and first demonstrates in great details how the result of this derivation most naturally consists of the equations of motion of Weyl transverse gravity (WT), rather than the Einstein equations of General Relativity (GR). Although classically sharing the same space of solution, the two theories are essentially different in their local symmetries (subgroup of spacetime volume preserving diffeomorphisms together with Weyl transformations for WT in place of full diffeos for GR). These differences reflect in a series of notable new aspects in the derivation. A full, nonlinear theory of Weyl transverse gravity requires an extra non-dynamical background metric structure, similarly a Rosen-style bi-metric theory, with equations of motion which reduce to Einstein's equations only in the so called unimodular gauge. Secondly and most notably, the possibility to have non-conserved local energy-momentum tensor in WT, due to non-vanishing WTDiff-invariant divergence of the equations of motion. Third, the different status of the cosmological constant in the theory, which represents an additional, global degree of freedom and plays a central role in the departure from standard spacetime thermodynamics in GR.

Along this line, the Thesis explores the peculiar features of Weyl transverse gravity as a concrete alternative theory to General Relativity to shed new light on the interplay of gravity and thermodynamics.

The first step in this direction, in Chapter 1, consists in a complete extension of the covariant phase space formalism for local, fully Diff-invariant theories of gravity to WTDiff-invariant gravity. The extended formalism is applied to derive the first law of black hole mechanics in WT gravity, with the aim to generalise the Wald entropy formula, and to study thermodynamics of causal diamonds in the new setting. The standard results in GR are consistently generalised to the background dependent case of Weyl transverse gravity and further extended to the case of any local, WTDiff-invariant theory. Differently from GR, the WTDiff-invariant first law of thermodynamics generically includes a varying cosmological constant. Apart from such extra term, the physical content of spacetime thermodynamics is equivalent to the GR framework, which is consistently recovered in the unimodular gauge regime.

The second step, in Chapter 2, reverses the relation between gravity and thermodynamics and considers the derivation of WTDiff-invariant gravity dynamics from a local equilibrium constitutive equation for causal horizon thermodynamics. Technically, the main shift from the seminal work by Jacobson consists in a change of local spacetime thermodynamic setting, in analogy with the more recent results of the literature in the field: from the local causal horizon of local Rindler wedges, the analysis moves to the spherical local causal horizons, realised as the null boundaries either of local approximate light cones or of local causal diamonds.

In this line, the work focuses on the thermodynamic description of the causal diamond, within the WTDiff-invariant setup. Two independent derivations of the gravitational dynamics from equilibrium are considered, both building on the request that the total flux of entropy thought the boundary of the causal diamond must vanish. What varies in the two approaches is essentially the nature of the entropy flux contributions. In the first derivation (physical process approach) the assumption $\Delta S_{rev} + \Delta S_C = 0$ considers the effect of the change in diamond boundary area due to some flux of incoming classical matter (assuming $S = \eta A$) together with the entropy carried by the matter (assuming Clausius law $\delta Q = T \Delta S_C$). This derivation is fully classical and succeeds in recovering the full, non-linear equations of motion of WTDiff-invariant theory of gravity. Interestingly, as a specific feature of the WTDiff-theory setting, one finds in this case an irreversible contribution due to the diamond volume change. The latter is interpreted as the irreducible entropy production due to the expansion of the causal diamond. The second derivation relies instead on the notion of entanglement "equilibrium condition" $\delta S_W + \delta S_{vN} = 0$. Here δS_W is a perturbation of the renormalised entanglement entropy (vacuum) associated with the horizon of a causal diamond, equal to Wald entropy for generic gravity theories, while δS_{vN} is the (quantum) matter von Neumann entropy perturbation. Differently from the physical process approach, the entanglement equilibrium approach only allows to recover the linearised traceless equations of motion for a local, WTDiff-invariant theory of gravity.

The third Chapter of the Thesis moves a step beyond spacetime thermodynamics in the semiclassical setting. This is the most exploratory part of the Thesis work. Here the thermodynamic paradigm newly includes quantum corrections to entropy of the local causal horizons. As a result, the derived equations governing the gravitational dynamics contain genuine quantum corrections. The work focuses to the leading order quantum correction to the entropy, which is a dimensionless universal term logarithmic in the diamond horizon area. Remarkably, the local equilibrium condition allows to obtain nontrivial insights into gravitational dynamics even in presence of the logarithmic term present. This is a first important insight on the emergent nature of the gravitational interaction. In particular, in the entanglement equilibrium approach the corrections to linearised gravitational equations result in the equations of motion of WTDiff-invariant quadratic gravity coupled to conformal quantum matter fields. Concerning the nonlinear equations, again the quantum correction terms are quadratic in the spacetime curvature, they contain at most second derivatives of the metric. These results are nonetheless affected by a number of conceptual issues which leave room for future work. Very interestingly, in particular, the logarithmic quantum corrections applied to the thermodynamic equilibrium paradigm lead to the breaking of the WTDiff invariance. On the one hand, this result questions the main point of the Thesis, that is the naturalness of the WTDiff-invariant theory as the most general outcome of horizon thermodynamics equilibrium. On the other hand, more structurally, the point seems to indicate that non-local quantum corrections are incompatible with any form of local effective gravitational description. In a sense, they exceed the equilibrium setting hence the correspondence with a fully local gravitational description is partially lost.

Finally, in Chapter 5, the Thesis explores the implications of the logarithmic quantum corrections for low energy quantum gravitational dynamics. This setting, called "the quantum phenomenological gravitational dynamics" is considered, in particular, in relation to the case of homogeneous, isotropic cosmological models. Here, the main result consists in a generalisation of the classical Raychaudhuri equation for a cosmological spacetime. A perturbative solution of the quantum corrected Raychaudhuri is explicitly obtained and applied to the case of a universe filled with a perfect fluid leading to a modified Friedmann equation. The corrections found are proportional to the squared Planck scale and linear in the Hubble parameter, indicating a possible relevance in a early universe scenario. In this case, the notable feature has to do with the sign of the quantum correction. Indeed, positive quantum corrections to horizon entropy are consistent with a Big Bang singularity resolution, whereas negative corrections are not. Also, interestingly, the analytical solutions obtained are perturbatively equivalent to the effective dynamics of the loop quantum cosmology.

3 Detailed Evaluation

- **Originality and Significance**: The research work of the candidate is extremely original. Within a very exploratory framework like spacetime thermodynamics, it realises a systematic analysis of a whole new scenario: it explores and support the idea of a natural connection between thermodynamics of spacetime and Weyl transverse gravity and uses it to develop a new framework of spacetime thermodynamics for a generic class of WTDiff-invariant gravitational theories. In this sense, the candidate produces a new mathematical apparatus required to achieve a consistent generalisation of the covariant phase space formalism to the WTDiff-invariant setting and use it as the cornerstone to develop the classical black hole thermodynamics à la Wald for this class of theories. The results found are then applied to generalise the derivation of gravitational dynamics from causal diamond horizon equilibrium thermodynamics. Further the idea of considering the universal logarithmic correction to the area law to the spacetime thermodynamic setting is new and very significant. This a very broad and consistent analysis. The original character of the work naturally leads to several open questions, a remarkable resource for future work on the one hand, but also a shaky ground when used to justify part of the argument in the last part of the work. For instance, the open issue concerning the apparent incompatibility of a local theory of gravity with the non-local character of the logarithmic corrections to the horizon entropy make the proposed scenario of quantum phenomenological gravitational dynamics uqesionable though of secure interest. Overall, the produced work is highly original and significant. It fills the gap of a consistent spacetime thermodynamics in the WTDiff-invariant framework and leads to extremely interesting open questions, with potential effects in diverse theoretical frameworks, like loop quantum cosmology.
- **Literature Review**: The literature review is quite complete and very comprehensive. The candidate adequately cover the existing research, from the seminal papers to the

most up-to-date and relevant sources in the literature. However, little extra attention could have been devoted to a wider theoretical framework where the interplay of gravity and thermodynamics plays a central role currently.

- **Research Methodology**: The research method is very rigorous both formally and conceptually. The whole Thesis project appears as the consistent result of a very focused research path. The research questions are clearly stated, as well as the limits and open points of the analysis. The results are clearly presented and well-supported by a complete derivation. The whole argument is self-contained.
- **Discussion and Conclusions**:The discussion and conclusions are quite essential but very complete. All the results are stated clearly and the associated implications discussed in details. Possibly a more detailed discussion on the potential and limits of the spacetime thermodynamic paradigm, with respect to the results found, would have added further value to the work.
- **Presentation and Writing**: Overall, presentation, organization, and writing of the Thesis work are very well realised. The writing is generally clear and concise.
- **Conclusion**. Overall, I consider the Ph.D. Thesis work of the candidate excellent in content and form. The work is able to turn a very precise and specific technical question into a series of important and general results and a number of interesting and significant open questions.

4 Questions/Comments to the Candidate

1) Role of the cosmological constant. As an additional global degree of freedom of Weyl transverse gravity, what is ultimately the role of Λ from a thermodynamic analogy? In particular, it would be nice to further characterise the role of such dof in the irreversible entropy production and its relation with the causal diamond volume change.
2) Concerning local energy *non* conservation Sec.1.1.2/5. It seems to be a defining feature of the proposed equilibrium derivation the fact that the contracted Bianchi identities do not enforce a divergenceless energy- momentum tensor in the WTDiff-invariant gravity theory. On the other hand, for any WTDiff-invariant Lagrangian (with any matter fields present and including non-minimal coupling) the local conservation of energy holds. What is eventually the role of $\mathcal{J}$ in thermodynamic description. Related terms appears several time, e.g. in the generalised first law, however the contribution is always discarded within the set of assumptions considered.

3) Could one describe physical process derivation and entanglement equilibrium derivation not as alternatives, but as pertaining to different regimes, for instance a classical equilibrium and a quantum thermodynamic equilibrium condition? This would be consistent with finding as a result the full non-linear equations of motion classically and the linearised equation in the purely quantum case. Further in this light, the logarithmic correction would appear as a higher order quantum correction to an area scaling entropy of the horizon due to vacuum entanglement. Any comment?

4) speculative/emergence. In the different framework of gauge gravity duality, in particular, in the case of $\sim AdS_2/\sim SYK$, very recent results support a connection between JT gravity and random matrix models in the double-scaling limit. In this case gravity is conjectured be dual to an ensemble average of quantum systems. Hence again an effective theory description, 2d gravity, associated to the equilibrium thermodynamic regime of some non-perturbative quantum theory. While the duality between theories leaves at the level of thermostatic equilibrium, thermodynamic variation of equilibrium may have to with gravitational dynamics as in the case you describe. Also, similar scenarios may appear regarding low energy quantum gravitational effects. Do you think the two frameworks may have something in common?