Review of Master Thesis

Gao Tongtong: Actuarial Reserving Methods for Non-Life Insurance

Topic:

The thesis compares the classical and stochastic methods (particularly, GLMM models) for calculating the claims reserves in non-life insurance. The comparison is performed by numerical study with real insurance data.

Survey:

The first chapter discusses the basic notation and the existing loss reserving estimation methods including the description of CL and B-F method. The second chapter deals with generalized linear models GLM and generalized linear mixed models GLMM. The third chapter establishes these models for loss reserving. In the fourth chapter the presented methods are applied to real-life data and comparison of the classical and stochastic methods is commented.

Contributions:

The author finds that the stochastic models are more effective in capturing random fluctuations in loss reserving data than the common methods based on classical principles.

Mathematical level:

The mathematical level of thesis is sufficient. However, incorrect mathematical presentations and errors corrupt the text from this point of view.

Sources and formal comments:

Bibliography has an acceptable form. However, some sections are without reference (in particular, the key Chapter 3 of thesis is without any reference as if it was an own contribution of the author). According to my opinion, the thesis is closely related to the reference Antonio and Beirlant (2007) which is referred only once in a negligible context. The graphical form of the thesis is standard. The level of English language is logically above standard (however, various grammatical errors can be found, e.g. 8^{6}: ... which significantly advancing the applications ..., 9^{1}: ... function is typically takes the form ..., 10_{1}: complement*o*ry, 28^{1}: criteria*s*). The translation of the abstract and keywords to Czech language contains some non-relevant terms probably due to the translator (generalizované lineární modely, rezerva na škody, řetězový žebřík). There is a lot of misprints in the text, both formal ones (even in the chapter title, see 8^{1}: Generalized ...) and objective ones (even though some sections follow closely the reference literature, e.g. Section 1.2, see below).

The contributions of the thesis are not summarized in the Conclusion (they are only briefly sketched in the Abstract: "... we find that stochastic methods are more effective in capturing random fluctuations in the data").

Comments and questions:

- 3^{8}: Which or who is the *self-insurer* and which is the role of such a subject in the context of IBNR reserves?
- 3^{18}: Symbols should be written in italics, e.g., I, and others.
- 3: In Table 1.1. the accident years and development years should start from 0 (not from 1).
- 4^{12}: In (1.4) it should be $k = i + 1$ (instead of $k = j + 1$).
- 4: In (1.6) it should be $\mathrm{E}(C_{i,j} \mid C_{i,j-1})$.
- 4_2: ... that the mean value of the ultimate claims amount $C_{i,J}$ fulfils ...
- 5^{1}: (1.8) holds only for $I = J$.
- 5^{10}: It should be f_j in (1.10) instead of f_i.
- 5^{11}: Why f_j in (1.10) is the weighted average of the individual development factors?
- 7: Which is the estimator of parameter μ_i in (1.24) concerning the B-F method (it is commented only briefly in 7_3)?
- 9^{9}: The grammatical structure is incorrect (it should be: "Since the expectation can be expressed as ...").
- 9_6: It should be $a(\varphi) = 1$
- $11^{7\text{-}8}$: Which is *canonical parameter* (this term has not been defined in the previous text)?
- 11: Binomial distribution in Table 2.2: Which is relation between parameters μ and p?
- 12: Starting from p. 12 the symbols ω and w are interchanged (probably the symbol ω should be everywhere).
- 13^{16}: Which is r (in relation to p)?
- $14^{3\text{-}4}$: Which is meaning of the conclusion that "there is evidence of a significant effect for the coefficient of interest" in the context of H_0: $\boldsymbol{C\beta} = \boldsymbol{r}$?
- 14-19: The presentation of GLMM models in Section 2.2 is difficult to understand. For instance, for some vectors one does not apply bold symbols (e.g. v_i, x_{ij}, z_{ij}, α) without declaring them as vectors, there are missing indices (e.g. in 15_{12} it should be ... + $\boldsymbol{z}_{ij}'\boldsymbol{v}_i$) and missing interrelations (e.g. between $\boldsymbol{v}$ in 15^{10} and $\boldsymbol{v}_i$ in 15^{1}) and others. One has to consult the source literature (e.g. Antonio and Beirlant (2007) to catch the main principles of GLMM including their estimation.
- 20-: Starting from p. 20 it should be $j \leq I - i + 1$ for upper triangles and $j > I - i + 1$ for upper triangles.
- 21_9: Why the number of parameters is $p = 2I + 1$?
- 25_{13}: Why the *random intercept model* is not constructed in the opposite way considering only the column specific effects v_j?
- 43, Table 4.21: The legend for some tables is not fully specific. E.g. Table 4.21 is constructed using negative binomial distribution GLM model for claim counts and the inverse Gaussian distribution GLM model for average claim amounts.
- $51_{7\text{-}1}$: The only explicit comparison of CL and GLM or GLMM methods is done according to the estimated IBNR reserve. It is sufficient to declare that "... we find that stochastic methods are more effective in capturing random fluctuations in the data"?
- It is pity that among rich offer of applications of GLMM in actuarial reserving only very

simple structure (3.12) is chosen across the whole thesis. Has the author tested other more complex structures?

- When one applies Mack model in the context of CL method then one can construct confidence intervals for IBNR reserves (see also R software). The approach in thesis based on GLM a GLMM consisting in multiplying estimated claim counts and average claim amounts eliminate the possibility to estimate explicitly the confidence reserve intervals?
- No information on the application R software applied in the thesis is provided (also the code in Appendix is given without any comments or explanations).
- Let us imagine that the author is a reserving actuary in a commercial insurance company. Which strategy covering the methods described in the thesis should be recommended to estimate the IBNR reserves (e.g. in the third party motor liability insurance)?

Conclusion:

The author has proved the capability to deal with non-trivial mathematics in the context of mathematics of non-life insurance and to apply the corresponding theory to real data. Therefore I recommend the text to be accepted as the Master Thesis.

4. 8. 2024 Prof. RNDr. Tomáš Cipra, DrSc.