This thesis researches Gabidulin codes over Galois rings and their application in cryptography. The first objective is to understandably explain the construction of Galois rings and their essential properties to the reader. This step is necessary to provide code theory over these rings and understand their differences from the standard one over finite fields. The cardinal rank and its induced metric are studied and utilised in linear codes. A significant part of the thesis is presenting an efficient decoding algorithm for the given error-correcting codes. The concluding part proposes a public key cryptosystem whose decryption is founded on the decoding algorithm.