Opponent report for the bachelor thesis

Volodymyr Sahan: Spectrum of the density operator

The thesis deals with rank-one and nuclear operators, their definitions and main properties. Then density matrix operators are defined and it is shown that they are nuclear. Finally, spectra of such operators are investigated in two special cases.

The main shortcoming of this work is that the spectrum of the density operator, which should be the main topic of the thesis, is only studied in the last two examples of the thesis. After reading the text I had the feeling that it is full of preliminaries and when the main object (the density operator) appears, then there are only two lemmas proving that the operator is nuclear and then two examples. So, it was not clear to me what is the main goal of the thesis. Moreover, it is not explained why the density operators should be interesting, it is only mentioned that they are used in quantum physics. In fact, the lack of explanation stretches throughout the thesis, there is almost no text except definitions, theorems and proofs.

On the other hand, the thesis has 18 pages which is just right for a bachelor thesis and it is well structured. It contains many propositions with proofs and the propositions and definitions are chosen and ordered correctly, so that everything needed is there. The proofs are correct and well written with some minor errors that I mention below.

Together, I think that the text meets the conditions set for a bachelor thesis and the assignment of the thesis was fulfilled, although it could have been fulfilled significantly better.

Some comments:

1. References should be made with more care. In Definition 1.0.4 a book is cited without mentioning a page. Theorem 3.0.1 and Lemma 3.0.2 are mentioned without proof and without reference.
2. Theorem 3.0.1 should be stated as equivalence not just one implication.
3. Lemma 3.0.5. In the last paragraph of the proof it is unclear how is T related to A, why $T^{1/2}$ exists and why the equalities hold.
4. The numbers s_n and orthonormal bases $\{\phi_n\}$, $\{\phi_n\}$ from Theorem 3.0.1

are not unique and they are different for different operators. In several places it is not clear whether the author is aware of this, it should be mentioned in some proofs explicitly, e.g. statement of Lemma 3.0.13 and proofs of Theorem 3.0.14, Lemma 3.0.15, Theorem 3.0.16.

5. Lemma 5.0.7. The argument z is missing in the integral.

Questions:

1. Can you explain the proof of Lemma 3.0.5?
2. Can you explain the equality $\left\|\sum_{n=k+1}^{N_A} s_n \psi_n \otimes \phi_n\right\|_1 = \sum_{n=k+1}^{N_A} s_n$ from the proof of Theorem 3.0.6? In particular, is it clear what are s_n, ϕ_n and ψ_n for the operators $A_k - A$?

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