Review by the supervisor of the bachelor thesis

Thesis title: Proving combinatorial identities via formal power series
Thesis author: Tomáš Sklenář

Summary of the thesis content

The thesis deals with formal power series and their applications in combinatorics. The first part of the thesis provides a thorough introduction of formal power series, establishing basic properties such as metric completeness. This is followed by an introduction useful operators, including composition, derivative, exponentiation and logarithm; it is shown that the operators defined as such satisfy the usual properties we expect. The second part of the thesis establishes well-known theorems in combinatorics, using the toolbox of formal power series. These results include Gauss' binomial theorem (Theorem 3.5), Jacobi's triple product formula (Theorem 3.8), the Rogers-Ramanujan identities (Theorem 3.12), and finally the Lagrange-Jacobi four square theorem (Theorem 3.1), which is the main result of the thesis.

Overall evaluation of the thesis

Thesis topic. The difficulty of the thesis is appropriate for a bachelor's thesis. Formal power series are an important tool in combinatorics; through the use of generating functions, combinatorial information can be encoded as a formal power series. The thesis demonstrates that the author has sufficient ability to understand and perform combinatorial arguments at the expected level of expertise.

Author's contribution. The author's contribution includes the formal definitions of the basic notions concerning formal power series, and providing proofs for the basic properties of formal power series. The author also vastly expanded and rewritten the proofs of the main results as well as their preceding lemmata; most of the proofs in the source [1] is either brief or entirely omitted. The author also added explanations for many of the combinatorial tricks used in the source [1], and provided examples relating formal power series to the more familiar Taylor series of common functions, such as the trigonometric functions. The author's contribution is clearly declared in the introduction of the thesis.

Overall, the author did a large amount of work on the thesis (on his own and during regular meetings with the consultant of the thesis Siu Hang Man who also prepared a draft of this report).

Mathematical level. The mathematical content is correct, and the definitions and the proofs are presented in a rigorous way.

Work with sources. The sources are correctly cited, and the thesis does not contain verbatim copied passages.

Formal editing. The presentation is largely clear, and is easily understandable. There are a small amount of typos and colloquialisms, but this does not interfere with the presentation of the arguments and mathematical rigour.

Comments and questions

1. p.8, line -18: It is somewhat unclear what this paragraph does. And what is α_m?

2. Chapter 2: There seems to be some confusion between α and a (resp. β and b) in the proofs.

3. Remark 3.6: The remark is perhaps slightly misleading, suggesting that the formulae in Theorem 3.5 hold when X is replaced by β. In fact, the Gauss product also depends on X, and hence is affected by the substitution.

Conclusion

I consider the thesis to be very good and I recommend that it be accepted as a Bachelor's thesis. I recommend the grade 1.

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