Errata

Na popud oponenta zde uvádím errata opravující některé nalezené chyby a alespoň trochu vylepšující přehlednost dlouhých rovnic. Je také doplněno označení rovnic pro perturbace s konvencí takovou, že $(\delta G_{RR} - \kappa \delta T_{RR})_{a,b,c}$ značí rovnici perturbace složky RR, při níž uvažujeme perturbace funkcí a, b, c. V případě Fisherovy metriky jsou pak a, b, c nahrazeny $Fisher$.

V (16) byla převzata chyba z výchozího textu, upraveny jsou tedy i poznámky:

$$p \equiv \left(1 + 2\kappa A^2\right)^{\frac{1}{2}} \geqslant 1. \tag{16}$$

Poznámka. : Platnost řešení se povedla ověřit v Mathematice pro $2A^2\kappa = p^2 - 1$, což je ekvivalentní (16). Pokud by bylo znaménko v (12) minus, dostali bychom $2A^2\kappa = 1 - p^2$, což je ve sporu (16), odkud by plynulo $p < 1$. Takto bylo objeveno použití jiné konvence úžení Riemannova tensoru.

Poznámka. : V textu [1] je chybně uvedeno $p \equiv \left(1 + \frac{4\kappa A^2}{r_0^2}\right)^{\frac{1}{2}} \geqslant 1$, což by spolu s $2A^2\kappa = p^2 - 1$ nutně vedlo k $r_0^2 = 2$.

Zde uvádím dodatečně rovnice (84)-(86) v obecné podobě, bez dosazení pozaďového řešení:

$$\frac{1 - rF'r' - 2F(r')^2 + Fr(\kappa r(\varphi')^2 + 4r'')}{r^2}\delta F - \frac{Fr'}{r}\delta F' + F\frac{2F((r')^2 + rr'') + rFr' - 2}{r^3}\delta r - \\ - \frac{FrF' + 2F^2r'}{r^2}\delta r' - 2\frac{F^2}{r}\delta r'' - \kappa F^2\varphi'\delta\varphi' = 0 = (\delta G_{tt} - \kappa\delta T_{tt})_{F,r,\varphi} \tag{D1}$$

$$\frac{1 - rF'r'}{F^2r^2}\delta F + \frac{r'}{Fr}\delta F' - \frac{2F(r')^2 + rF'r - 2}{Fr^3} + \frac{rF' + 2Fr'}{Fr^2}\delta r' - \\ - \kappa\varphi'\delta\varphi' = 0 = (\delta G_{RR} - \kappa\delta T_{RR})_{F,r,\varphi} \tag{D2}$$

$$r(\kappa r(\varphi')^2 + 2r'')\delta F + 2rr'\delta F' + r^2\delta F'' + 2(F'r' + \kappa Fr(\varphi')^2 + rF'' + Fr'')\delta r + \\ + 2Fr\delta r'' + 2\kappa Fr^2\varphi'\delta\varphi' = 0 = (\delta G_{\theta\theta} - \kappa\delta T_{\theta\theta})_{F,r,\varphi} \tag{D3}$$

V rovnicích (84)-(87) byly nalezeny závažné věcné chyby a byly tak kompletně předělány a byla ověřena jejich platnost pro triviální perturbace. (87) je nyní uvedena v nedosazeném tvaru:

$$
\begin{aligned}
&2A\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\left[(p+1)R_-^2+2(p^2-1)R_+R_--(p-1)R_+^2\right]\delta F+\\
&+ApR_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\left[(p+1)R_-+(p-1)R_+\right]\delta F'-\frac{4A(R_+-R_-)\left[(p+1)R_-+(p-1)R_+\right]}{\sqrt{R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}}}\delta r+\\
&+4Ap^2R_+R_-\frac{R_++R_-}{\sqrt{R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}}}\delta r'+\frac{4Ap^2R_+^2R_-^2}{\sqrt{R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}}}\delta r''-\\
&-p(p^2-1)R_+R_-(R_++R_-)\delta\varphi'=0=(\delta G_{tt}-\kappa\delta T_{tt})_{F,r,\varphi}
\end{aligned}
\tag{84}
$$

$$
\begin{aligned}
&2A\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\left[(p+1)R_-^2+2(p^2-1)R_+R_--(p-1)R_+^2\right]\delta F+\\
&+AR_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\left[(p+1)R_-+(p-1)R_+\right]\delta F'-\\
&-\frac{4Ap(R_--R_+)\left(\frac{R_-}{R_+}\right)^{\frac{1}{p}}\sqrt{R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}}\left[(p+1)R_--(p-1)R_+\right]}{R_+R_-}\delta r+\\
&+4Ap^2(R_++R_-)\left(\frac{R_-}{R_+}\right)^{\frac{1}{p}}\sqrt{R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}}\delta r'+\\
&+p(p^2-1)(R_--R_+)R_+R_-\delta\varphi'=0=(\delta G_{RR}-\kappa\delta T_{RR})_{F,r,\varphi}
\end{aligned}
\tag{85}
$$

$$
\begin{aligned}
&2Ap\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\left[(p+1)R_-+(p-1)R_+\right]\delta F'+Ap^2R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\delta F''+\\
&+\frac{4A(R_--R_+)\left[(p+1)^2R_--(p-1)^2R_+\right]}{R_+R_-\sqrt{R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}}}\delta r+\frac{8Ap(R_+-R_-)}{\sqrt{R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}}}\delta r'+\frac{4Ap^2R_+R_-}{\sqrt{R_+R_-\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}}}\delta r''+\\
&+2p(p^2-1)(R_+-R_-)\delta\varphi'=0=(\delta G_{\theta\theta}-\kappa\delta T_{\theta\theta})_{F,r,\varphi}
\end{aligned}
\tag{86}
$$

Poznámka. : v (86) se člen s δF vynuluje.

$$\begin{aligned} &2(rF'r' + 2F(r')^2 + Frr'' - 2)\delta r - \\ &- r\left[(2(r')^2 + \kappa r^2(\varphi')^2 + 4rr'')\delta F + 2(rr'\delta F' + rF'\delta r' + 2Fr'\delta r + Fr\delta r'')\right] = 0 \end{aligned} \tag{87}$$

Následující rovnice jsou graficky upraveny pro lepší přehlednost:

$$\begin{aligned} &2\left\{-4A^2[(1+p)R_- - R_+][R_- + (p-1)R_+] + p^2(p^2-1)R_+^2R_-^2\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}(\Lambda + V)\right\}\delta F + \\ &+ 2A^2R_+R_-(R_- - R_+)\delta F' + A^2p^2R_+^2R_-^2\delta F'' + \\ &4Ap(p^2-1)R_+R_-(R_- - R_+)\left(\frac{R_-}{R_+}\right)^{\frac{1}{p}}\delta\varphi' + 2p^2(p^2-1)R_+^2R_-^2\delta V = 0 = (\delta G_{tt} - \kappa\delta T_{tt})_{F,\varphi} \end{aligned} \tag{45}$$

$$\begin{aligned} &2\left\{-4A^2[(1+p)R_- - R_+][R_- + (p-1)R_+] + p^2(p^2-1)R_+^2R_-^2\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}(\Lambda + V)\right\}\delta F + \\ &+ 2A^2R_+R_-[(3+2p)R_-(3-2p)R_+]\delta F' + A^2p^2R_+^2R_-^2\delta F'' + \\ &+ 4Ap(p^2-1)R_+R_-(R_- - R_+)\left(\frac{R_-}{R_+}\right)^{\frac{1}{p}}\delta\varphi' - 2p^2(p^2-1)R_+^2R_-^2\delta V = 0 = (\delta G_{RR} - \kappa\delta T_{RR})_{F,\varphi} \end{aligned} \tag{46}$$

$$\begin{aligned} &\left\{4A^2[(1+p)R_- - R_+][R_- + (p-1)R_+] + p^2R_+2R_-\right\}\delta F + \\ &+ 2A^2R_+R_-[(1+p)R_- - (1-p)R_+]\delta F' + A^2p^2R_+^2R_-^2\delta F'' - \\ &- 4Ap(p^2-1)R_+R_-(R_- - R_+)\left(\frac{R_-}{R_+}\right)^{\frac{1}{p}}\delta\varphi' - 2p^2(p^2-1)R_+^2R_-^2\delta V = 0 = (\delta G_{\theta\theta} - \kappa\delta T_{\theta\theta})_{F,\varphi} \end{aligned} \tag{47}$$

$$ApR_+R_-[(p+1)R_- + (p-1)R_+]\delta F' = 0, \tag{48}$$

$$\begin{aligned} &2A(R_- - R_+)[(p+1)R_- + (p-1)R_+]\delta F - \\ &- pR_+R_-\{[(p+2)R_- + (p-2)R_+]\delta F' + pR_+R_-\delta F''\} = 0 \end{aligned} \tag{50}$$

$$\begin{aligned} &2A(R_- - R_+)[(p+1)R_- + (p-1)R_+]\delta F - \\ &- pR_+R_-\{[(3p+4)R_- + (3p-4)R_+]\delta F' + pR_+R_-\delta F''\} = 0 \end{aligned} \tag{51}$$

$$Ap\{16r_0R\delta F - R_+R_-[2(r_0 - 2R)\delta F' - R_+R_-\delta F'']\} = 0. \tag{53}$$

$$\begin{aligned} &2\Big\{-4A^2[(1+p)R_-^2 + (2p^2 - p - 2)R_+R_- + (1-p^2)R_-^2] + \\ &+p^2(p^2-1)R_+^2R_-^2\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}(\Lambda + V)\Big\}\delta F_{arg} + 2A^2p(2p-1)R_+R_-(R_- - R_+)\delta F'_{arg} + \\ &+ A^2p^2R_-^2R_+^2\delta F''_{arg} + 4Ap^2(p^2-1)R_-^2(R_- - R_+)\delta\varphi' + \\ &+ 2p^3(p^2-1)R_-^3R_+\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\delta V = 0 = (\delta G_{tt} - \kappa\delta T_{tt})_{F_{arg},\varphi} \end{aligned} \tag{56}$$

$$\begin{aligned} &2\Big\{-4A^2[(1+p)R_- - R_+][(p-2)R_- - 2(p-1)R_+] - \\ &p^2(p^2-1)R_+^2R_-^2\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}(\Lambda + V)\Big\}\delta F_{arg} - 2A^2pR_+R_-[(4p+1)R_- + R_+]\delta F'_{arg} - \\ &- A^2p^2R_-^2R_+^2\delta F''_{arg} + 4Ap^2(p^2-1)R_-^2(R_+ - R_-)\delta\varphi' + \\ &+ 2p^3(p^2-1)R_-^3R_+\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\delta V = 0 = (\delta G_{RR} - \kappa\delta T_{RR})_{F_{arg},\varphi} \end{aligned} \tag{57}$$

$$\begin{aligned} &-4A^2[(p^2+1)R_-^2 + 2(p-1)R_+R_- + (p^2-1)R_+^2]\delta F_{arg} - \\ &-2A^2pR_+R_-[(3p-1)R_- - (p-1)R_+]\delta F'_{arg} - \\ &- A^2p^2R_+^2R_-^2\delta F''_{arg} - 4Ap^2(p^2-1)R_-^2(R_+ - R_-)\delta\varphi' + \\ &+ 2p^3(p^2-1)R_-^3R_+\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\delta V = 0 = (\delta G_{\theta\theta} - \kappa\delta T_{\theta\theta})_{F_{arg},\varphi} \end{aligned} \tag{58}$$

$$\begin{aligned} &A\Big\{2(R_- - R_+)[(p^2 - 2r - 1)R_- + R_+ + (2-3p)pR_+]\delta F_{arg} + \\ &+ pR_+R_-\{[(5p-2)R_- + (2-3r)R_+]\delta F'_{arg} + pR_+R_-\delta F''_{arg}\}\Big\} = 0 \end{aligned} \tag{61}$$

$$A\Big\{2(R_- - R_+)[(-3+p(3p-2))R_- - (3+p)(p-1)R_+]\delta F_{arg}+ \\ + p^2 R_+ R_-[(7R_- - R_+)\delta F'_{arg} + R_+ R_- \delta F''_{arg}]\Big\} = 0 \tag{62}$$

$$Ap\Big\{8r_0[2rR + r_0(p-1)(2p+1)]\delta F_{arg} - \\ - R_+ R_+[(4R - 8pr_0 + 6r_0)\delta F'_{arg} + R_+ R_- \delta F - arg'']\Big\} = 0, \tag{64}$$

$$2\left[2A^2(R_- - R_+)^2 + p^2(p^2-1)R_+^2 R_-^2 \left(\frac{R_+}{R_-}\right)^{\frac{1}{p}} (\Lambda + V)\right]\delta F_1 + \\ + A^2 p R_+ R_-[(5+2p)R_- + (2p-5)R_+]\delta F'_1 + A^2 p^2 R_+^2 R_-^2 \delta F''_1 - \\ - 4A^2[(3+2p)R_-^2 + 2(p^2-3)R_+ R_- + (3-2p)R_+^2]\delta F_2 + \\ + A^2 p R_+ R_-[(3+2p)R_- + (2p-3)R_+]\delta F'_2 + 4Ap(p^2-1)R_+ R_- \left(\frac{R_-}{R_+}\right)^{\frac{1}{p}} \delta\varphi' + \\ + 2p^2(p^2-1)R_+^2 R_-^2 \delta V = 0 = (\delta G_{tt} - \kappa \delta T_{tt})_{F_1,F_2,\varphi} \tag{67}$$

$$4A^2 p(R_+^2 - R_-^2)\delta F_1 + A^2 p R_+ R_-[(2p+5)R_- + (2p-5)R_+]\delta F'_1 + A^2 p^2 R_+^2 R_-^2 \delta F''_1 + \\ + 2\Bigg(-2A^2[(p+2)R_-^2 + 2(p^2-2)R_+ R_+ - (p-2)R_+^2] + \\ + p^2(p^2-1)R_+^2 R_-^2 \left(\frac{R_+}{R_-}\right)^{\frac{1}{p}} (\Lambda + V)\Bigg)\delta F_2 + A^2 p R_+ R_-[(2p+1)R_- + (2p-1)R_+]\delta F'_2 + \\ + 4Ap(p^2-1)R_+ R_-(R_- - R_+)\left(\frac{R_-}{R_+}\right)^{\frac{1}{p}} \delta\varphi' - \\ - 2p^2(p^2-1)R_+^2 R_-^2 \delta V = 0 = (\delta G_{RR} - \kappa \delta T_{RR})_{F_1,F_2,\varphi} \tag{68}$$

$$2A^2(R_- - R_+)[(p+2)R_- + (p-2)R_+]\delta F_1 + A^2 p R_+ R_-[(2p+3)r_- + (2p-3)R_+]\delta F'_1 + \\ + A^2 p^2 R_+^2 R_-^2 \delta F''_1 + 2A^2 p R_+ R_- \left(4 + \frac{R_-}{R_+} - \frac{R_+}{R_-}\right)\delta F_2 - A^2 p R_+ R_-(R_- - R_+)\delta F''_2 - \\ - 4Ap(p^2-1)R_+ R_-(R_- - R_+)\left(\frac{R_-}{R_+}\right)^{\frac{1}{p}} \delta\varphi' - 2p^2(p^2-1)R_+^2 R_-^2 \delta V = 0 = (\delta G_{\theta\theta} - \kappa \delta T_{\theta\theta})_{F_1,F_2,\varphi} \tag{69}$$

$$
\begin{aligned}
&2(R_- - R_+)^2[(p+1)R_- + (p-1)R_+]\delta F_2 - \\
&- pR_+R_-[2(p+1)(2p+3)R_-^2 + 4(4p^2-3)R_+R_- + 2(p-1)(2p-3)R_+^2]\delta F_2' - \\
&- p^2R_+^2R_-^2[(6p+5)R_- + (6p-5)R_-]\delta F_2'' - p^3R_+^3R_-^3\delta F_2''' = 0.
\end{aligned} \tag{71}
$$

$$
\begin{aligned}
&2\left[A^2(2p^2+p-1)(R_- - R_+)^2 + p^2(p^2-1)R_+^2R_-^2\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}(\Lambda+V)\right]\delta F_{arg1} + \\
&+ A^2pR_+R_-[(6p+1)R_- - (2p+1)R_+]\delta F'_{arg1} + A^2p^2R_+^2R_-^2\delta F''_{arg1} + \\
&+ 2A^2\{-(3+2p)(p+1)R_-^2 + 2[p(3-2p)+3]R_+R_- + (p+1)(2p-3)R_-^2\}\delta F_{arg2} - \\
&- A^2pR_+R_-[(2p+3)R_- + (2p-3)R_+]\delta F'_{arg2} + \\
&+ 4A^2pR_-^2[(p^2-1)(R_- - R_+)]\delta\varphi' + 2p^3(p^2-1)R_-^3R_+\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\delta V = 0 = (\delta G_{tt} - \kappa\delta T_{tt})_{F_{arg1},F_{arg2},\varphi}
\end{aligned} \tag{74}
$$

$$
\begin{aligned}
&-2A^2(R_- - R_+)\{(p+1)R_- - [p(2p+3)-3]R_+\}\delta F_{arg1} - \\
&- A^2pR_+R_-[(6p+1)R_- - (2p+1)R_+]\delta F'_{arg1} - A^2p^2R_+^2R_-^2\delta F''_{arg1} + \\
&2\Bigg\{A^2[p(2p-5)+5]R_+^2 + 2A^2(2p^2+p-5)R_+R_- + \\
&+ (p+1)R_-^2\left[A^2(5-2p) - p^2(p-1)R_+^2\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}(\Lambda+V)\right]\Bigg\}\delta F_{arg2} - \\
&- A^2pR_+R_-[(2p+1)R_- + (2p-1)R_+]\delta F'_{arg2} - 4Ap^2(p^2-1)R_-^2(R_- - R_+)\delta\varphi' + \\
&+ 2p^3(p^2-1)R_-^3R_+\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\delta V = 0 = (\delta G_{RR} - \kappa\delta T_{RR})_{F_{arg1},F_{arg2},\varphi}
\end{aligned} \tag{75}
$$

$$
\begin{aligned}
&-2A^2(R_- - R_+)[(2p^2+1)R_- - (2p(p-1)+1)R_+]\delta F_{arg1} - \\
&- A^2pR_+R_-[(6p-1)R_- + (2p-1)R_+]\delta F'_{arg1} - A^2p^2R_+^2R_-^2\delta F''_{arg1} - \\
&- 2A^2[R_-^2 + 2(2p^2+p-1)R_+R_- - (2p-1)R_+^2]\delta F_{arg2} + A^2pR_+R_-(R_- - R_+)\delta F'_{arg2} + \\
&+ 4Ap^2R_-^2(p^2-1)(R_- - R_+)\delta\varphi' + 2p^3(p^2-1)R_-R_+\left(\frac{R_+}{R_-}\right)^{\frac{1}{p}}\delta V = 0 = (\delta G_{\theta\theta} - \kappa\delta T_{\theta\theta})_{F_{arg1},F_{arg2},\varphi}
\end{aligned} \tag{76}
$$

$$4(R_- - R_+)^2\{[p(4p^2-2p-3)-1]R_- - (4p^3-2p^2-p-1)R_+\}\delta F_{arg2}+$$
$$+\,pR_+R_-\{(28p^2-6p-2)R_-^2-[8p(p-1)-4]R_+R_- + 2(2p+1)(p-1)R_+^2\}\delta F'_{arg2}+$$
$$+\,p^2R_+^2R_-^2[(12p-1)R_- + R_+]\delta F''_{arg2} + p^3R_+^3R_-^3\delta F'''_{arg2} = 0. \tag{78}$$

Grafická úprava rovnic Fisherovy metriky:

$$\left\{-16+8\exp(\lambda)[2+\kappa r^2(2\Lambda+\chi^2U^2)] + r\lambda'(16+3r\nu')-\right.$$
$$\left.-r[8\kappa r(U')^2+\nu'(12+5r\nu')+6r\nu'']\right\}\delta\nu+$$
$$+\,2r(4-r\lambda')\delta\nu' + 4r^2\delta\nu'' + [8-3r\lambda'(4+r\nu') + r^2(8\kappa(U')^2+5(\nu')^2+6\nu'')]\delta\lambda+$$
$$+\,2r(4-r\nu')\delta\lambda' + 16\exp(\lambda)\kappa\chi^2r^2U\delta U = 0 = (\delta G_{tt}-\kappa\delta T_{tt})_{Fisher} \tag{88}$$

$$r[4\nu'+r(-3\lambda'\nu'+5(\nu')^2+6\nu'')]\delta\nu + 2r(r\lambda'-4)\delta\nu' - 4r^2\delta\nu''+$$
$$+\left\{-24+8\exp(\lambda)[2+\kappa r^2(2\lambda+\chi^2U^2)] + 3r\lambda'(4+r\nu') - r[\nu'(8+5r\nu')+6r\nu'']\right\}\delta\lambda+$$
$$+\,2r\delta\lambda' + 16\kappa r^2(\exp(\lambda)\chi^2U\delta U + U'\delta U') = 0 = (\delta G_{RR}-\kappa\delta T_{RR})_{Fisher} \tag{89}$$

$$\begin{aligned}&[8r\nu'+r^2\nu'(\nu'-\lambda')+2r^2\nu'']\delta\nu + 2r(r\lambda'-2r\nu'-4)\delta\nu' - 4r^2\delta\nu''+\\ &\left\{8+r[8\kappa r(U')^2+4\nu'+3r(\nu')^2-3\lambda'(4+r\nu')+6r\nu'']\right\}\delta\lambda + 2r^2\nu'\delta\lambda'+\\ &+16\exp(\lambda)\kappa\chi^2r^2U\delta U = 0 = (\delta G_{\theta\theta}-\kappa\delta T_{\theta\theta})_{Fisher}\end{aligned} \tag{90}$$

Bibliografie

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