Reviewer's opinion of the master thesis

Daria Dunina

Wreath product of operadic categories

The thesis under review consists of three chapters

1: Preliminaries
2: Wreath product of operadic categories
3: Application to Boardman-Vogt tensor product of operads

I do not think that the notation of the first chapter as Preliminaries is the best one. It makes impression that we can find there notations and maybe some notions, which is not the case. This chapter is no doubt also very important part of the thesis. We find here definition of operadic category. I am not going to describe this category here, but I think that it is good to mention that for morphisms in this category it is possible to introduce fibers. The operadic category was introduce already in 2014 by M.Batanin and M.Markl (Operadic categories and Duoidal Deligne's conjecture, Advances in Math., 2014), but in this original paper the authors were not formal enough and there appeared something which was not completely correct (no mistake, no gap). Daria Dunina improved what was necessary and put everything in order. You can recognise her changes everywhere where you find the morphism *inc*. Let us mention that slightly different approach to this formal problem we can find in the paper R.Garner, J.Kock, and M.Weber: Operadic categories and décalage.

The second chapter introduces the wreath product of two operadic categories. The wreath product was introduced for several structures in algebra and category theory and it is not surprising that there was effort to introduce it also for operadic categories. This notion is due to M.Batanin and M.Markl. In the thesis it was first necessary to prove that the wreath product of two operadic category is again an operadic category. D.Dunina carefully verifies the five axioms with the necessary precision using her morphism *inc*. She proves that the wreath product is associative and presents an example showing that it need not be commutative. Finally she shows that the wreath product of the category of k-trees and the category of l-trees (category of n-trees was introduced in Chap.1) is isomorphic with the category of $(k + l)$-trees.

The last third chapter is probably most interesting, but on the other hand it is not easy to describe it. We take an operadic cateory O and an O-operad $\mathcal{P}$ in *Set* and perform the operadic Grothendieck construction. It can be easily recognized in texts because of the notation $\int_O \mathcal{P}$, not very common in category theory. But D.Dunina changes the notation and uses simpler notation $\mathbb{I}(\mathcal{P})$. I apprecite that the definition of the Grothendieck construction presented here is relatively simple. The main result in this chapter is Prop.39 showing an epimorphism from the Boardman-Vogt tensor product of single colored operads $\mathcal{X} \otimes_{BV} \mathcal{Y}$ to the wreath product of $\mathbb{I}(\mathcal{X})$ with $\mathbb{I}(\mathcal{Y})$.

Generally, my opinion of the thesis is very high. I am aware of the fact that D.Dunina had possibility to discuss with M.Batanin and M.Markl and maybe some other people as well. But nevetheless she had to learn and understand a lot of material, and this material was not at all easy. I would say that some parts of category theory are maybe easier, but other are real jungle. It is also necessary to mention here that the research presented in the thesis is not some classical staff but very contemporary research of high quality. At the end of the thesis D.Dunina presents questions and topics for further research. I shall present also one silly question. Is there of some interest to study functors from general category to operadic category?

Olomouc, 3rd Septtember 2024   doc.RNDr. Jiří Vanžura, CSc

P.S. I would like to add here a short historical remark. I think that it was around 1963 when doc. Karel Drbohlav at MFF UK delivered first lectures on category theory. In the audience there were only members of the of the faculty (I remember Věra Trnková and Aleš Pultr), I was the only student. During a lecture doc. Drbohlav several times addressed me and asked "Do you understand?" I remember that he complained. The source he was following used the notation "first morphism f, then morphism g" we write fg. But he preferred the more standard notation gf. These permanent changes required a lot of effort.