CHARLES UNIVERSITY

FACULTY OF SOCIAL SCIENCES

Institute of Economic Studies

Government Bond Duration: Theory and Evidence

Master's Thesis

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Study programme: Economics and Finance

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Year of the defence: 2024

Declaration

1. I hereby declare that I have compiled this thesis using the listed literature and resources only.
2. I hereby declare that my thesis has not been used to gain any other academic title.
3. I fully agree to my work being used for study and scientific purposes.

In Prague on July 30, 2024 Bc. Vladimír Bajer

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Abstract

Over the past four years, the COVID-19 pandemic has led to economic shutdowns and uncertainty, resulting in inflationary pressures in 2021 and 2022, compounded by geopolitical tensions such as Russia's invasion of Ukraine. These events collectively influenced the government bond markets. This thesis investigates, whether duration and convexity measures remain reliable for assessing interest rate risk for government bonds in the Czech Republic, Germany and the USA, using quarterly data. We applied the fixed effects and Two-Stage Least Squares methods to analyze the data across various models, categorized by country and bond type. Our findings indicate that duration measures were generally reliable, even amid fluctuating yields and rising policy interest rates, though their accuracy varied yearly. Incorporating convexity measures into the regressions improved the precision of duration metrics. Additionally, our analysis confirmed the reliability of duration metrics for short-term government bonds in the US and Germany. These findings underscore the resilience and utility of traditional duration metrics, particularly when complemented by convexity measures, in assessing interest rate risk under diverse economic conditions, suggesting avenues for future research.

Keywords

government bonds, duration, interest rate risk

Title

Government Bond Duration: Theory and Evidence

Abstrakt

V posledních čtyřech letech vedla pandemie COVID-19 k ekonomickým uzávěrám a nejistotě, což vedlo k inflačním tlakům v letech 2021 a 2022, které byly umocněny geopolitickým napětím, jako je invaze Ruská na Ukrajinu. Tyto události společně ovlivnily trhy státních dluhopisů. Tato práce zkoumá, zda jsou míry durace a konvexity stále spolehlivými metodami pro posouzení úrokového rizika státních dluhopisů v České republice, Německu a USA, s použitím čtvrtletních dat. Aplikovali jsme metody fixních efektů a dvoustupňové nejmenší čtverce (Two-Stage Least Squares), abychom analyzovali data v různých modelech, kategorizovaných podle země a typu dluhopisu. Naše zjištění naznačují, že míry durace byly obecně spolehlivé, i přes kolísající výnosy a rostoucí úrokové sazby, ačkoli se jejich přesnost v jednotlivých letech lišila. Zapojení měr konvexity do regresí zlepšilo přesnost měr durace. Navíc naše analýza potvrdila spolehlivost měr durace pro krátkodobé státní dluhopisy v USA a Německu. Tato zjištění podtrhují odolnost a užitečnost tradičních měr durace, zvláště když jsou doplněny o měry konvexity, při posuzování úrokového rizika v různých ekonomických podmínkách, což otevírá možnosti dalšího výzkumu.

Klíčová slova

vládní dluhopisy, durace, úrokové riziko

Název práce

Durace státních dluhopisů: teorie a praxe

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Acronyms

CNB	The Czech National Bank
CPI	Consumer Price Index
CZE	The Czech Republic
ECB	The European Central Bank
ESG	Environmental, Social, and Governance
GER	Germany
GMM	Generalized Methods of Moments
FE	Fixed Effects
FED	The Federal Reserve System
OLS	Ordinary Least Squares
RE	Random Effects
TSLS	Two-Stage Least Squares
USA	The United States of America

Master's Thesis Proposal

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Proposed Topic:

Government Bond Duration: Theory and Evidence

Motivation:

After the COVID-19 pandemic began to fade slowly, the Czech Republic has observed a sharp increase in consumption and inflation commenced to rise uncontrollably, yet even more after Russia's military invasion of Ukraine at the beginning of 2022 and the unstable situation in the energy markets. All these factors made the Czech National Bank expand interest rates in order to cope with this unprecedented situation. Years prior to that, the country had not observed such frequent changes in the interest rates. Moreover, and this is not just the case in the Czech Republic, increasing interest rates had a direct effect on the increase in government bond yields.

One of the most popular measures of the timing of a bond's cash flow is the duration. One shall distinguish between Macaulay duration (often simply referred to as duration) and modified duration, and take emphasis on their disparity. Macaulay considers both interim cash flows as well as the final payment, weighing each by present value. The modified duration evaluates the price sensitivity when the yield to maturity alters. The modified duration is directly derived from the Macaulay duration, it is yet mathematically one concept. Based on the theory, there is a direct relationship between the duration and its price sensitivity to changes in market interest rates (Reilly & Sidhu, 1980) as their computations are rather complementary. Besides, the second derivative of the duration, the convexity, encourages beyond the clarification of this concept.

Duration theory has been the subject of several studies for decades, from Boquist et al. (1975) focusing on the assessment of the risk of bonds and how duration helps evaluate government bond return, to Chance (1990) examining the duration of zero-coupon bonds and their default risk, or Cox et al. (1979) duration's role in the measurement of basis risk. Furthermore, Hatchondo & Martinez (2009) established a framework of sovereign default including borrowing of long-term duration bonds which contrasts with previously performed research.

Focusing on privately distributed bonds and securities, Ambasta et al. (2010) examined the empirical duration of corporate bonds and numerous potential clarifications of the large reversal in interest rate sensitivity. This thesis will aim to recompute the duration for various government bonds and further perform manifold regressions on whether the theoretical rules were satisfied during the studied period, e.g., will government bond prices with increasing yields fall accordingly?

Hypotheses:

1. Hypothesis #1: The duration theory is not an accurate measure of the interest rate risk of government bond/interest rate sensitivity in times of low interest rates.

2. Hypothesis #2: The duration theory is more accurate in times of higher interest rates.

3. Hypothesis #3: The duration theory can be applied also to government variable bonds.

Methodology:

Several economists measured the duration of bonds adopting their own framework. Jacoby (2003) derived a model for the risk-adjusted duration of corporate bonds. Furthermore, Hong & Sarkar (2004) computed the duration of callable corporate bonds with a contingent-claims model incorporating call and default risks. In this thesis, the government bond duration is calculated and further estimated in order to confirm or disprove the duration theory as illustratively shown for instance by Chua (1984).

The data were collected on a half-year basis, beginning in September 2020 and ending in September 2023, thus 7 periods in total. This occurs due to data availability for the yield curves which are obtained from Reuters. Moreover, the thesis will focus on government bonds issued by three countries – the Czech Republic, the USA and Germany (EU). Thus we will be able to compare bonds issued also in different currencies and work with panel data. All the relevant information concerning each bond will be extracted from the official websites of stock exchanges, ministries of finance or complementary parties of given countries.

First, for each bond issued, we will compute their prices and durations consequently. Secondly thanks to different interest rates and bond yields, we will be able to perform plentiful econometric evaluations on whether the rules of duration are applicable and eventually how big are the differences between expected and actual values. In addition, we will try to clarify possible reasonings for these disparities and link them to subsequent bond theory.

We will test whether this can be applied in times of lower interest rates (our dataset commences in 2020), also of higher ones, and compare it across countries and types of bonds, including variable bonds. As mentioned earlier, we will be working with panel datasets, thus we consider different estimation techniques such as fixed-effects, random effects or GMM to account for potential endogeneity. We will also aim to support the results by implementing vector autoregression, commonly used in research in this field due to autogenous issues.

Expected Contribution:

As it is quite apparent that the vast majority of papers and studies focusing on duration theory are from the previous decades, this master thesis would provide new evidence of whether the theory can be still applied to today's bonds issued by governments across the world. In practice, this theory is commonly used in the business and financial sector as a feasible tool. We will try to verify how precisely this concept operates and what are the sources of inaccuracies, specifically on government bonds in our studied countries (the Czech Republic, the USA and Germany/EU). In addition, using the most recent available data gives us extra insight into different interest rates and yield levels in the span of four years.

Also, we will be able to test the theory on both lower and higher interest rates as the span of the last year has provided us with quite varying levels of these rates. Thanks to these fluctuations, it will be possible to explore what started happening to the bond prices when interest rates commenced to rise.

Outline:

1. Introduction
2. Literature review – an overview of past studies and analyses, explaining the contribution of this thesis and why it is important
3. Theoretical background – a demonstration of theory concerning the bond duration, important relationships related to it and crucial assumptions used in the further examination
4. Data – a description of the studied dataset, the behaviour and explanations of distinct patterns
5. Methodology – explanations of methods used and their suitability, depiction of tests performed to ensure no methodological bias, description of equations regressed
6. Empirical results – a presentation of obtained results, answering of proposed hypotheses
7. Discussion – deliberating on results, possible explanations of the outcome, linking results with previous studies and theoretical background, stating limitations of the thesis and their magnitude
8. Conclusion – compiling the whole research, briefly answering the research questions, suggesting implications for future studies

Core Bibliography:

Ambastha, M., Dor, A. B., Dynkin, L., Hyman, J., & Konstantinovsky, V. (2010). Empirical duration of corporate bonds and credit market segmentation. The Journal of Fixed Income, 20(1), 5-27.

Boquist, J. A., Racette, G. A., & Schlarbaum, G. G. (1975). Duration and risk assessment for bonds and common stocks. The Journal of Finance, 30(5), 1360-1365.

Chance, D. M. (1990). Default risk and the duration of zero-coupon bonds. The Journal of finance, 45(1), 265-274.

Chua, J. H. (1984). A closed-form formula for calculating bond duration. *Financial Analysts Journal*, *40*(3), 76-78.

Cox, J. C., Ingersoll Jr, J. E., & Ross, S. A. (1979). Duration and the measurement of basis risk. Journal of Business, 51-61.

Dunetz, M. L., & Mahoney, J. M. (1988). Using duration and convexity in the analysis of callable bonds. Financial Analysts Journal, 44(3), 53-72.

Fooladi, I. J., Roberts, G. S., & Skinner, F. (1997). Duration for bonds with default risk. Journal of Banking & Finance, 21(1), 1-16.

Galliani, C., Petrella, G., Resti, A., & Cazan, F. (2014). The liquidity of corporate and government bonds: drivers and sensitivity to different market conditions. Joint Research Centre Technical Reports, 1, 38.

Hatchondo, J. C., & Martinez, L. (2009). Long-duration bonds and sovereign defaults. *Journal of International Economics*, *79*(1), 117-125.

Jacoby, G. (2003). A duration model for defaultable bonds. Journal of Financial Research, 26(1), 129-146.

Mejstřík, M., Pečená, M., & Teplý, P. (2015). Bankovnictví v teorii a praxi/Banking in Theory and Practice. Charles University in Prague, Karolinum Press.

Reilly, F. K., & Sidhu, R. S. (1980). The many uses of bond duration. Financial Analysts Journal, 36(4), 58-72.

Sarig, O., & Warga, A. (1989). Bond price data and bond market liquidity. *Journal of Financial and Quantitative Analysis*, *24*(3), 367-378.

Sarkar, S., & Hong, G. (2004). Effective duration of callable corporate bonds: Theory and evidence. *Journal of Banking & Finance*, *28*(3), 499-521.

van Binsbergen, J. H., & Schwert, M. (2022). Duration-based valuation of corporate bonds. Jacobs Levy Equity Management Center for Quantitative Financial Research Paper.

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Magda Pečená

Author **Supervisor**

1. Introduction

Government bonds are a cornerstone of the financial markets, serving as a critical instrument for both public finance and investment portfolios. As debt securities issued by national governments, these bonds are pivotal in funding government activities, stabilizing economies, and providing a benchmark for other debt securities. Among the key concepts in measuring interest rate risk and management of government bonds is duration, a measure that captures the sensitivity of a bond's price to changes in policy rates.

Understanding bond duration is essential for both policymakers and investors. For policymakers, duration analysis aids in managing public debt and crafting monetary policy. For investors, it informs risk management strategies and portfolio optimization. Duration is a critical tool for assessing interest rate risk, which is especially pertinent in the context of fluctuating economic conditions and varying monetary policies across different countries.

The period from 2020 to 2023 has been marked by significant economic upheaval and unprecedented fiscal and monetary responses across the globe. In the wake of the COVID-19 pandemic and Russia's invasion of Ukraine, governments across the world, including the Czech Republic, Germany and the USA, which we will closely study in this thesis, implemented extensive fiscal stimulus measures and monetary policy adjustments to mitigate the economic fallout. These interventions, increased global uncertainty, increased prices, and higher inflation, have led to substantial fluctuations in policy interest rates and bond yields, highlighting the critical importance of understanding government bond duration. Studying government bond duration during this volatile period provides valuable insights into the effectiveness of duration as a tool for measuring and managing interest rate risk and optimizing bond portfolios in rapidly changing economic environments. Moreover, it underscores the necessity to adapt strategies to navigate the complexities introduced by such extraordinary circumstances.

Our goal is to investigate whether the duration metric has remained a reliable and adequately accurate measure of interest rate risk in the three above-mentioned countries during the period from 2020 to 2023. This analysis is conducted by calculating modified duration and convexity for all types of government bonds, with the data segmented by year to capture annual variations and trends. By examining these metrics across different bond categories, we aim to assess their effectiveness in predicting price sensitivity to policy rate changes amidst unique economic conditions.

To address our research question, we employed several panel data estimation methods. The panel dataset encompasses both cross-sectional units and time series data. Methods considered included Pooled OLS, Fixed Effects, and Random Effects models. After evaluating the strengths and limitations of each method, we determined that the Fixed Effects model was the most suitable option. To further validate our results and address potential endogeneity issues, we confirmed our findings using the TSLS technique.

After regressing various model variants, we concluded that government bond duration remains a pertinent metric for assessing interest rate risk, even amidst economic volatility and rapidly increasing policy rates. Our findings largely corroborated the hypothesis that an increase in government bond yield by one percentage point results in a corresponding price decrease by one percentage point for each year of the bond's modified duration. This relationship held true for government bonds from the Czech Republic, Germany, and the USA during the period from 2020 to 2023.

Moreover, we observed that incorporating convexity measures into the regressions enhanced the precision of the duration metrics, reflecting the quadratic nature of the price-yield relationship. While most coefficients were significant, they were predominantly higher than the expected value, indicating a stronger curvature of the relationship between bond's yield and price. This provides insight into the asymmetry of price movements and associated risks.

Additionally, our analysis revealed that the duration metric is reliable for US and German short-term government bonds too. These bonds inherently exhibit short duration due to their brief maturities and absence of coupon payments. The minimal fluctuations in short-term rates can cause the duration not to capture significant risk, as rate changes are negligible, as was the case for the Czech Republic. Furthermore, short-term bonds are typically utilized for liquidity purposes rather than for hedging interest rate risk, potentially inclining to diminish the relevance of duration in this context.

The accuracy of the duration metric varied throughout the studied period. For some years, the metric was robust, albeit less sensitive than anticipated. On the other hand, the highest sensitivities were recorded in 2023. However, the results varied depending on the country and the type of bond. Once policy rates surpassed a certain threshold, especially in the USA and Germany, the duration metric stabilized and approximated the expected -1 value.

This thesis is structured as follows: The Theoretical Background section introduces the fundamental concepts of duration theory. In the Literature Review section, relevant studies on duration are discussed. The Data section outlines the variables included in the regression analysis and provides summary statistics for these variables over the four studied years. The Methodology section presents the estimation methods used and discusses the assumptions necessary to ensure unbiased and reliable results. The Empirical Analysis and Results chapter reveals the outcomes of various regression analyses. Following this, the Discussion section examines the results, offers potential explanations, and addresses the limitations of the thesis. Finally, the Conclusion section summarizes the key findings of the thesis and suggests avenues for future research.

2. Theoretical background

We commence our exploration by explaining the basic concepts of duration theory. In this chapter, we narrow our focus to the formalized aspects of bond duration theory, emphasizing its application to government bonds. We begin by clearly defining government bonds and subsequently delve into the intricacies of duration theory, elucidating its relationship to interest rate risk. Furthermore, we highlight the crucial role of bond duration in investment analysis, drawing nuanced comparisons with corporate bonds. Throughout, we address the challenges inherent in this field, providing insight into overcoming theoretical hurdles. This chapter unfolds progressively through these themes, offering a structured understanding of the theoretical foundations that underpin our examination of government bond duration.

Government bonds, often referred to as sovereign bonds or treasuries, are debt securities issued by a government to raise capital and cover deficits. Investors purchase these bonds, essentially lending money to the government, in exchange for periodic interest payments and the return of the principal amount upon maturity (Fabozzi & Mann, 2011). Government bonds assume a crucial role within the financial market, acting as a benchmark for diverse policy rates. Recognized as a fundamental instrument in determining the risk-free rate, they exert significant influence on the pricing of other financial assets. The stability and creditworthiness inherent in government bonds establish them as a cornerstone for evaluating risk and informing investment decisions throughout the financial landscape (Andritzky, 2012).

Furthermore, the significance of government bonds extends beyond the financial market, playing a crucial role for both investors and the broader economy. Firstly, they are often regarded as a safe haven for investors during times of economic uncertainty. Their low default risk (related to developed countries) makes them an attractive option for preserving capital. Secondly, the yields on government bonds influence policy interest rates throughout the economy. Central bankers consider these rates to implement monetary policy, impacting borrowing costs. Next, governments utilize bond issuances as a primary mechanism to fund public projects and cover budgetary deficits. The ability to borrow through bond markets provides flexibility in managing economic policies. Lastly, fluctuations in prices and yields of government bonds serve as indicators of broader economic conditions. In times of economic stress, increased demand for government bonds may signal a fight for safety. (Schuknecht, 2004). In summary, government bonds serve as a linchpin in financial markets, providing a safe investment option, and contributing to economic stability through funding mechanisms.

2.1. Bond duration

Duration serves both as a metric for assessing risk and a measure of time. Frederick Macaulay initially developed it in the 1930s (Macaulay, 1938). Since then, several distinguished economists independently rediscovered the concept of duration (Zipf, 2003). The fundamental equation for Macaulay duration, often referred to simply as duration, is derived as follows:

$$D = \frac{\sum_n \left(\frac{1}{(1+r)^n} t_n \left(c_n + M_n \right) \right)}{\sum_n \left(\frac{1}{(1+r)^n} \left(c_n + M_n \right) \right)} \tag{1}$$

Here, *r* represents the bond yield used to determine its price, *t* stands for the number of periods for cash flow payments, *c* symbolizes the payment received from the coupon at the time *n* and *M* signifies the maturity payment at the time *n*. Equation 7 can be equivalently expressed as:

$$D = \frac{\sum n * PVC_n}{t * PVTC} \tag{2}$$

where *PVC* represents the present value of the cash flow at time *n*, calculated based on the yield to maturity of the instrument. Moreover, *t* stands for the frequency of payments per year, and finally, *PVTC* stands for the present value of the total cash flow (which besides equals the sum of the bond price and accrued interest). Another approach solves duration using the following argument:

$$D = \frac{\frac{\Delta P}{P}}{\frac{\Delta \mathrm{r}}{1+r}} \tag{3}$$

where *P* symbolizes the bond's price, *⊿* symbolizes the change, and *r* stands for interest rate. Equation 3 can be interpreted as the percentage change in bond price corresponding to a per cent change in policy interest rates (Mejstřík et al., 2015). To summarize the equations above, Macaulay duration measures risk by providing insights into the reactivity of a bond's price to changes in policy rates.

Having established the Macaulay duration, now we shift to more justifiable modifications without percentage change in policy interest rates. Modified duration introduces adjustments for changes in yield, offering a more dynamic perspective on interest rate sensitivity. It might be expressed as follows:

$$D_{mod} = \frac{\frac{\Delta P}{P}}{\Delta r} \tag{4}$$

Combining with Equation 3, we can express modified duration also using Macaulay's duration:

$$D_{mod} = \frac{D}{1 + \frac{r}{t}} \tag{5}$$

Here, r stands for yield, and t represents the frequency of payments per year. Modified duration is therefore an adjusted version of Macaulay duration. Unless Macaulays, it is a unitless measure. It represents the percentage change in bond price per one per cent change in yield. In summary, while Macaulay duration focuses on time-based characteristics, modified duration enhances the measure by incorporating interest rate sensitivity, making it a valuable tool for assessing and managing interest rate risk in bond investment.

Another commonly used measure of interest rate risk is effective duration. It is akin to modified duration but extends its applicability to callable bonds, encompassing bonds with option features. Specifically, it considers alternations in cash flows triggered by the activation of the call feature. The formula for effective duration is typically expressed as follows:

$$D_{eff} = \frac{P_{-} - P_{+}}{2P_0 \Delta r} \tag{6}$$

where P_{-} stands for the bond's price in the event of a decrease in yield, P_{+} represents the bond's price in the event of an increase in yield, P_0 states the bond's original price, and lastly, Δr represents the change in yield. Effective duration provides investors with a more realistic measure of interest rate risk, especially in the presence of features that can alter a bond's cash flows under different policy rate scenarios (Jorion, 2007). However, it is important to note that duration measures, including effective duration, are approximations. They may not fully capture the interest rate risk of bonds with a call feature. The complexity of callable bonds, where cash flows change significantly if the bond is called before maturity, means that effective duration simplifies the impact of optionality on bond prices.

Lastly, yet another modification of the duration theory can be defined, which encourages beyond the clarification of this concept. Convexity is mathematically the second derivative of the duration. It measures the curvature or non-linear relationship between a bond's price and changes in its yield or policy interest rate. It provided additional information about how a bond's price reacts to fluctuations in policy rates. It gives insight into the degree of curvature in the price-yield relationship. The formula is typically expressed as:

$$K = \frac{1}{P}\frac{\partial^2 P}{\partial^2 r} \tag{7}$$

which consists of a share of the second derivative of the bond price with respect to yield and bond price. Convexity is valuable because it provides a more accurate estimate of bonds's price change in response to interest rate movements rather than duration alone. Most bonds exhibit positive convexity, meaning that as yields increase or decrease, the percentage change in the bond's price is not perfectly linear (Rebonato & Putyatin, 2018). This is preferred by investors in general as it implies a more favourable risk-return profile in response to policy rate changes. Figure 1 below provides an example of a relationship between a bond's yield and price. It compares the predicted relationship of this relationship based on the duration metric versus an actual relationship curve reflecting the convexity feature.

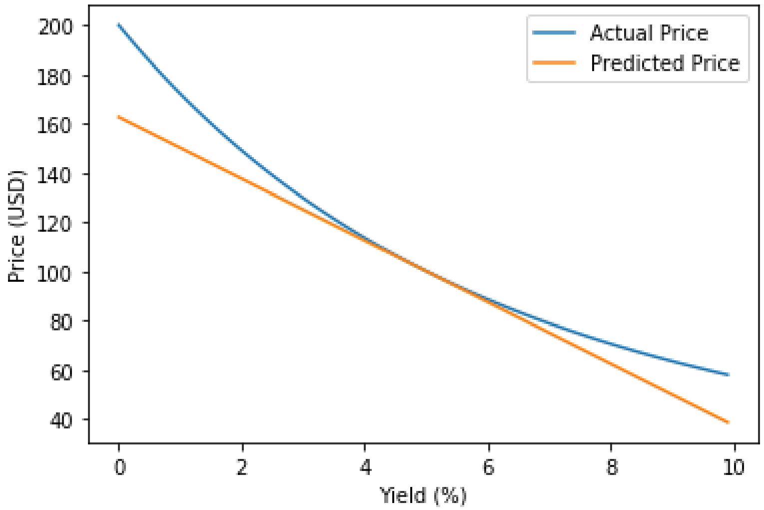

Figure 1 – An example of a bond's duration vs. convexity (predicted vs. actual price under current yield of 5%)

Source: Bond Valuation and Analysis in Python. DataCamp (2024)

Please note that in this subchapter, interest rate and bond yield are often used interchangeably. However, they have distinct nuances. Interest rates typically refer to a general level, whereas bond yields represent a specific return on investment. As demonstrated in the next chapter, when policy interest rates rise, bond yields correspondingly increase. Both terms convey the idea of changes in the cost of borrowing or the return on the investment, influencing the bond's price. However, it is essential to understand the nuances between them.

This subchapter has shown how to compute Macaulay's duration, and its adjustments such as modified or effective duration, and further introduced a second derivation of duration, convexity. Chapter 2 hereafter explains the importance of duration in investment analysis, contributing with both practical and academic findings, compares duration between corporate and government bonds and concludes the theoretical part of the thesis with gaps in current research.

2.2. Importance in investment

The duration plays a pivotal role in assessing risk and understanding temporal dynamics in the bond market. These measures contribute to an understanding of the bond's response to interest rate movements. Based on the equations above and yet provided information, it is clear that several factors influence the duration of a bond, and understanding these factors is crucial for making informed investment decisions. Here are some key factors that ought to explain bond duration (as shown in Equation 1):

- **Time to maturity** – the longer the time remaining until a bond matures, the higher its duration. The duration of a bond, when repaid at maturity, cannot exceed the remaining time to maturity. According to Ingersoll et al. (1978), duration is particularly sensitive to changes in policy interest rates for bonds with longer maturities.
- **Coupon rate** – bonds with higher coupon rates typically have shorter durations. That is due to the higher coupon payments providing an earlier return to principal, reducing the sensitivity to changes in policy rates. Duration quantifies the period required for a bond's price to be reimbursed by its internal cash flows, primarily just from coupons (Hyman et al., 2014).
- **Frequency** – bonds with more frequent coupon payments generally have shorter duration because the present value of more frequent cash flows is less sensitive to

changes in policy rates, resulting in quicker recovery of initial investment (Kraft & Munk, 2007).

- **Yield to maturity** – duration is by definition inversely related to the bond's yield to maturity. As yield increases, durations shall decrease and vice versa. This relationship reflects the fact that changes in policy rates have a more significant impact on the present value of future cash flows for bonds with lower yields (Chen et al., 2007).
- **Maturity payment** – the presence and timing of the maturity payment also influence duration, as Hopewell & Kaufman (2017) claim. Bonds with larger maturity payments or those with payments occurring later in the bond's life ought to have longer durations.

In summary, duration serves as a crucial risk metric, aiding investors in making informed decisions and managing the potential risks associated with changes in interest rates. Understanding these factors allows investors to assess the interest rate risk associated with a bond and make decisions aligned with their risk tolerance and investment objectives. These factors are either supported or disproved in the empirical analysis of this thesis.

2.3. Corporate vs government bond duration

Up to this point, we have introduced a theory of bond duration and its key features. Nevertheless, it is crucial to recognize the distinct characteristics that set corporate and government bonds apart. It is noteworthy that this thesis exclusively focuses on treasuries in its research segment. This subchapter aims to underscore the fundamental differences in the duration profiles of both types of bonds and to provide a rationale for the exclusive focus on government bonds. Gaining insights into these distinctions is vital for investors navigating the complexities of fixed-income markets. Corporate bonds, as unique entities in the fixed-income landscape, exhibit characteristics that significantly influence their duration. In comparison with sovereign bonds, we endeavour to highlight the most crucial nuances between these two categories.

Credit risk and stability in duration profiles

A paramount factor in corporate bond duration is credit risk, the potential of a borrower defaulting on debt payments. Higher credit risk tends to increase a bond's sensitivity to changes in interest rates and extend its duration. On the other side, higher yields compensate for the elevated risk. On the contrary, government bonds often exhibit greater stability in duration, being backed by sovereign status. The creditworthiness, especially that of economically robust nations, contributes to a consistent reliable cash flow stream, influencing duration dynamics (Babbel et al., 1997).

Interest rate sensitivity

Secondly, government bonds, especially long-term treasuries, are highly sensitive to policy rate fluctuations. Changes in policy interest rates impact the present value of future cash flows, affecting the duration of government bonds. Similarly, the sensitivity of corporate bonds to interest rate movements depends not only on the general level of policy rates but also on the specific terms of the bonds, including their credit spread and its related risk (Kraft & Munk, 2007).

Market dynamics and liquidity

Lastly, government bonds often exhibit higher liquidity. The greater liquidity contributes to market efficiency and can influence how interest rate changes are reflected in bond prices. Contrariwise, liquidity in the corporate bond market can vary significantly based on the specific issuer. This variability in liquidity can impact the efficiency of market pricing and the duration dynamics (Ericsson & Renault, 2006).

Rationale for focusing on government bonds

We have outlined the key distinctions between sovereign and corporate bonds. In our empirical analysis, we focus exclusively on government bonds. This decision was primarily influenced by several factors. Firstly, treasuries, particularly those issued by financially stable governments, are widely regarded as risk-free assets. Additionally, treasuries exhibit high sensitivity to changes in policy rates, a significant feature during the years under study. This characteristic enables a more precise analysis, avoiding the added complexity of factoring in the credit risk associated with corporate bonds. The absence of credit risk in treasuries simplifies various aspects of the analysis. Moreover, government bonds are frequently employed as indicators of economic conditions. The issuance of treasuries is directly influenced by government policies, allowing us to scrutinize how the government decisions are transmitted to bond yields and gain insight into broader economic trends. Lastly, the availability and consistency of data play a crucial role. Government bond data are more easily accessible and consistent compared to corporate bonds, mitigating potential challenges and biases associated with data availability.

2.4. Limitations around existing theories

Bond duration theory, while widely used in the field of finance, is not without its challenges. Understanding these challenges is essential for researchers to critically assess the limitations of bond duration theory.

Duration theory assumes parallel shifts in the yield curve, meaning that interest rates across all maturities change by the same amount. Yield curve shifts can be non-parallel, and different maturities may experience varying rate changes. This can lead to inaccuracies in predicting bond price changes. Our research ough to answer these questions. As we state in our thesis proposal, we believe the duration theory is more accurate in times of higher policy rates, and we expect it is not an accurate measure in times of policy interest rates (Hyman et al., 2014).

Further, while duration provides a linear approximation of bond price changes to interest rate changes, it does not account for the curvature in the bond price-yield relationship. Convexity adjustments (see Chapter 2.1) attempt to address this, but they add complexity to the calculations and may not fully capture the non-linear aspects of bond price changes (Hyman et al., 2014). In the regression analysis, we try to employ convexity measures as part of the robustness check.

Duration models – for instance, Hatchondo & Martinez (2009) or Boquist et al. (1975) – often focus on interest rate changes without explicitly considering broader macroeconomic factors. Economic events, inflation trends, and geopolitical developments can impact bond prices, and these factors are not fully captured by traditional duration measures.

Critics such as Shiller (2003) or Kahneman & Tversky (2013) among others questioned the assumption of rationality in financial models. Duration models often assume rational investor behaviour and efficient markets. In reality, investor behaviour is influenced by psychological factors, and markets cannot be perfectly efficient. Incorporating behavioural aspects into duration models is a complex challenge.

In conclusion, while bond duration theory is a widely utilized concept in finance, its limitations necessitate critical examination. The upcoming chapter will try to react to most of these controversies brought up above.

In Chapter 2, we have explored bond duration theory, with a specific emphasis on its application to government bonds. The chapter underscored the significance of duration in investment analysis, discussing factors influencing a bond's duration. Additionally, we compared corporate and government bond duration, highlighting areas where consensus is lacking. The decision to exclusively focus on government bonds in the empirical analysis was justified based on factors such as their risk-free status, sensitivity to interest rate changes, and data availability. Furthermore, by providing a comprehensive exploration of existing literature in the upcoming chapter, we encounter diverse perspectives on bond duration theory, encompassing a range of insights and viewpoints.

3. Literature Review

In this chapter, we provide a comprehensive overview of existing literature on bond duration. Recognizing the vastness of the subject, we selectively chose a subset of research that specifically aligns with our investigation into the theory of government bonds. The selection process focused on studies cited significantly in other works. Despite some being authored in the 20th century, we acknowledge potential constraints in capturing subsequent academic developments. Additionally, a considerable portion of relevant research on bond duration has traditionally centred around corporate bonds. Thus, we highlight pivotal conclusions from these studies. Lastly, we delve into studies directly addressing government bond duration to offer a nuanced understanding of this specific sector. In conclusion, our literature review sets the stage for our research goal of examining the impact of percentual change in interest rates on a bond's price, and vice versa.

As duration is widely utilized by investors and financial analysts, numerous analyses focus on duration, especially concerning privately distributed bonds. Boquist et al. (1975) advocate for the incorporation of the duration of common risk measurements for an asset. This enables investors to make informed decisions within their portfolios, establishing a consistent factor that relates the percentage change in bond price to alterations in yield. In this thesis, we will assess the enduring validity of this pattern for government bonds, considering that the study is nearly 50 years old.

Furthermore, in the study by Chance (1990), the examination of the duration of a zero-coupon bond (i.e., does not provide periodic interest payments), considering default risk, revealed results consistent with the previous paper. The author applied the contingent claim approach, originating from Merton (1977), commonly used in risk management and pricing. Similarly, the duration was found to be less than maturity, a point we besides others aim to validate in the empirical section of this thesis. Additionally, Fooladi & Skinner (1997) established a comprehensive expression for duration in the presence of default risk. They explored potential errors introduced by the omission of default risk in the two primary applications of duration – interest rate price elasticity and immunization. Their conclusion suggests the necessity for adjusted measures when dealing with bonds with default risk. Since our thesis focuses on government bonds considered default risk-free, we allow overlooking the effect in the empirical part. While acknowledging the age of the studies, considered somewhat obsolete, the cases presented below are more current, taking into account previously proved or disproved theories.

3.1. Corporate bond duration

In the exploration of bond duration, a significant facet lies in understanding the dynamics unique to corporate bonds. This subchapter delves into recent studies, with a primary focus on corporate bond duration, shedding light on the nuanced interplay of factors influencing these financial instruments. Beginning with Ambastha et al. (2010), the findings below lay the foundation for a thorough comprehension of the unique duration effects evident in the corporate bond market.

In recent studies, Ambastha et al. (2010) explored the domain of corporate bonds, specifically investigating the phenomenon of credit market segmentation. Through the analysis of interest rates and bond prices, they posited that different segments or sectors within the credit market exhibit varying behaviours, resulting in distinct duration effects. This underscores the complex dynamics inherent in the corporate bonds market.

Galliani et al. (2014) primarily investigated the liquidity of both corporate and government bonds and their responsiveness to various market conditions. Nonetheless, while analysing the liquidity, they utilised the bond's duration and detected that the longer the duration is, the effect of illiquidity is greater. Further, Hyman et al. (2015) explored how coupon levels impact the pricing of corporate bonds by investigating the relationship between coupon and spread, as well as the influence of duration. Findings indicate that discount bonds, with lower spreads and better default risk protection, exhibit a "spread convexity" effect leading to an outperformance of discount bonds over premiums, especially during periods of high default risk.

Some research papers had a similar goal as one of ours – recomputing the duration of bonds. Sarkar & Hong (2004) calculated the effective duration of callable bonds (i.e., gives the issuer the right, not the obligation, to redeem the bond before its maturity date) by employing a contingent-claims model that integrates both default risk and call risk. Utilizing monthly bond price and interest data, cross-section regression analysis is applied and reveals that, with the exception of low-grade bonds, the call feature tends to shorten the duration. The difference between these and our work is that we try to confirm or negate a pattern of already created theory, they rather provide unique approaches to compute duration. Further, an article written more recently by Xie et al. (2009) also linked together bond duration and default risk.

Their findings reveal that call risk diminished the duration of default-free bonds, similar outcomes as the paper mentioned above. Nonetheless, they claim that the effect on duration is diminishing as bond ratings decline (i.e. a higher discount rate), and additionally, the call risk's impact is less pronounced for discount bonds.

As Domanski et al. (2017) presented, the interest risk adjusted significantly in 2014 as long-term policy rates in Europe experienced a significant and historically low level. Their investigation focused on managing duration mismatch for the investors (in this concrete case, in the German insurance sector). As long-term interest rates decrease, the negative duration gap on the portfolio of funds widens. This goes in line with our hypotheses, i.e. changes in interest rates affect duration variously in diverse rate levels.

The analysis of corporate bond duration highlights intricate dynamics, such as credit market segmentation, liquidity impacts, and the role of coupon levels. Moreover, investigations into interest rate adjustments align with our hypotheses, underscoring the diverse effects of interest rate changes on bond duration.

3.2. Government bond duration

Having delved into the nuanced dynamics of corporate bond duration, we now turn our focus to the realm of government bonds. Our goal is to uncover unique factors influencing these financial instruments and broaden our understanding of the broader bond market dynamics.

The studies mentioned above predominantly present findings related to corporate bonds. In the study by Hatchondo & Martinez (2009), the authors extended the framework employed in recent studies by introducing government borrowing through long-duration bonds. This modification resulted in substantially higher and more volatile interest rates, suggesting that the inclusion of long-duration bonds is a valuable tool for future research. Moreover, a separate study focused on the duration-based valuation of corporate bonds. By examining the correlation between returns and the duration of government bonds, the research revealed unique return patterns compared to conventional excess return definitions, such as those exceeding Treasury bills. Notably, these findings underscore the importance of accounting for the non-stationary interest rates environment when conducting asset pricing tests (van Binsbergen & Schwert, 2022).

Several authors have conducted empirical tests on duration within specific samples, such as a particular stock exchange/capital market. For instance, Ivanovski et al. (2013) inspected the valuation of treasury bonds on the Macedonian Stock Exchange, assessing the sensitivity of bond prices to changes in interest rates through measures like duration, modified duration, and convexity. Their key finding suggested limited sensitivity of these treasury bonds to interest rate changes. Similarly, Jin et al. (2018) explored the holdings of Government of Canada bonds by institutional investors, revealing that both bond duration and investor types significantly influence variations in interactions between changes in yields and net purchases. Our approach differs from these studies, primarily by encompassing various sets of bonds from different exchange markets.

A different study demonstrated that through an examination of government bond market indices, both duration and spread account for a significant portion of the price variance and covariance observed across these markets. These findings align with the commonly used insight for the effective management of bond risk (de Jong, 2023). Furthermore, Di Asih & Abdurakham (2021) assessed interest rate risk, specifically its impact on bond price fluctuations. Utilizing the duration and convexity model provided estimates of the bond price response to changes in interest rates (treating Indonesian Government Bonds), more specifically, an increase in interest rates has a relatively modest impact on risk for a bond.

In addition to the aforementioned insights, duration theory significantly influences studies examining yield curves, such as Singleton's (2003) research on Japanese government bond markets. Singleton's study highlights the sub-optimal nature of duration-based hedging strategies, impacting risk management across various instruments, including corporate bonds. Finally, Kraft & Munk (2007) contribute by combining the durations of corporate and treasury bonds within an intensity-based credit risk framework. In contrast to prior studies, their findings suggest conditions under which the duration of a corporate coupon bond may exceed that of a similar treasury bond, providing a valuable upper bound on corporate coupon bond duration.

Our exploration of government bonds shed light on critical insights. The inclusion of long-duration bonds enhances a higher interest rate volatility, and empirical studies highlight the limited sensitivity of treasury bonds to interest rate changes. Additionally, research on risk management and duration theory contributes to a holistic understanding of government bond dynamics.

In conclusion, this chapter underscores the nuanced roles that bond duration plays in the analysis of interest rate risk, bond yields, and price fluctuations. The prevailing consensus across the majority of articles embraces the established duration theory of bonds, utilizing its implications in diverse research contexts. Notably, the investigations scrutinized in the literature do not align with the precise research object we have delineated for our study. In the upcoming chapters, our focus will be on confirming a fundamental premise: specifically, examining the impact of a one per cent increase in interest rates on a bond's price, and vice versa. To achieve this, we will analyse government bond yields and prices from recent years, marked by significant fluctuations in interest rates. The subsequent chapter will further elaborate on these foundations and discuss the data used in our analysis.

4. Data

In this chapter, we provide an in-depth analysis of the collected data, which serves as the foundation for our estimations. The significant rise in inflation since 2021 has spurred widespread adjustments in policy rates by central banks, prompting the investigation conducted in this study. Our primary objective in this thesis is to assess the applicability of the duration theory amidst fluctuating policy rates, and bond yields respectively. We commence by presenting evidence of inflationary trends, focusing particularly on our three selected countries: the Czech Republic, the United States of America, and Germany. Subsequently, we examine the evolution of policy rates, illustrating how these changes directly influence yield curves. Additionally, we provide a comprehensive description of the dataset used in our estimations. This includes an overview of the variables and various types of bonds considered, followed by detailed descriptive statistics and other information the data gives, facilitating a deeper understanding of its characteristics.

4.1. Motivation and development

The global economy has experienced unprecedented turbulence in recent years, driven by a multitude of factors. Events such as the COVID-19 pandemic and geopolitical conflicts like Russia's invasion of Ukraine have severely disrupted global supply chains, and reallocation of consumption and labour, leading to supply shortages and heightened inflationary pressures. Moreover, economic recovery efforts, alongside expansionary monetary policies and fiscal measures implemented by governments worldwide, have contributed to the complex landscape of inflation dynamics (Di Giovanni et al., 2023). These diverse influences have collectively resulted in inflationary trends not witnessed in decades in developed economies. In Figure 2 below, we present changes in the consumer price index for our studied countries during the years 2020-2023.

At the onset of our studied period, Germany experienced low inflation, with periods of deflation observed in the later 2020, particularly in the third and fourth quarters. These deflationary episodes reflected domestic and external factors like the temporary reduction of the value-added tax introduced in August 2020 or the global oil price collapse (Guttmann & Guttmann, 2022). In contrast, the Czech Republic consistently recorded higher inflation rates than Germany, attributed largely to economic and energy crises, with significant impacts observed, notably in the food sector. The United States saw the earliest peak in inflation, reaching 6.6% in September 2022, followed by Germany, which peaked at 8.8% in November

2022. The Czech Republic exhibited the highest peak, hitting 18.0% in September 2022. Post-peak, both the USA and Germany experienced a gradual moderation in inflation rates, varying slightly below 4% by December 2023. However, the Czech Republic witnessed a more rapid decline from its peak, stabilizing at 6.9% by the end of the period, indicative of ongoing inflationary pressures. The disparities underscore the distinct trajectories and magnitudes of inflation across the three countries, shaped by unique economic conditions and policy responses.

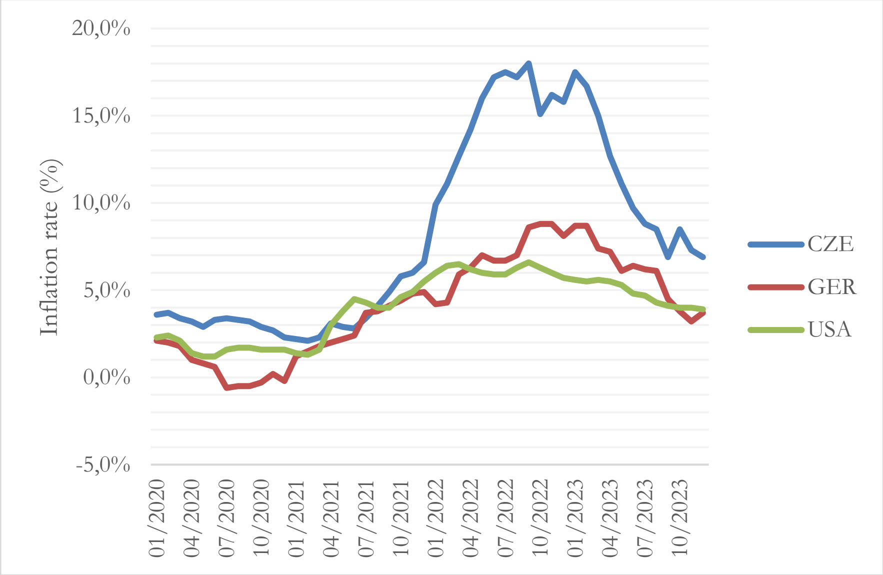

Figure 2 – Inflation rate (change on previous year's month)

Source: Author's computation based on multiple sources (2024)

In order to decrease inflation, central banks often raise their policy interest rates to reduce inflationary pressures. Higher policy rates can dampen consumer spending and investment, leading to a decrease in aggregate demand, economic activity and, subsequently, a slowdown in price growth. Figure 3 provides a policy rates development by the Czech National Bank, the European Central Bank and the Federal Reserve System in our studied period.

After the COVID-19 surge across the world, central banks used policy interest rates decrease mainly to boost consumer spending and investment, ensuring financial stability by providing liquidity to financial markets, mitigating unemployment or in the case of the Czech Republic, weakening domestic currency to promote exports (Sims & Wu, 2021). By mid-2020, these rates had dropped to near zero. In the case of the ECB, the main refinancing rate was already set at zero since 2016 and did not change until mid-2022. By that period, they even set the deposit facility rate below zero (-0.5%). Throughout 2021, the policy rates remained low as the central banks aimed to support ongoing economic recovery efforts. In June 2021, CNB began increasing its policy interest rates, ahead of the other two. The rise was steep and continuous, reaching 7% in June 2022. This was their last increase, likely to be linked to the exchange of members of the Bank Board, followed by a different approach to dampen inflation. The ECB maintained its zero rates, starting to raise them in July 2022. These increases were also gradual and reached their peak in September 2023 at 4.5%. Similarly, the FED kept its rates low and began to increase them in February 2022. The rate hikes by the FED were more pronounced compared to the ECB, reaching 5.5% in July 2023.

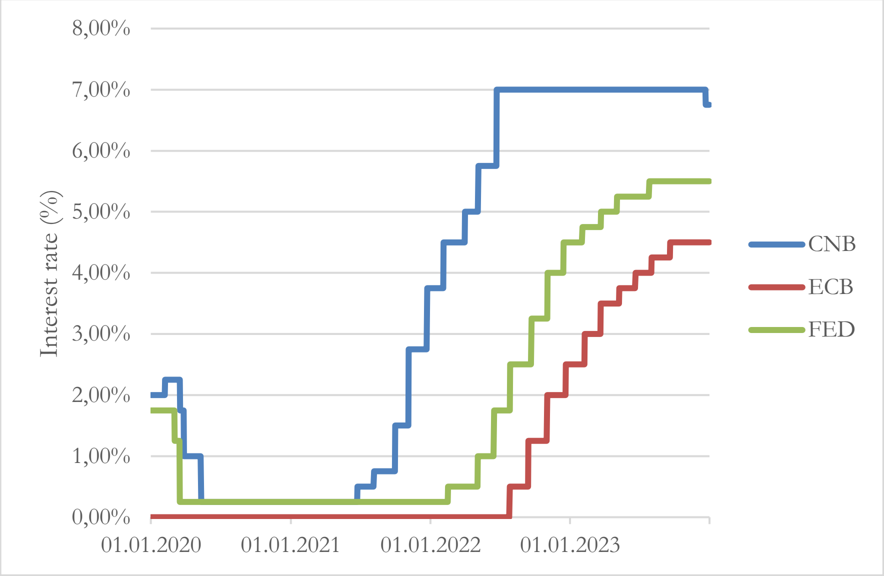

Figure 3 – Main refinancing interest rate (set by a central bank)

Source: Author's computation based on CNB, ECB, FED (2024)

While all three banks initially reduced rates to support the economy, they shifted to increasing rates from late 2021 onwards to combat rising inflation shown in Figure 2. These policy rate changes are directly linked to changes in bond yields, as shown by for instance Fabozzi & Fabozzi (2021), Wu & Xia (2020), Leombroni et al. (2021) and many others. They claim that as policy rates rise, the bond yield typically will rise too. Newly issued bonds have higher coupon rates, and existing bonds with lower ones become less attractive, causing their prices to fall. Since bond yield is inversely related to bond prices (Purnomo et al., 2022), as bond yields increase, prices decrease. Furthermore, if investors expect central banks to continue raising policy rates or have higher inflation expectations, these can lead to higher yields likewise. It is worth noting that while policy rates are short-term, their impact on yields is intertwined with long-term economic expectations (Gomez-Cram & Yaron, 2021).

In Figure 4-7, we present the yield curves for government bonds at the year-ends of each studied year. The data used to graphically portray the yield curves were retrieved from the respective central banks' websites (see References). Note that while central banks set policies, the market determines bond yields, and central banks, or markets, collect and report the data afterwards. For a small number of missing data points, we employed an extrapolation method using yields from the Refinitiv Eikon database. This database allowed us to obtain daily yields for every bond used in the subsequent estimations (see Chapters 4.2-4.3 for further details).

Figure 4 represents yield curves as of December 31st 2020. We observe that all three exhibit almost "normal" shapes, i.e., an upward positive slope with minor discrepancies for shorter maturities. This shape is the most common, signalling lenders to expect higher compensation for a longer maturity bond, which is also exposed to greater risk. For all described maturities, Czech government bonds have the highest yields out of the three presented, with 30Y bonds reaching 2% yield p.a. However, due to low policy rates, the yield does not surpass higher values. The US curve was the most conventionally upward-sloping of the three. Notably, German bonds with maturities shorter than seven years had negative yields, primarily due to the zero policy interest rate set by the ECB, which implemented unconventional monetary policies such as quantitative easing or negative deposit policy rates to combat low inflation. This was supported by the expected economic weaknesses of market participants and safe-haven demand during this period (Heider et al., 2021).

Figure 4 – Government bond yields as of 31st December 2020

Source: Author's computation based on multiple sources (2024)

As mentioned at the beginning of this chapter, the CNB was the first of three to start increasing its policy rates, responding to the earliest and most pronounced inflationary pressures observed. This notably affected the Czech government yield curve by the end of

2021, as shown in Figure 5. As German and US curves remained in the positive steep shape (larger difference between short-term and long-term return expectations), Czech appeared as inverted. It occurs due to the perception that policy rates will decline in the future, most likely due to a decline in inflation. An inverted shape can be perceived as an indicator of an economic downturn, although it may also reflect expectations of a 'return to normal' economic conditions. Nonetheless, the yields of Czech government bonds were still significantly higher compared to the other two, 1Y yield above 4%, and the 10Y and longer falling just under 3%. The US yield was approximately 1% higher than the German for shorter maturities, and the difference tightened as the maturities extended. German bonds reported negative yields only for maturities up to 3Y, even though the ECB's main refinancing rate remained zero.

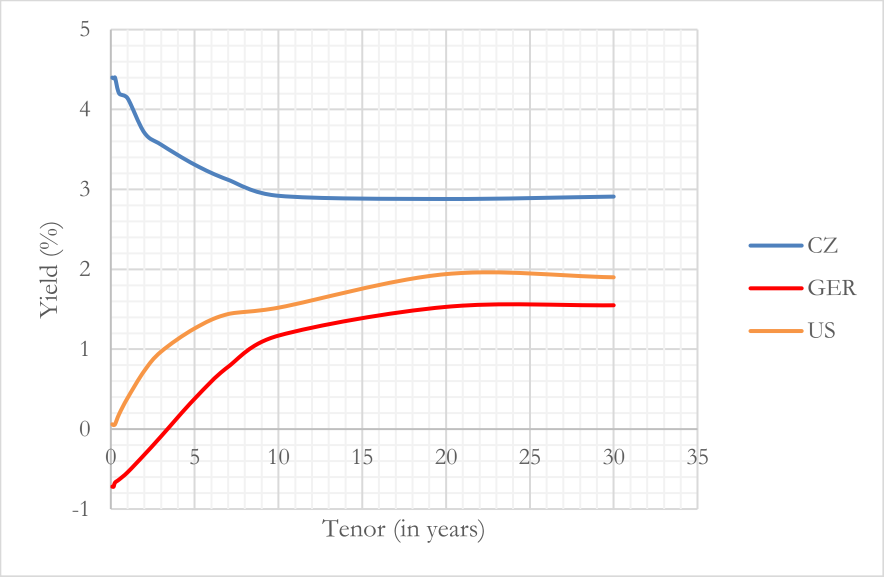

Figure 5 – Government bond yields as of 31st December 2021

Source: Author's computation based on multiple sources (2024)

Next, Figure 6 demonstrates yield curves as of 31st December 2022. As described earlier, the Czech yield curve remains in its inverse shape. The yields have shifted higher, ranging between 5-7.25 percentage points. This indicates investors' concerns about the Czech economy's short-term outlook, which were mainly fuelled by the consequences of the pandemic and Russia's aggression in Ukraine in the first quarter of 2022, leading to higher yields to compensate for the increased risk. In contrast, both the US and German curves strikingly altered their frame and values. The German government yield curve could be described as humped, a phenomenon occurring when medium-term yields are higher than both short-term and long-term yields. Importantly, the German yield abandoned the negative values, with the lowest yield for short-term bonds just above 2% and the highest yield just under 4% for 10-year bonds, further decreasing for even longer maturities. However, the yield was generally still lower compared to the US curve. It is also significantly higher than the previous year, with its shape almost flat, reaching its peak at 4.76% for a bond maturing in 6Y, and later oscillating around 4%. In summary, the rise in inflation and interest rates in 2022, is evident across all three markets, proving the effect of policy rates on bond yields, and further bond prices.

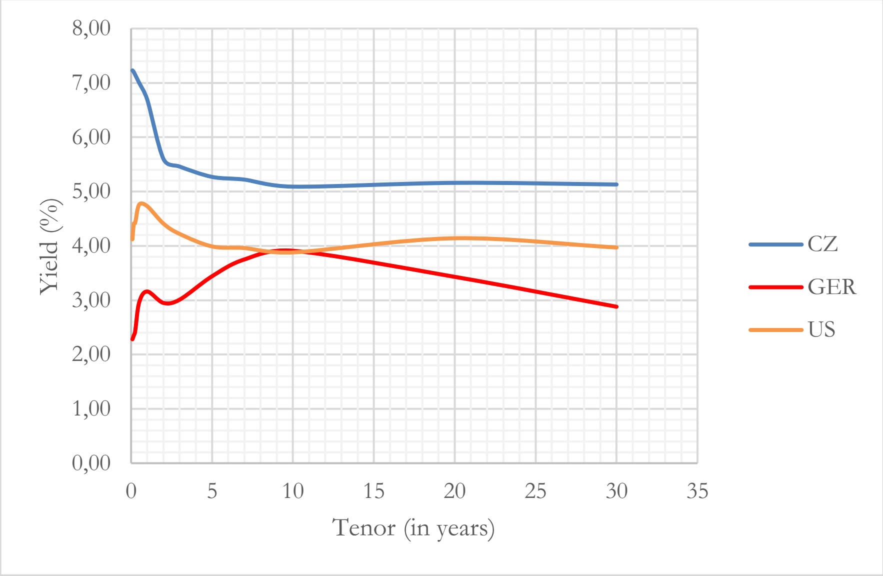

Figure 6 – Government bond yields as of 31st December 2022

Source: Author's computation based on multiple sources (2024)

Lastly, Figure 7 presents the respective government yield curves as of 31st December 2023. For the first time in our studied years, the Czech yield is not the highest among the three. The US yield curve exhibits an inverse shape, reaching above 5.5% for 1-month bonds and decreasing to fluctuate around 4% for longer maturities, similar to the previous year. The curve's flattening suggests that while the short-term rates remain high due to ongoing monetary tightening, long-term rates reflect expectations of eventual economic stabilization. The Czech government yield curve also retains its inverse shape. However, the decrease in values suggests some easing in inflation concerns and stabilization in CNB policies, as supported by Figures 2 and 3. The yield peaked at 5% for 1-year bonds and varied around 4% for longer maturities, indicating a difference of approximately 1-2% compared to the previous year. The German yield curve has similarly shifted to an inverse shape, peaking at 4% for 1-month bonds, then continuing with a negative "hump", with the lowest value around 2% for 2-year bonds, before rising to almost stabilized values for long-maturity bonds between 3-3.5%. This indicates moderate investors' expectations about long-term growth.

To conclude, the trends observed in the yield curves reflect the broader economic conditions and central bank policies, particularly the increases in policy rates over the past four years. As demonstrated in this sub-chapter, all three studied markets showed noteworthy changes in their government bond yields, driven by rising in policy interest rates in response to unprecedented inflation levels. Consequently, we will further investigate whether the duration metrics remain valid for government bonds and if they continue to be a reliable method for investors to manage interest rate risk.

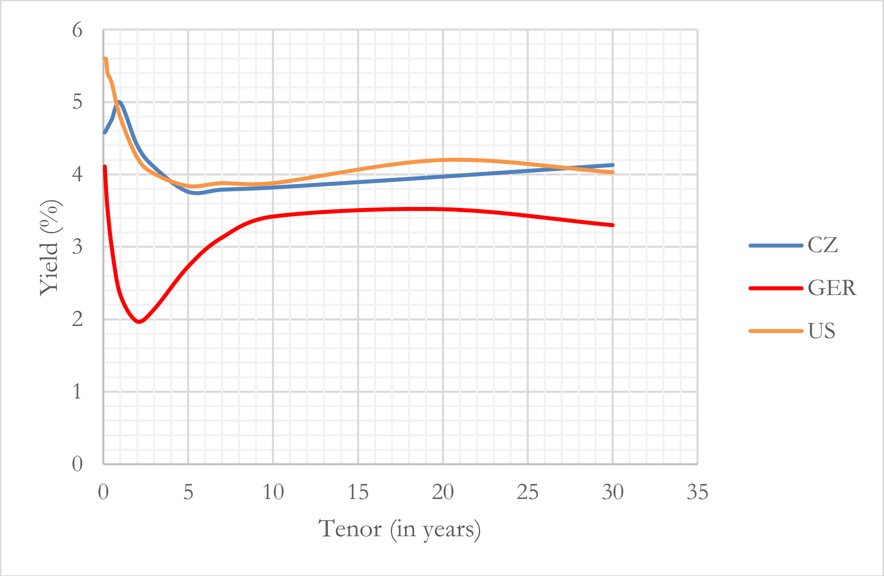

Figure 7 – Government bond yields as of 31st December 2023

Source: Author's computation based on multiple sources (2024)

4.2. Dataset

This subchapter presents an overview of the dataset further used for the estimations. Our full dataset consists of 7,584 observations, specifically 972 government bonds over 2 to 16 time periods. The data was collected on a quarterly basis, starting on 31st March 2020 and ending on 31st December 2023. We chose not to include earlier periods because the three central banks related to the thesis did not significantly alter policy interest rates before this timeframe. Our goal is to estimate duration metrics during periods of interest rate changes. Although collecting data on a more frequent basis (monthly, daily) would increase the number of observations, we opted for quarterly data collection. This decision is based on our estimation approach, which relies on time differences. We believe that changes in bond yields affect prices, but the market requires some time to adjust. For more details, see Chapter 5 – Methodology.

Each bond in our dataset has a different number of observations due to varying maturities and tenors, resulting in an unbalanced panel dataset. It is important to note we were not able to gather data for all bonds issued or not matured within the studied four years. Bonds excluded from our final dataset were either not publicly traded (often traded "over-the-counter") or were excluded due to information sensitivity. Additionally, we excluded bonds with varying coupon rates as their duration calculation differs from the standard fixed coupon rates. The duration is from definition very low (the repricing is short, so is the duration, hence sensitivity is low), and thus features of these bonds could not be applied uniformly. Varying coupon bonds may have interest rate sensitivities that change over time, depending on how the coupon is adjusted. This variability complicates the estimation of future cash flows, which is integral to the duration calculations. Moreover, their durations are typically short due to frequent coupon adjustments, leading to frequent reevaluation. Maintaining a homogenous bond sample (i.e., fixed coupon rates) allows for more accurate comparisons and generalizations regarding interest rate risk. However, we included all available data from various sources, allowing us to estimate the full (available) population. Thus, these discrepancies are due to the natural features of bond issuance and trading, meaning we do not need to address this unbalanced panel data in later estimations. To be precise, we ruled out approximately 20% of Czech government bonds, less than 1% of German bonds, and around 7% of US Treasuries for the reasons mentioned above.

As we work with government bonds issued by three independent governments, their strategy, frequency, amount or types of bonds quite differ. Below, let us disclose the types of bonds for each country, as they will be also to some extent estimated according to their category (for more details, see Chapter 6 -Empirical Analysis and Results).

Czech Republic

To ensure the completeness of Czech government bonds, we went through the emission calendars prepared by the Ministry of Finance. Based on them, we distinguish the following types of bonds:

- *"Středně- a dlouhodobé dluhopisy" (SDD)* – medium and long-term bonds with maturity between 1 and 50 years, coupon rates between 0 and 6.2% with annual payments. 34

bonds (differentiated by identifiers – "ISINs") included in our dataset, 7 excluded (see above).

- *"Státní pokladniční poukázky" (SPP)* – short-term bonds with a maturity of under 1 year, for our purpose SPPs suitable only for those with maturity greater than 90 days (due to quarterly data collection and estimating time differences), zero coupon. 13 bonds included in our dataset, 5 excluded.

The Czech government further issues *"Dluhopisy Republiky",* 6-year bonds either with fixed, inflation-based or reinvesting coupons, *"Spořící státní dluhopisy"*. However, these are not traded publicly and thus cannot be used for our study.

Germany

In order to guarantee all-encompassing coverage of German government bonds, we examined the issuance calendar provided by the Deutsche Finanzagentur, i.e., the German Finance Agency, the body providing services for the German federal government regarding debt management. Our datasets consist of the following kinds of bonds (all with annual coupon payment frequency):

- *"Bobl"* – medium-term bonds with 5-year maturities, coupon rates mostly zero, not higher than 2.5%, all 17 included in the dataset.
- *"Bubil"* – short-term bonds with a maturity of either 6 months or 1 year, zero coupon rate, 50 bonds included, 1 excluded (no publicly available disclosed price information).
- *"Bund"* – long-term bonds with maturities between 7 and 30 years, varying coupon rates, all 56 included.
- *"Green"* – medium to long-term bonds, issued since 2020 to promote the market for green financial products, reporting about the green expenditure of the Federal budget that is allocated to them, coupon rates from zero to 2.3%, all 7 included.
- *"Schatz"* – 2-year maturity bonds (some sources classify them as either Bobls or Bubils), coupon rates reaching 3.1% at most (also zero-coupon variants), all 22 bonds (based on ISINs) included in our dataset.

The USA

To ensure the completeness of US government bonds (often referred to as "Treasuries"), we went through the list of issued treasuries provided by the United States Department of the Treasury (the equivalent of the Czech Ministry of Finance). Apart from the previous countries, their coupons are repaid semiannually (coupon rates still disclosed per annum). The studied dataset includes the following types of treasuries:

- *"Bill"* – short-term bonds maturing under 1 year (shortest maturity 3 days, for our purpose used those maturing in 13 weeks or later), zero coupon rates, 215 bonds (characterized based on CUSIP identifier) included in the dataset, 29 excluded (see reasoning earlier in the sub-chapter).
- *"Note"* – bonds with medium maturity, ranging between 2 and 10 years, various coupon rates (depending on the original issue date), 453 bonds included, 12 excluded.
- *"Bond"* – long-term government bonds with a maximum of 30.5-years maturity, various coupon rates, 105 government bonds included (based on CUSIPs), 20 excluded from the dataset.

4.2.1. Variables

At the outset of sub-chapter 4.2, we presented an overview of our dataset, encompassing various types of government bonds of the Czech Republic, Germany, and the USA. The primary source for price and yield data was the Refinitiv Eikon database. Additional details such as coupon rate or maturity were sourced from the respective bodies for each country, the Czech Ministry of Finance, the US Department of Treasury, and the German Finance Agency. A complete list of sources is provided in the References section. Below, we outline the variables included in the dataset, which are further used in the regression analyses:

Price of bond

In this study, one of the key variables analysed is the price of government bonds (P), as it features both dependent and independent variables in our estimations. This variable reflects the market price at which bonds are traded, quoted as a percentage of the bond's face value. Data for this variable are sourced from Refinitv Eikon (2024), ensuring accurate market prices.

Bond yield

The bond yield (r) is a fundamental measure in financial markets, representing the return an investor receives on a bond investment. It is expressed as a percentage and reflects the annual return relative to the bond's current market price and coupon payment. In this study, the bond yield serves as a critical explanatory variable in regression analyses. Similar to the price, data for bond yield are retrieved from Refinitiv Eikon (2024).

Coupon rate

Although not directly entering the regressions, we include the coupon rate (C) here since it plays a key role in computing the bond's duration and convexity. The coupon rate represents the annual interest payment as a percentage of the bond's face value and is predetermined at the time of issuance. In essence, the coupon rate serves as the contractual yield promised to bondholders by the issuer. Data for coupon rates are obtained from the issuance calendars of each respective country and can be found in the Reference section.

Time-to-maturity

Similarly to coupon rate and not directly entering the regression, time-to-maturity (T) plays a significant role in calculating both duration and convexity, as well as classifying the bonds into short-term, medium-term and long-term, each with distinct attributes. Time-to-maturity represents period remaining until the bond issuer repays the principal amount to bondholder. In modelling analysis, it serves a key factor in models such as discounted cash flow. Data are retrieved from Refinitiv Eikon (2024).

Duration

In sub-chapter 2.1, various methods for calculating a bond's duration (D) were presented, specifically in Equations 1-3. However, due to data availability and the aforementioned variables, we proceeded with duration computation as in Equation 8 below:

$$D = \sum_{t=1}^{T} \frac{\frac{C}{(1+r)^{t}}}{P} * t + \frac{\frac{M}{(1+r)^{T}}}{P} * T \tag{8}$$

Additionally, both Czech and German government bonds have annual coupon payments, allowing us to apply Equation 8. However, US bonds have semiannual coupons, necessitating an adjustment, as described in Equation 9:

$$D = \sum_{t=1}^{2T} \left(\frac{\frac{C}{2}}{P} * \frac{1}{\left(1+\frac{r}{2}\right)^{t}} \right) * t + \left(\frac{M}{P} \frac{1}{(1+\frac{r}{2})^{2T}} \right) * 2T \tag{9}$$

Duration is widely used in bond portfolio management to optimize portfolio composition and hedge against the interest rate risk. In our study, the duration variable serves as the last step in obtaining modified duration, our crucial independent variable, and the focus of this thesis.

Modified duration

Modified duration (D_{mod}) enters all regressions as a crucial part of independent variable. Unlike Macaulay's duration, which represents the weighted average time until all of the bond's cashflows are received, modified duration incorporates bond's yield into calculation (see Equation 5). By factoring in the yield, modified duration accounts for the relationship between prices and changes in interest rates, making it a more accurate measure of interest rate sensitivity.

Convexity

Convexity (K) was also expressed in sub-chapter 2.1, as shown in Equation 7. Similarly to duration, we had to adjust to compute it correctly, as described in Equation 10:

$$K = \frac{C}{P} * \sum_{t=1}^{T} \frac{t(t+1)}{(1+r)^{t+2}} + \frac{M}{P} * \frac{T(T+1)}{(1+r)^{T+2}} \tag{10}$$

Additionally, Equation 10 had to be adjusted for US bonds due to semiannual coupon payments, resulting in Equation 11:

$$K = \frac{\frac{C}{2}}{P} * \sum_{t=1}^{2T} \frac{t(t+1)}{\left(1+\frac{r}{2}\right)^{t+2}} + \frac{M}{P} * \frac{2T(2T+1)}{(1+\frac{r}{2})^{2T+2}} \tag{11}$$

Unlike duration, convexity captures the curvature of the bond price-yield curve. By incorporating convexity into portfolio analysis, investors can better assess the risk-return trade-offs and make more informed decisions. In our study, it serves as a significant part of independent variables, complementing the information provided by duration.

4.2.2. Summary statistics

So far, we have introduced the variables used in our regressions, either directly or indirectly. In this sub-chapter, we provide an overview of the summary statistics, specifically focusing on the mean, standard deviation, median, and the 25th and 75th percentiles. We present these summary statistics for the complete dataset, which includes data from the three studied countries, as well as for each country separately. This approach allows us to highlight notable differences between the countries. Table 1 below shows the summary statistics for our full dataset, i.e., 7,584 observations.

	Mean	Standard deviation	25th percentile	Median	75th percentile
Price (%)	101.22	14.92	95.88	99.96	103.84
Yield (%)	1.93	2.02	0.18	1.65	3.89
Coupon rate (%)	1.67	1.53	0.25	1.50	2.50
Time-to-maturity	6.74	8.22	1.13	3.38	7.63
Duration	5.91	6.71	1.13	3.32	7.27
Modified duration	5.79	6.57	1.09	3.28	7.18
Convexity	134.03	274.57	2.30	14.90	66.54

Table 1 – Summary statistics (full dataset)

Source: The author's computation, sources for each variable listed above.

Both price and yield display a relatively tight distribution around the mean, with the median close to it, indicating a symmetric distribution. However, both also suggest a right-skewed distribution, as the median is lower than the mean. The coupon rate has a moderate spread with noticeable differences between the lower and upper quartiles, indicating variability in the coupon rates (increased in later periods with high policy rates). Time-to-maturity, duration and modified duration, all exhibit high standard deviations, pointing out to a diverse set of bonds of interest rate sensitivity. Lastly, convexity shows the highest variability among all metrics. Further, we compare this with Table 2 (below), which represents the summary statistics for Czech government bonds.

	Mean	Standard deviation	25th percentile	Median	75th percentile
Price (%)	95.69	14.13	88.57	97.69	100.82
Yield (%)	3.16	2.04	1.15	3.65	4.92
Coupon rate (%)	1.91	1.89	0.10	1.25	2.75
Time-to-maturity	6.67	7.53	1.70	4.88	8.52
Duration	5.62	4.86	1.70	4.68	8.02
Modified duration	5.44	4.71	1.66	4.54	7.72
Convexity	114.34	260.93	4.53	29.64	89.25

Table 2 – Summary statistics (Czech government bonds)

Source: The author's computation, sources for each variable listed above.

Table 2 was computed from a total of 385 observations. Based on these summary statistics, Czech bonds have a lower average price and higher average yield compared to the entire dataset. This indicates that they may be offering higher returns with lower pricing. The coupon rate, with a higher mean, suggests a wider range of rates but with a concentration of lower coupon rates. Duration and modified duration are lower on average, with less variation, indicating a generally less interest rate sensitive set of bonds. Lastly, the lower average convexity suggests that Czech bonds have a less pronounced curvature in the price-yield relationship. Overall, higher yields, lower prices, and lower variability in duration and convexity compared to the broader dataset could indicate a more stable bond market in the Czech Republic.

	Mean	Standard deviation	25th percentile	Median	75th percentile
Price (%)	105.35	20.90	97.86	100.48	105.35
Yield (%)	0.70	1.62	-0.64	-0.01	2.29
Coupon rate (%)	1.11	1.78	0.00	0.00	1.70
Time-to-maturity	6.51	7.54	0.98	3.88	8.38
Duration	6.01	6.76	0.98	3.83	8.13
Modified duration	5.96	6.70	0.99	3.81	8.15
Convexity	112.43	257.27	1.96	18.85	79.36

Table 3 – Summary statistics (German government bonds)

Source: The author’s computation, sources for each variable listed above.

Our dataset consists of total 1,227 observations of German government bonds. These bonds exhibit a higher average price and lower average yield compared to both overall dataset and the Czech bonds. Additionally, the median yield of German bonds is negative, reflecting various factor such as ECB monetary policies, demand for safe assets, inflation expectations, and market and global conditions. Moreover, the median coupon rate is zero, indicating a significant proportion of disclosed bonds are zero-coupon. The average time-to-maturity is similar to that of the entire dataset, with slightly less variability. Furthermore, convexity is even lower than that of the Czech bonds, on average, indicating less sensitivity to interest rate changes. Overall, German bonds exhibit a market characterized by higher stability and lower returns.

	Mean	Standard deviation	25th percentile	Median	75th percentile
Price (%)	100.69	13.12	95.83	99.89	103.72
Yield (%)	2.11	1.98	0.26	1.87	4.04
Coupon rate (%)	1.78	1.42	0.50	1.75	2.63
Time-to-maturity	6.80	8.40	1.09	3.25	7.05
Duration	5.90	6.80	1.10	3.18	6.78
Modified duration	5.78	6.65	1.08	3.11	6.70
Convexity	139.92	278.70	2.26	13.66	57.41

Table 4 – Summary statistics (US government bonds)

Source: The author's computation, sources for each variable listed above.

Table 4 presents summary statistics for US government bonds from our dataset, comprising 5,972 observations, which represents 78% of our complete dataset. Trends and characteristics observed in US bonds closely mirror those in Table 1, reflecting their significant representation. US government bonds exhibit a slightly lower average price compared to German bonds and have higher average prices compared to Czech bonds. Similarly, the average US yield and coupon rate fall between the other two countries, indicating a balanced position. While the duration and modified duration of US government bonds suggest similar sensitivities to interest rate changes, their convexity is the highest out of the three, hinting at slightly higher risk associated with US bonds. The average time-to-maturity for US government bonds is comparable to that of German and Czech bonds, indicating similar maturity profiles across the three groups. Same applies to duration and modified duration, suggesting comparable sensitivity.

As we provided the statistical description of our dataset, in the next sub-chapter we closely analyse some of data patterns such as variable changes over time, the relationship between variables and comparative analysis between countries.

4.2.3. Data analysis

The last part of the Data chapter provides an analysis of the most important patterns and trends of our dataset, we try to describe variables behaviour and how it can affect the results in the upcoming part of this thesis.

Firstly, Figure 8 illustrates the price development of three long-term bonds, one from each country: a Czech 50Y SSD, maturing in 2057, a US 30Y bond maturing in 2040, and a

German 30Y IBL maturing in 2046. Overall, all three bonds exhibited an initial period of growth until late 2020 or early 2021. The Czech bond reached the highest price nearly 173%. It depends on when the bond was originally issued and what was the coupon rate. This was further driven primarily by aggressive monetary easing policies implemented by central banks in response to the COVID-19 pandemic in 2020. This initial growth was followed by a significant decline throughout 2022, with the Czech bond experiencing the steepest drop, falling to around 90 percentage points of its original face value. This decline was largely due to rising inflation and policy interest rate hikes in order to curb inflation. Additionally, geopolitical risks heightened market volatility, further impacting bond prices. As new economic conditions and policies took effect, the bond prices showed some recovery in 2023. However, they did not return to their initial peaks, stabilizing around 100-110% of their face value. The graph underscores the sensitivity of long-term bond prices to macroeconomic policies, illustrating the broader impacts of these factors on global financial markets.

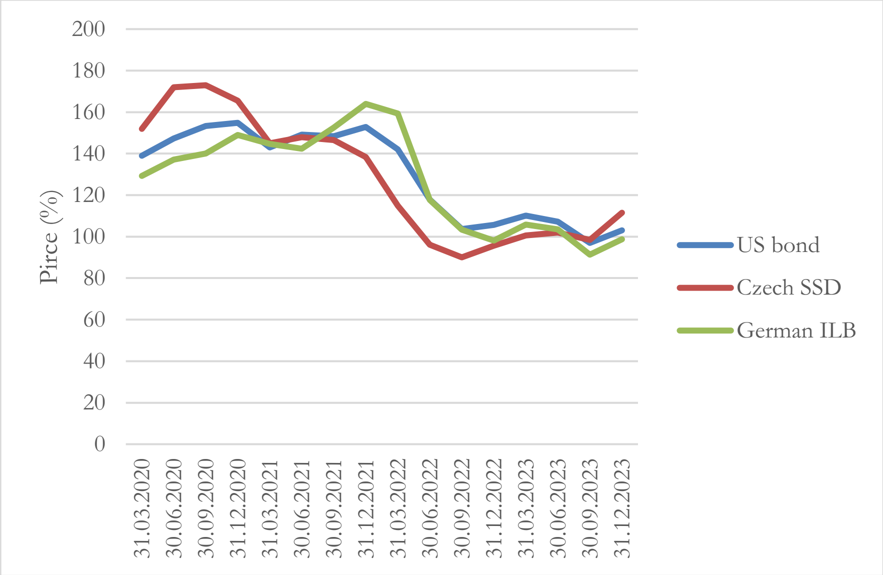

Figure 8 – Price development of long-term government bonds

Source: Author's computation from Refinitiv Eikon (2024) database

Further, Figure 9 illustrates the price development of short-term bonds over our studied period, with each bond maturing annually in January. All bonds maintained strong and stable prices throughout 2020 and the first half of 2021, with prices ranging between 99.5% to 100.5% of their face value due to supportive monetary policies. However, starting in late 2021, bond prices began to decline sharply, with the Czech short-term bonds falling below 98%. Once again, this decline was primarily driven by rising inflation – higher required returns. German Bubils resisted the price fall for a longer period but eventually dropped below 98% at the beginning of 2023. Meanwhile, US bills reached the general lowest point just above 96% during the same period. As inflationary pressures eased and markets adjusted to the new policy interest rate environment by the end of 2023, this was reflected in the bond prices, which rebounded to near their original face values, approaching 100%. The improved economic conditions demonstrated the effectiveness of central banks' policies.

Overall, the variation in prices is significantly lower compared to Figure 8, depicting long-term bonds. This lower variation is mainly due to the expected interest rate sensitivity and lower duration of short-term bonds, as their future cash flows are discounted over a shorter period (additionally, these bonds have zero coupon rates). Other factors, such as lower reinvestment risk or greater resilience in volatile economic environments, also contribute to these differences.

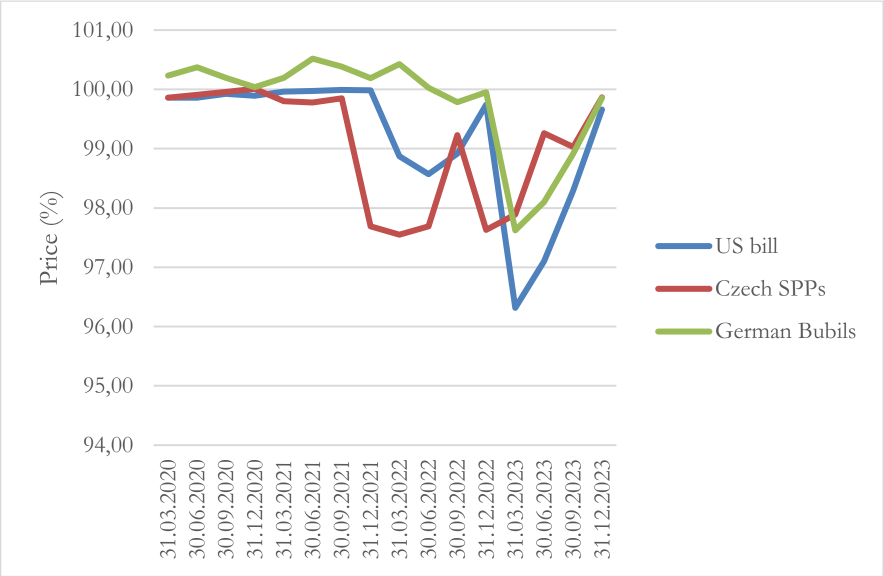

Figure 9 – Price development of short-term government bonds

Source: Author's computation from Refinitiv Eikon (2024) database

In Figure 10 below, we plotted prices against yields to explore their relationship in a dataset containing over 7,000 observations, which could be complex to visualise comprehensively. Therefore, we randomly selected 400 and plotted this sample. The plot reveals a noticeable negative correlation between the two variables, as expected – higher bond yields generally correspond to lower bond prices. However, the data points also show a considerable dispersion, suggesting that factors beyond yields influence bond prices. These factors include maturity, country-specific conditions, changes in interest rates, and broader economic sentiment. Notably, some bond yields in our dataset even turned negative, mirroring trends observed in the German yield curve. This phenomenon was influenced by actions like liquidity injections by the European Central Bank and high demand for safe-haven assets amidst economic uncertainty following the COVID-19 pandemic and concerns over deflation.

In summary, the price analysis revealed the effect of policies on bond prices across different maturities and countries. Long-term bonds showed significant fluctuations in prices, while short-term exhibited greater stability despite economic volatility. Figure 10 highlihts the inverse relationship between prices and yields, underscoring the complexities in navigating global bond markets.

Figure 10 – Relationship between bond's price and yield

Source: Author's computation from Refinitiv Eikon (2024) database

Moreover, we delve into the distribution of coupon rates, focusing on key metrics such as spread, central tendency, outliers etc. This analysis offers valuable insights into the development of coupon rates over our studied period. It is important to note we excluded short-term bonds from our sample due to their zero-coupon nature, which could skew our results. However, even among medium and long-term bonds, we observe instances of zero-coupon rates, as illustrated in Figure 11.

From 2020 to 2022, there is a noticeable downward trend in the median coupon rates, falling from 1.88% to 1.38%, suggesting a general decline in the coupon rates offered on new bond issuances due to low benchmark policy interest rates or increased demand for bonds. Notably, Czech government bonds had fewer issuances compared to German and US bonds, which dominated the dataset due to higher demand for safer investments. The slight increase in the median to 1.75% in 2023 indicates a shift in market conditions, likely due to higher inflation expectations and changes in monetary policies. Throughout the period, zero-coupon bonds are consistently present, as indicated by the lower whisker reaching 0% each year (in the case of zero-coupon bonds, we account for them to be sold with a discount which reflects their yield). Each year shows a significant rise in coupon rates, with a notable number of outliers, especially in the first periods, reaching as high as 8.75% in 2020, 8% in 2021 and 7.63% in 2022, among long-term maturing bonds. Despite the general trends, there were still issuances with higher coupon rates. All four means are higher than the median, implying a right-skewed distribution. The interquartile rate (IQR) provides further insights, extending from 1%-2.63% in 2020 to 0.38%-3% in 2023, reflecting greater variability. These trends and diversity provide a comprehensive view of government bond market dynamics during 2020-2023.

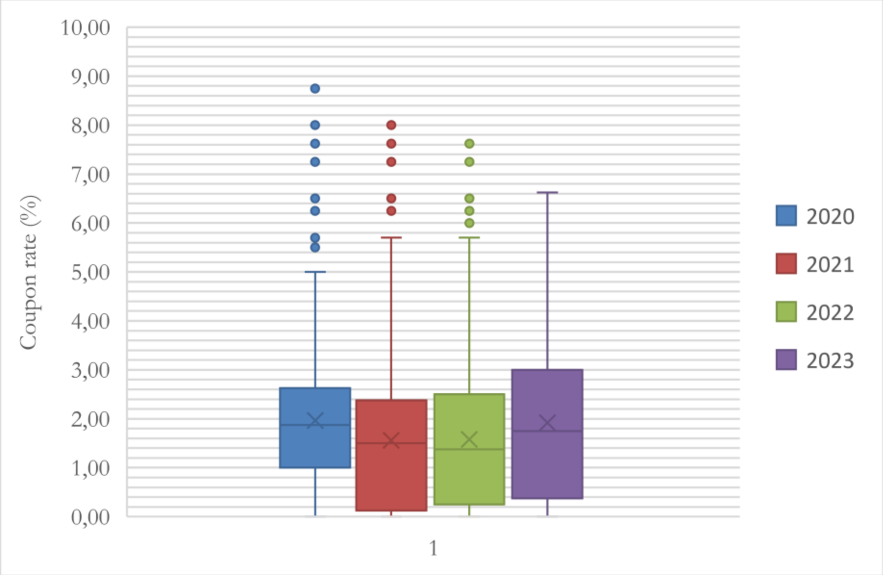

Figure 11 – Coupon rate distribution (ST bond excluded)

Source: Author's computation, based on multiple sources (2024)

The last two graphs in this chapter focus on the duration. Firstly, Figure 12 shows a relationship between time-to-maturity and bonds' duration. We have excluded the ST bonds (maturing in less than a year) for better visualization and clarity. The plot demonstrates a positive correlation between these variables, as time-to-maturity increases, the duration generally increases as well. This is supported by the linear trend line added, consistent with financial theory. Interestingly, for some data points show duration higher than their time-to-maturity, which can be explained by previously mentioned negative yields that prolong the repayment period for investors. Despite these anomalies, most observations show that duration is lower than time-to-maturity, as evidenced by the trend line's slope being less than 45 degrees. Additionally, there are significant deviations from the general trend, notably among Czech LT bonds with an original maturity of 50 years. Lastly, the spread observed in data points with similar maturities indicates heterogeneity, which needs to be eventually treated before performing estimations. For more details on dealing with this heterogeneity, see Chapter 6.

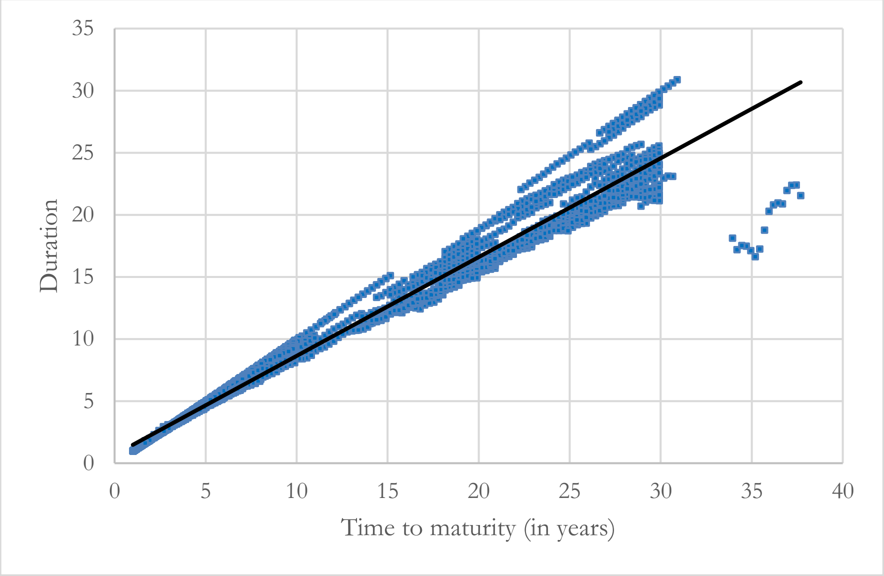

Figure 12 – Scatter plot time-to-maturity vs duration

Source: Author's computation, based on multiple sources (2024)

The last graph in Chapter 4, Figure 13 below, depicts the relationship between a bond's duration and convexity. As with Figure 11, we were not able to project all observations due to dataset size constraints. Therefore, we randomly selected a representative sample of 400 observations for better visualization and clarity. The graph reveals a positive and increasingly convex relationship between duration and convexity. Convexity tends to increase at an accelerating rate as duration increases. This indicates that a bond's sensitivity to interest rate changes grows more rapidly as its duration becomes longer. This accelerating sensitivity, known as convexity, is a critical factor in assessing the interest rate risk of bonds. Furthermore, the spread in data points indicates variability in convexity for bonds with similar duration. This variability can be attributed to differences in coupon rates, yields and other bond-specific characteristics and periods. This plot aids in comprehending the intricate dynamics of bond pricing and interest rate risk. For a detailed estimation analysis of how convexity directly impacts price changes, please refer to Chapter 6.

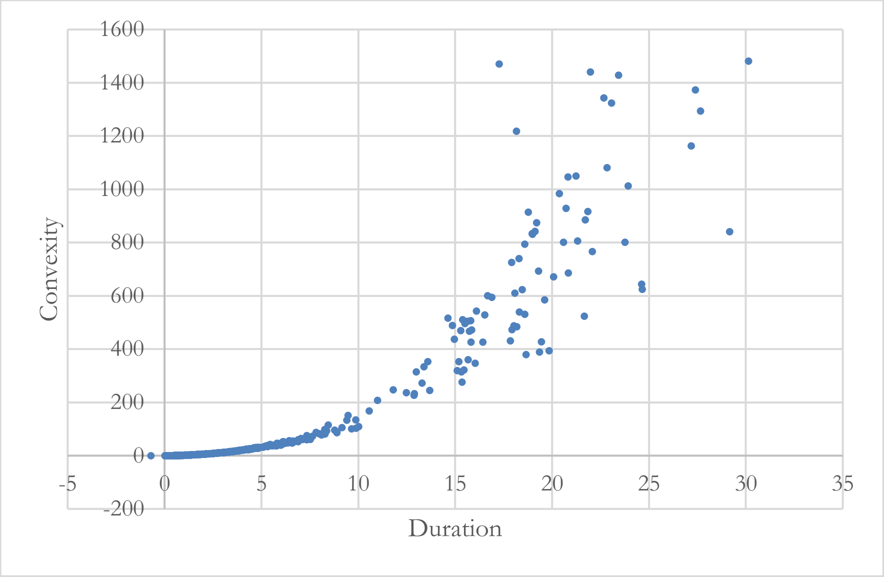

Figure 13 – Scatter plot duration vs convexity

Source: Author’s computation, based on multiple sources (2024)

In this extensive data chapter, we first motivated this study by discussing how increasing inflation reflected changes in policy rates set by central banks, directly affecting government bond yield curves. We further introduce the variables used in our regressions, presented their summary statistics, and provide numerous graphs to inspect the behaviour of our observations. We believe that the unprecedented shifts in bond yields have significantly impacted bond prices. Therefore, in the upcoming chapters, we will closely examine whether duration metrics still hold as relevant measures for assessing interest rate risk.

5. Methodology

In the previous chapter, we introduced the dataset used to estimate whether the duration metrics held from 2020 to 2023, during periods of changing interest rates resulted in fluctuations in government bond yields and, consequently, their respective prices. This chapter specifies the concrete equations used in our models, details the estimation methods employed and the rationale behind their selection, and explains the statistical tests performed to ensure model fitness. Lastly, we propose our hypotheses.

5.1. Model

Duration is one of the most common features used by investors to determine interest rate risk. Therefore, it is crucial to assess if it has remained a reliable approach in recent years. As indicated in Chapter 4, our collected data is in the form of panel data. The general regression model is as described below:

$$y_{it} = \beta_o + \beta_1 x_{1it} + \cdots + \beta_n x_{nit} + a_i + u_{it}; \tag{12}$$

where represents the outcome variable (in our case, change in the price of bond *i* in period *t+1*, as we work with time differences, period *t* is not utilised as a sole period), further x_{1it} to x_{nit} stand for explanatory variables, a_i represents the time-invariant fixed-effect of each government bond, and u_{it} symbolizes the idiosyncratic error (time-variant and individual-specific). Enumerating Equation 12 in the first version of the model, it is expressed as:

$$\Delta P_{i,t+1} = \beta_0 + \beta_1 (P_{i,t} * \Delta r_{i,t+1} * D_{mod;i,t}) + a_i + u_{it} \tag{13}$$

To comply with the duration metrics, β should equal -1. In other words, as the yield of the bond changes by one point, its respective price should change in the opposite direction by one point for each year of the modified duration. In Chapter 2, however, we explain that convexity plays a significant role in this dynamic, meaning the magnitude of price movement differs at various yield levels. Despite investors typically not accounting for convexity (due to complicated calculations and previously stable interest rates), we introduce a second model, expressed as in Equation 14:

$$\Delta P_{i,t+1} = \beta_0 + \beta_1 (P_{i,t} * \Delta r_{i,t+1} * D_{mod;i,t}) + \beta_2 (P_{i,t} * K_{i,t} * \Delta r_{i,t+1}^2) + a_i + u_{i,t} \tag{14}$$

Even after expanding the estimation equation, we still believe, based on theory, β_1 should equal -1, and the results should be even more precise. Regarding the parameter β_2, theoretically, it should equal 0.5, assuming price changes in positive and negative direction are included. However, given that our dataset and studied period mainly exhibit one-directional development, we cannot express this parameter to be precise. Furthermore, the added convexity term is intended to confirm or reject the precision of duration metrics and is not the primary focus of our study.

5.2. Estimation Method

To provide relevant, unbiased, and precise results from our proposed models, we have considered several panel data estimation techniques: Pooled OLS, Fixed Effects and Random Effects estimators (Wooldridge, 2010).

The first mentioned, the Pooled OLS method, treats all observations as part of a single cross-sectional dataset and does not account for potential correlations within entities over time. Ignoring these time or individual effects can lead to omitted variable bias, especially if these effects significantly influence the dependent variable. Conversely, the Fixed Effects (FE) model accounts for individual-specific effects by allowing each entity to have its own intercept. To determine the suitability of Pooled OLS, we performed the Breusch-Pagan Lagrange Multiplier Test. Under the null hypothesis, no individual-specific effects are present, indicating that the pooled OLS method is appropriate. However, in all model variations, we rejected this null hypothesis, indicating the presence of individual-specific effects and necessitating a decision between the FE and Random Effects (RE) models.

The Random-effects model addresses unobserved effects by introducing a random intercept for each entity. If the individual effects are uncorrelated with the explanatory variable, the RE model provides more efficient estimates than the FE model because it utilizes both within-entity and between-entity variation. To compare the RE model with the FE model and check for a correlation between individual effects and explanatory variable, we conducted the Hausman test. The null hypothesis of the Hausman test states that individual effects are uncorrelated with the explanatory variable, making the RE model preferred. However, across all model variants, we consistently reject the null hypothesis, indicating that individual effects are correlated with the explanatory variable, thus favouring the FE model.

Based on this statistical evidence, we proceed with the Fixed Effects model. This choice ensures the consistency and reliability of our estimates, crucial for accurately understanding bond duration. By controlling for time-invariant characteristics, reducing omitted variable bias, and being robust to unobserved heterogeneity, the FE model provides a solid foundation for our analysis. Nonetheless, we must follow the assumptions listed below to obtain reliable estimates and address potential endogeneity concerns using additional methods such as Two-Stage Least Squares (2SLS) (Greene, 2012).

5.3. Assumptions

While applying the estimation methods and interpreting the results, it is crucial to ensure that all model assumptions are satisfied. This ensures the integrity of variables and supports reliable estimation of coefficients. To achieve this, we identify and list all relevant assumptions, and detail how each one is verified and upheld.

Random sample

As previously discussed at the beginning of Chapter 4, we initially planned on including the whole population instead of a sample. However due to data availability a negligible share of bonds could not be included due to data availability. We included all the possible observations from the period 2020 to 2023, and thus we can consider our sample rather full population as we were not sampling any data.

No multicollinearity

In our analysis, we assess the presence of multicollinearity, which occurs when independent variables in a regression model are highly correlated. In Equation 13 of our model, only one independent variable, a combination of price, change in yield, and modified duration, is included, mitigating concerns of multicollinearity that would arise if these effects were studied separately. However, in Equation 14, two such combinations are used as independents variables. To evaluate multicollinearity, we calculated the correlation between these variables, resulting in a coefficient of 0.57. Multicollinearity is typically flagged by correlations exceeding 0.7 or 0.8. Given our correlation falls below this threshold, we conclude that multicollinearity is not a significant concern in our estimations.

Homoskedasticity

Moreover, the assumption ensures that the variance of error terms remains constant across entities and over time. Heteroskedasticity, which occurs when the variance of residuals varies systematically, violates this assumption and undermines the efficiency of estimation. To detect potential heteroskedasticity in our models, we conduct the Breusch-Pagan test for each estimation. Under the null hypothesis of the test, the variance of the residuals is constant, validating the assumption of homoskedasticity. However, rejection of the null hypothesis indicates the presence of heteroskedasticity, necessitating corrective measures. To address this issue, we employ serial correlation robust error when applicable. This approach helps ensure reliable estimates of the coefficients and their standard errors despite the presence of heteroskedasticity.

No serial correlation

An important assumption in our models is that the error terms are not serially correlated over time for each entity. Serial correlation would violate the assumption of independence, which is crucial for the validity of statistical tests and confidence intervals. To check for serial correlation, we perform the Breusch-Godfrey test. Under the null hypothesis, there is no serial correlation in the residuals. If we fail to reject the null hypothesis, we conclude that serial correlation is not present, and we proceed accordingly. Conversely, if we reject the null hypothesis, we apply the corrective measures such as using robust standard errors that adjust for serial correlation. By conducting this test, we gain insight into the temporal dependencies in the residuals and ensure the reliability of our estimates.

Normality

The normality assumption is generally not strictly necessary in the context of panel data analysis, as long as the coefficient estimates are unbiased and consistent. It becomes more relevant for inference purposes. If the errors are not normally distributed but the sample size is large, the Central Limit Theorem implies that these tests are asymptotically valid (Hsiao, 2022). Therefore, we perform the Anderson-Darling normality test. If the null hypothesis that the data follow a normal distribution is rejected and our sample can be considered not large (we set a threshold of 100 observations), we correct for this issue by implementing robust standard errors. While normality is not a critical assumption, checking the distribution of residuals remains a good practice.

Strict exogeneity

Lastly, the assumption of no endogeneity is one of the most crucial and easily violated assumptions in regression analysis. This assumption requires that the independent variables in our models are not correlated with their respective residuals. If endogeneity is present, the estimated coefficients are biased and inconsistent. As we described earlier, we will not be using the Pooled OLS method. Based on Hausman test results we chose fixed effects over random effects. This model accounts for the time-invariant characteristics of studied entities. By controlling for these fixed characteristics, the model reduces bias from omitted variables that are constant over time.

Furthermore, a critic could argue that exogeneity cannot be guaranteed since price variables enter both the dependent and independent variable parts of our models. We believe this issue is mitigated by the fixed effects estimator, as mentioned above. However, to support this claim, we include a robustness check in our result by estimating the same models using the Two-Stage Least Squares (2SLS) method, employing an instrumental variable to address endogeneity. For these instrumental variables, we use the price variable lagged in time. The first stage regression is as follows:

$$P_{i,t} * \Delta r_{i,t+1} * D_{mod;i,t} = \beta_0 + \beta_1 \left(P_{i,t-1} * \Delta r_{i,t+1} * D_{mod;i,t}\right) + \eta_{i,t} \qquad (15)$$

where $P_{i,t}$ stands for the lagged price in time, and $\eta_{i,t+1}$ stands for the error term of the first stage regression. The second stage is represented as follows:

$$\Delta P_{i,t+1} = \beta_0 + \beta_1 \left(\widehat{P_{\imath,t}} * \widehat{\Delta r_{\imath,t+1}} * \widehat{D_{mod;\imath,t}}\right) + a_i + u_{i,t} \qquad (16)$$

Where $\widehat{P_{\imath,t}} * \widehat{\Delta r_{\imath,t+1}} * \widehat{D_{mod;\imath,t}}$ are the predicted values from the first stage regression. In the same manner as we adjusted Equation 13, we apply the 2SLS method to Equation 14 as well. By using this method, the obtained results should be consistent and unbiased even in the presence of endogeneity.

5.4. Hypotheses

Before presenting the results of the estimations, let us state the hypotheses we have set. These hypotheses are based on the theoretical framework introduced so far in this thesis. By systematically testing these hypotheses, we seek to provide robust insights into the implications of government bond duration on price changes. We aim to address the key research questions concerning the government bond duration.

Hypothesis #1: Government bond duration is a relevant metric in times of changing interest rates. Specifically, for a dataset containing all three studied countries – the Czech Republic, the USA and Germany. If the yield changes by 1 percentage point, the respective bond price changes by 1 percentage point in the opposite direction for each year of the modified duration.

Hypothesis #2: Government bond duration is a more precise measure when accounting for convexity. We believe that, despite unprecedented volatility in bond yields, the theory holds and becomes more precise when convexity is included in the measures.

Hypothesis #3: The duration metric is not precise for short-term government bonds. As the durations of these bonds are quite low, sometimes even negative, we believe the coefficients will not be one and will be rather volatile across the studied countries.

Hypothesis #4: The accuracy of the duration metric varies at different levels of policy rates, and thus yields. Historically, duration has been an accurate measure during periods of fairly stable and low policy rates. We assume the coefficients will vary each studied year as policy rates move upwards.

These hypotheses are central to our analysis and will be tested using the fixed-effects and 2SLS methods to account for potential endogeneity issue. In the next chapter, the findings will help us to determine whether our proposed hypotheses are confirmed or rejected.

6. Empirical analysis and results

Having described our data sources and methodological approach for assessing the reliability of duration metrics in predicting the interest rate risks during periods of varying inflation, we now present the empirical results. Our analysis begins by examining government bonds categorized by their country of origin, exploring how duration varied over the study period before aggregating these findings utilizing the fixed-effects model detailed in Chapter 5 ensures robust analysis. Additionally, we conduct a thorough robustness check employing the TSLS method to address potential endogeneity concerns, and further propose and estimate two alternatives to our models. This chapter concludes by evaluating our hypotheses and demonstrating the achievement of our research objective.

Given that our estimations involve time differences, the number of observations in our samples is naturally reduced as we exclude the initial period (zero) of each bond issue. Furthermore, we will present two tables for each sub-sample: one with an equation including only modified duration (as denoted in Equation 13) and another with an equation including modified duration along with the convexity measure (as explained in Equation 14). This separation allows for clearer differentiation between the linear (duration) and nonlinear (convexity) components of interest rate sensitivity. It enhances clarity, facilitates a more comprehensive explanation, and supports comparison between models.

6.1. Czech government bonds

Firstly, we present the results for Czech government bonds. Table 5 below displays the regression outcome of the quarterly price change on the bond yield change for each year of its respective duration. The first column represents the regression results for the entire study period, followed by columns showing the results for each year studied separately.

Based on the presented results, the β_1 for the Czech SDDs is -0.929. Compared to the expected -1, we can state that the duration metrics are quite precise in determining the interest rate sensitivity during the studied years for Czech medium and long-term government bonds. Furthermore, the coefficients for all presented versions are negative and highly significant. The magnitude of the coefficients suggests that the bond's price change is slightly less sensitive to yield changes in 2021 and 2022 compared to 2020 and 2023. This outcome suggests that increases in the central bank's policy interest rates might have played a significant role, as during 2021-2022 the CNB gradually increased its rates as a reaction to the inflation developments. We expected otherwise but it turned out that rate increases did not affect the prices to be more sensitive but showed higher resilience.On the other hand, the policy rates were rather stable in 2020 and 2023 (see Figure 2).

Table 5 – Fixed effects estimation, Czech SDDs

			ΔP		
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.929*****	-1.101***	**-0.864*****	-0.845***	**-1.113*****
	(0.013)	(0.027)	**(0.009)**	(0.024)	**(0.023)**
Observations	**321**	58	**82**	84	**97**
Adjusted R^2	**0.945**	0.990	**0.992**	0.938	**0.893**
Note:					$^{*}p^{**}p^{***}p<0.01$

In 2020, the reported coefficient is -1.101, suggesting a rather unstable market, with the coefficient varying from the desired -1 more than in overall. If the coefficient is higher than the expected -1, it indicates that the bond's price is more sensitive to changes in yield than predicted. Specifically, a change of policy rate and thus in yield results in a change in the bond's price, which is more than expected by investors. In a low-interest-rate environment, bond prices shall not react as strongly to small changes in yield. In 2021 and 2022, changes in the economic environment, inflation expectations and monetary policies did not heighten the bond's sensitivity to yield changes. In 2023, the value returned to almost the same level as in 2020. All estimations recorded quite high adjusted R-squared, indicating a good model fit. Overall, the relationship between bond price changes and yield changes remains strong, and the degree of sensitivity varies slightly. So far, we have assumed (as it is mostly by investors for simplicity) a linear relationship between the variables. However, this is not always the case, as discussed in the previous parts of this thesis. Therefore, in Table 6 below, we introduce the convexity into the estimations and compare the results to those in Table 5.

Based on the presented result, the β_1 coefficient increased (in absolute values) and moved closer to the -1 value throughout the studied period when convexity was included. This suggests that Czech bonds indeed exhibit a nonlinear relationship, as expected, and that duration metrics provide sufficient evidence to measure interest rate risk, even in periods of varying interest rates. Comparing the yearly results with Table 5, all yearly coefficients moved closer to the desired value. This adjustment indicates that when accounting for convexity, the reported interest rate sensitivity is higher and more accurate. More interestingly, in 2020, when the highest changes in policy rates occurred in the Czech Republic, the coefficient of modified duration was almost perfect with a value of -0.994, demonstrating a very precise outcome. In 2023, even though the duration coefficient was -1.068, it is still sufficiently close to -1, considering the significant macroeconomic events that transpired in prior periods.

Table 6 – Fixed effects estimation, Czech SDD’s incl. convexity

			ΔP		
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-1.014***** **(0.012)**	-1.024*** (0.007)	**-0.961***** **(0.004)**	-0.994*** (0.037)	**-1.068***** **(0.030)**
$P * C * \Delta r^2$	**0.464***** **(0.038)**	0.636*** (0.021)	**0.493***** **(0.014)**	0.415*** (0.086)	**0.320***** **(0.085)**
Observations	**321**	58	**82**	84	**97**
Adjusted R^2	**0.964**	0.999	**0.998**	0.955	**0.895**
Note:					*p**p***p<0.01

Before we elaborate on the convexity part of the equation, let us first remind ourselves what value we expect to obtain from the β_2 coefficient. As explained in Chapter 2, convexity is the measure of the second derivative of the bond price with respect to yield. Therefore, the coefficient for a multiple of price, convexity, and yield squared shall be around 0.5, as the quadratic term involves halving the effect to account for the curvature over a yield change. However, our primary aim is to examine the duration behaviour, with convexity added solely as a specification rather than the main focus of the study.

Based on Table 6, all convexity coefficients turned out to be positive and statistically significant, varying between 0.32 and 0.636. Despite the larger variation compared to duration coefficients, the presented results are considered rather satisfying too. Especially in 2021, when the coefficient is almost perfect at 0.5, again surprisingly as this year began with the upward shifts of policy rates, and therefore government bond yield. The significance of these coefficients underscores the importance of considering both duration and convexity when analyzing government price movements. Yet as our yearly samples are quite lower in magnitude, we will see in German and US samples if these outcomes hold too.

Furthermore, we also estimated the same regression models using the Czech SPPs, short-term government bonds. However, as their sample was already quite small (i.e., 30 observations), we only regressed these as one period. We would implement at least treatment variables for years to see the annual effects, however, this approach is not feasible since we use the fixed effects.

Table 7 – Fixed effects estimations, Czech SPPs

	ΔP	
$P * D_{mod} * \Delta r$	-1.937^{***} (0.002)	-3.067^{***} (0.001)
$P * C * \Delta r^2$		42.824^{***} (0.029)
Observations	17	17
Adjusted R^2	0.813	0.998
Note:		$^{*}p^{**}p^{***}p<0.01$

Based on Table 7, we can assert that duration metrics are not an accurate measure of interest rate risk for Czech short-term government bonds. The regression results indicate that when yields change by 1%, bond prices move in the opposite direction by 2% or 3%, indicating higher sensitivity than duration would predict. This finding is significant, as relying solely on duration for short-term bonds could potentially lead to substantial losses during periods of high bond yields. When accounting for convexity, the duration coefficients further decrease, reaching values up to -3.067. In contrast to the results from long-term bonds, the convexity coefficient is substantially above the expected 0.5. This high value suggests a pronounced curvature in the price-yield relationship, indicating that for larger yield changes, government bond price adjustments are significantly impacted by convexity. It is important to note that after excluding the initial period of each bond, we are left with only 17 observations. Consequently, drawing broad conclusions for the population over an extended time span may be misleading. To conclude the analysis of the Czech sub-sample, we will re-estimate the data, combining both SDDs and SPPs into a single sample.

Table 8 below presents the results for the entire Czech dataset. As mentioned earlier, since the volume of Czech short-term bonds is more than 18 times lower than that of the SDDs sample, we did not expect significant differences compared to Table 6. We include only the regressions that account for convexity, as the analysis in Table 6 demonstrated its significant role in explaining the relationship between yield changes and price movements.

Table 8 – Fixed effects estimation, Czech government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-1.014***** **(0.012)**	-1.024*** (0.007)	**-0.961***** **(0.004)**	-0.994*** (0.036)	**-1.068***** **(0.030)**
$P * C * \Delta r^2$	**0.464***** **(0.037)**	0.636*** (0.021)	**0.493***** **(0.014)**	0.416*** (0.084)	**0.320***** **(0.085)**
Observations	**338**	61	**89**	87	**101**
Adjusted R^2	**0.962**	0.999	**0.998**	0.955	**0.891**
Note:					*p**p***p<0.01

Comparing the results to those in Table 6, we observe that the coefficients are nearly identical, underscoring the consistency of our findings. Minor variations in the standard errors may reflect fluctuations in short-term bond performance. To conclude, the aggregate results corroborate our previous findings. The near -1 duration coefficients indicate that duration metrics are generally accurate for Czech government bonds, but the significant and negative convexity coefficients emphasize their importance in the analysis. The high adjusted R-squared values across the models support the robustness of the theoretical frameworks, providing confidence in their practical implications for investors.

6.2. German government bonds

Building on our analysis of Czech government bonds, we now turn our attention to evaluating the accuracy of duration metrics in measuring interest rate risks for German government bonds. This investigation follows a structured approach similar to the previous sub-chapter. We begin by examining the results for long-term bonds, followed by medium-term and short-term bonds, and conclude with aggregate findings.

Table 9 below presents the results of estimations for German Bunds, long-term government bonds. The coefficients obtained closely resemble those reported in Table 5 for Czech SDDs. The overall coefficient across the four years is -0.867, varying more than in the Czech case, suggesting the importance of the exploration of yearly trends. In 2020, the duration coefficient was -1.008, falling through 2020 and 2022, reaching -0.707, a quite notable variation, yet with sensitivity largely below what we would initially expect. Notably, this increase does not align with the period when the ECB maintained near-zero policy interest rates until mid-2022 (refer to Figure 3). Subsequently, the coefficient in 2023 returned to values slightly above -1. These findings affirm that the sole duration metric could not be used during this period as a reliable metric of interest rate risk for German Bunds. Future analyses will incorporate convexity to further refine our understanding.

Table 9 – Fixed effects estimation, German Bunds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
P * Dmod * Δr	**-0.867*****	-1.008***	**-0.908*****	-0.707***	**-1.021*****
	(0.015)	(0.007)	**(0.010)**	(0.024)	**(0.006)**
Observations	**636**	122	**167**	171	**176**
Adjusted R^2	**0.969**	0.984	**0.997**	0.959	**0.981**
Note:					*p**p***p<0.01

After presenting the estimation results including the convexity measure in Table 10 below, we observe that the duration coefficients approach the desired -1 value even more closely. Particularly notable is the coefficient for 2021 and 2022, which were -1.010. This suggests that duration metrics became increasingly reliable for German Bunds, especially when ECB policy rates started to alter. On the other hand, in 2020, with many bonds reporting even negative yields, the coefficient varied the most, accompanied by the highest standard errors among the years studied, but still sufficiently close to the desired value. Turning to the convexity coefficients, all years reported statistically significant and positive results. They differed rather significantly, until 2023 when reporting a value of 0.742. At a low interest rate environment, the present value of future cash flows becomes more sensitive to changes. As we have seen in the Data chapter, yield curves can become flatter or even inverted, and market expectations about future changes can be more volatile. Even small changes can result in significant price changes due to the shape of the price-yield curve. Overall, these findings augmented by convexity, provide a robust measure of interest rate risk for German Bunds.

In our analysis, we further present the results for medium-term bonds, specifically Bobls and Green bonds. This time, we include the convexity measure from the outset, as our previous analyses have demonstrated its significant impact on changes in government bond prices. This approach also enhances the clarity of this chapter and avoids unnecessary repetition.

Table 10 – Fixed effects estimation, German Bunds, incl. convexity

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.995*****	-0.940***	**-1.010*****	-1.009***	**-1.019*****
	(0.003)	(0.022)	**(0.003)**	(0.015)	**(0.002)**
$P * C * \Delta r^2$	**0.959*****	2.685***	**1.354*****	0.974***	**0.742*****
	(0.027)	(0.792)	**(0.046)**	(0.044)	**(0.025)**
Observations	**636**	122	**167**	171	**176**
Adjusted R^2	**0.994**	0.985	**1.000**	0.995	**1.000**
Note:					*p**p***p<0.01

Based on Table 11, we observe that medium-term bonds exhibit behaviour nearly equivalent to that of long-term bonds, except for 2020. In this case, the reported coefficient is -1.1, suggesting a higher sensitivity in prices in the low-interest rate environment. The rest of the years vary from the long-term coefficients by not more than 0.01. These outcomes provide robust evidence that the duration metric is an accurate measure of interest rate risk, with the most heightened sensitivity in 2020, where the values still attain the desired -1. Regarding convexity, the coefficients are generally statistically significant and positive, reinforcing the importance of its inclusion in the analysis. The convexity coefficient for 2020 is notably higher in magnitude compared to other years, indicating a strong curvature in a price-yield relationship. The persistent positive values of convexity suggest adjustments in bond prices in line with what duration would predict. This behaviour will be explored in greater detail in the upcoming chapter.

Table 11 – Fixed effects estimation, German medium-term government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-1.012*****	-1.102***	**-1.017*****	-0.996***	**-1.022*****
	(0.010)	(0.062)	**(0.001)**	(0.037)	**(0.003)**
$P * C * \Delta r^2$	**0.988*****	5.905	**0.844*****	0.951***	**0.750*****
	(0.030)	(14.054)	**(0.017)**	(0.092)	**(0.024)**
Observations	**205**	31	**52**	58	**64**
Adjusted R^2	**0.987**	0.959	**0.999**	0.983	**0.999**
Note:					*p**p***p<0.01

Moving to Table 12 and the analysis of short-term government bonds, the results provide different outcomes, as predicted based on values for Czech ST bonds. The duration coefficients notably deviate from the desired -1 value. The overall duration coefficient is -1.324, which is significantly higher compared to that of German LT bonds. For the individual years, the coefficients vary, with the highest deviation in 2022 at -2.383, indicating exceptionally high sensitivity to yield changes. In contrast, the coefficients for 2020 and 2021 are closer to -1 and almost the same as in the case of medium-term government bonds. Regarding convexity, the results are quite mixed. Notably, only 2022 presents a positive and statistically significant coefficient. The rest are negative, and the overall or 2023 even not statistically significant, which was not previously observed in our results. This suggests an unusual effect where bond prices increase with larger yield changes. As Grieves et al. (2010) suggest, negative convexity can occur if investors are particularly risk-averse or if there is a significant shift in demand and supply. The results indicate additional negative adjustments beyond what would be expected from duration alone. The adjusted R-squared values remain rather high, but they suggest less explanatory power for this type of bond compared to previous estimation models. In summary, German short-term bonds exhibit significantly higher sensitivity to yield changes, as reflected by the coefficients. This complex behaviour warrants further comparison to Czech and US short-term bonds in the subsequent chapter.

Table 12 – Fixed effects estimation, German short-term government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-1.324***** **(0.120)**	-1.100*** (0.025)	**-1.088***** **(0.038)**	-2.383*** (0.150)	**-1.172***** **(0.044)**
$P * C * \Delta r^2$	**-1.604** **(4.032)**	-158.891*** (23.216)	**-96.223***** **(26.772)**	33.939*** (5.634)	**-5.316** **(3.263)**
Observations	**234**	42	**63**	68	**61**
Adjusted R^2	**0.853**	0.984	**0.920**	0.784	**0.908**
Note:					*p**p***p<0.01

Before concluding this sub-chapter, let us briefly present the results for all German bonds, as presented in Table 13. The aggregate results affirm the reliability of duration as a measure of interest rate risk for German government bonds. This consistency is observed across individual years, with coefficients ranging from -0.947 to -1.019. The year 2020 stands out with reduced sensitivity to yield changes and higher variation, aligning with our earlier analyses. Regarding convexity, the same year differs, where the coefficient is the largest in magnitude, in line with medium and long-term bonds. Overall, these findings reinforce the accuracy of duration metrics in measuring interest rate risk. We aim to confirm this pattern in the next sub-chapter, which will cover the largest part of our overall dataset: US government bonds.

Table 13 – Fixed effects estimation, German government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.997*****	-0.946***	**-1.011*****	-1.010***	**-1.019*****
	(0.003)	(0.021)	**(0.003)**	(0.014)	**(0.001)**
$P * C * \Delta r^2$	**0.965*****	2.496***	**1.354*****	0.978***	**0.743*****
	(0.024)	(0.775)	**(0.045)**	(0.041)	**(0.022)**
Observations	**1,075**	195	**282**	297	**301**
Adjusted R^2	**0.992**	0.983	**1.000**	0.993	**0.999**
Note:					*p**p***p<0.01

6.3. US government bonds

In this sub-chapter, we will maintain the structure as before, starting with long-term bonds, and continuing with medium- and short-term bonds, and then aggregating the results. As with the German bonds, we immediately include estimations that account for convexity.

Table 14 below presents the results of US long-term government treasuries. Notably, the estimated duration coefficients are similar compared to the previous results for German and Czech bonds. The overall coefficient is below the desired -1 value, at -0.942, indicating that US government bonds were less sensitive to changes in yield than the duration metric would predict. This discrepancy might be explained by several factors such as market imperfections (liquidity issues, transaction costs, market frictions) or the omission of important variables in the theoretical framework. Additionally, when changes in interest rates are relatively small, they may affect bond prices differently compared to the larger changes observed in Germany. Of the four studied periods, 2023 showed the greatest deviation from the -1 value, contrasting German LT bonds. We would expect 2020 to be the most variable year due to previous results, however, they are rather reported in 2022 and 2023 when the FED was continuously increasing its policy rates, which occurred earlier than in the ECB, as shown in Figure 3.

Regarding the convexity, it once again turned out to be positive, in line with the theoretical framework. The convexity coefficients were all significant, with the overall period showing a coefficient of 0.674. This suggests that convexity played a significant role in determining the changes in prices of US treasuries. Overall, the US long-term bonds appeared more resilient to interest rate changes than expected. In the following paragraphs, we will evaluate whether this statement also applies to US notes and bills.

Table 14 – Fixed effects estimation, US long-term government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.942***** **(0.004)**	-1.003*** (0.002)	**-0.967***** **(0.003)**	-0.980*** (0.007)	**-0.934***** **(0.004)**
$P * C * \Delta r^2$	**0.674***** **(0.027)**	1.595*** (0.022)	**0.778***** **(0.015)**	0.870*** (0.044)	**0.514***** **(0.012)**
Observations	**1,256**	223	**317**	344	**372**
Adjusted R^2	**0.997**	1.000	**1.000**	0.994	**0.998**
Note:					*p**p***p<0.01

Comparing the results of US notes in Table 15 to US long-term bonds, the overall coefficient is higher in magnitude, specifically -1.054. This provides robust evidence that duration is a reliable measure of interest rate risk. Based on this outcome, the prices of US notes were slightly more sensitive to changes, as a coefficient lower than -1 suggests. Interestingly, the 2023 coefficient resulted more closely to the desired value, comparing to the long-term bonds, implying the US notes changes in prices could be reliably predicted by the duration metrics even during this period.

Further, the significance of the overall convexity coefficient suggests previous claims that its inclusion improves our models' explanatory power. Although the convexity coefficient is mostly higher for the US notes than for the US bills, it implies additional adjustments in bond prices due to non-linear effects. A notable magnitude is observed in 2023, where there was not a highly sensitive response to changes in bond yields with a duration coefficient as mentioned. Although it is far from the desired 0.5, it indicates a substantial impact during this period of economic volatility. This highlights the impact of specific economic conditions and policy changes, particularly rising inflation and the Federal Reserve's rate increases. Lastly, the high adjusted R-squared indicates that the theoretical framework applied is effective in explaining bond price movements. This emphasizes the need to account for both linear and nonlinear components when assessing interest rate sensitivity in different market conditions.

Table 15 – Fixed effects estimation, US notes

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-1.054^{***}**	-1.003^{***}	**-0.967^{***}**	-0.980^{***}	**-1.037^{***}**
	(0.004)	(0.002)	**(0.003)**	(0.007)	**(0.002)**
$P * C * \Delta r^2$	**1.443^{***}**	1.595^{***}	**0.778^{***}**	0.870^{***}	**1.180^{***}**
	(0.050)	(0.022)	**(0.015)**	(0.044)	**(0.077)**
Observations	**3,673**	725	**969**	975	**1,004**
Adjusted R^2	**0.964**	0.970	**0.999**	0.988	**0.997**
Note:					$^{*}p^{**}p^{***}p<0.01$

The last group of government bonds in our sample are US bills, which are short-term bonds with zero coupons maturing in less than one year. Based on Table 16, we can assert that duration metrics, unlike for Czech and partially German short-term bonds, when including convexity measures, do reliably estimate interest rate sensitivity for US short-term government bonds. Even though the overall coefficient resulted in a value of -0.941, below the desired value, all coefficients varied between -0.934 and -0.98. This can be explained by factors such as market liquidity, economic conditions, central bank policies etc. The US bond market is one of the largest and most liquid in the world. This high liquidity ensures that the bond prices and yields are more stable and react more predictably. Moreover, the US economy tends to have more stable interest rate environment, especially for the short-term rates (Schestag et al., 2016).

Moreover, the overall convexity coefficient is not statistically significant, likely due to large standard errors in 2020, not previously observed in this period. Despite this, we still believe the including convexity is crucial, as proven in previous tables of estimation results. The insignificance of the duration coefficient in 2020, not observed in other years, suggests that the presented 0.834 coefficient cannot be considered reliable due to the high standard error. This unreliability may be due a small sample size, market changes, and the overall unsuitability of our proposed model for short-term government bonds. In 2023, the higher volatility in the previous table is not reflected here, providing almost perfect convexity coefficient, indicating sensitivity similar to our theoretical expectations. Finally, the rather surprising result is supported by a rather odd adjusted R-squared in 2023.

Table 16 -Fixed effects estimation, US bills

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.941*****	-0.956***	**-0.967*****	-0.980***	**-0.934*****
	(0.299)	(0.031)	**(0.003)**	(0.007)	**(0.004)**
$P * C * \Delta r^2$	**0.774**	0.834	**0.778*****	0.870***	**0.514*****
	(17.300)	(25.943)	**(0.015)**	(0.044)	**(0.012)**
Observations	**358**	77	**87**	95	**99**
Adjusted R^2	**0.452**	0.940	**0.952**	0.962	**-1.349**
Note:					*p**p***p<0.01

Table 17, the last table in this subchapter, summarises the overall coefficients for all maturities of US government bonds. In comparison to the overall duration coefficients of Germany and the Czech Republic, the obtained value of US bonds is lower in magnitude. This indicates that US government bonds were more resistant to policy rate changes. The coefficient of -0.962 is still close to the ideal value of -1, supporting our hypothesis that the duration metric, including convexity, is a reliable estimation even during periods of changing interest rates and market volatility. The least precise outcome is observed in 2023, a period when the FED kept increasing its policy rates, the same was observed in the case of the ECB, whereas the CNB kept them unchanged until the year-end when they began their decreasing. Despite this, the data from 2023 still provides stable evidence of the accuracy of the duration metric.

Table 17- Fixed effects estimation, US government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.962*****	-1.009***	**-0.973*****	-1.042***	**-0.953*****
	(0.004)	(0.004)	**(0.003)**	(0.006)	**(0.004)**
$P * C * \Delta r^2$	**0.742*****	1.871***	**0.803*****	1.083***	**0.564*****
	(0.027)	(0.137)	**(0.014)**	(0.033)	**(0.014)**
Observations	**5,287**	1,025	**1,373**	1,414	**1,475**
Adjusted R^2	**0.990**	0.987	**0.999**	0.992	**0.995**
Note:					*p**p***p<0.01

In this subchapter, we have provided a detailed description of our estimation results, emphasizing the nuances across different groups of bonds, categorized by countries and maturities. Furthermore, in Chapter 5, we have identified the endogeneity problem as a potential source of bias in our results. To ensure the robustness of our models and the validity of our assumptions, the following subchapter will present the results obtained using an alternative estimation method – Two-Stage Least Squares (TSLS) with an instrumental variable, specifically the lagged bond price.

6.4. Robustness check

The previous exhaustive description of results has given us an indication of whether the duration metric was reliable. Believing it to be so, as we will elaborate in the Discussion chapter, we first want to validate these outcomes through TSLS estimations. Including all variants of all models again would be confusing and redundant, so we decided to present only country-level estimations to demonstrate that the results hold using an alternate technique.

Fixed effects control for time-invariant variables that could be correlated with both dependent and independent variables, thus reducing potential omitted variable bias. This model also controls for common shocks or trends that affect all bonds similarly over time, helping to mitigate time-varying unobserved variables. However, if any endogeneity remains, primarily due to price entering both parts of the equations, we use the TSLS method. Since the instrumental variable is exogenous, the variation used to estimate the effect of the endogenous variable is not contaminated. It isolates the the part of endogenous variable that is not correlated with the error term, providing a consistent estimate of the causal effect.

Referring to Tables 18-20, we can confirm the results are indeed very similar to those obtained in Tables 8, 13, and 17. For German bonds, the TSLS overall coefficient differs by only 0.009 points, a negligible difference, still showing the expected decrease in government bond price when bond yield increases. For US bonds, the difference is only 0.006, indicating that bond prices are slightly less sensitive to changes in policy rates when using the TSLS method. The biggest difference is observed in the case of Czech bonds, where the original coefficient was above -1, and the TSLS coefficient is reported at -0.985, still a precise outcome compared to the theoretical framework.

Table 18 – TSLS estimation, Czech government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.985*****	-1.000***	**-0.939*****	-0.953***	**-1.068*****
	(0.014)	(0.021)	**(0.023)**	(0.036)	**(0.048)**
$P * C * \Delta r^2$	**0.365*****	0.708***	**0.411*****	0.306***	**0.045**
	(0.040)	(0.099)	**(0.101)**	(0.068)	**(0.194)**
Constant	**0.337*****	-0.219***	**-0.078**	0.469***	**0.694*****
	(0.049)	(0.056)	**(0.057)**	(0.127)	**(0.108)**
Observations	**338**	61	**89**	87	**101**
Adjusted R^2	**0.959**	0.987	**0.985**	0.958	**0.896**
Note:					*p**p***p<0.01

Table 19 – TSLS estimation, German government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-1.006*****	-0.065	**-1.150*****	-1.023***	**-1.013*****
	(0.007)	(0.131)	**(0.035)**	(0.010)	**(0.007)**
$P * C * \Delta r^2$	**0.986*****	42.891***	**2.874*****	1.031***	**0.582*****
	(0.039)	(6.981)	**(0.421)**	(0.037)	**(0.039)**
Constant	**-0.145*****	-0.353***	**-0.412*****	-0.018	**0.381*****
	(0.018)	(0.038)	**(0.028)**	(0.036)	**(0.026)**
Observations	**1,075**	195	**282**	297	**301**
Adjusted R^2	**0.979**	0.668	**0.954**	0.992	**0.986**
Note:					*p**p***p<0.01

Table 20 – TSLS estimation, US government bonds

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.956*****	-0.991***	**-0.975*****	-1.011***	**-0.930*****
	(0.003)	(0.006)	**(0.004)**	(0.006)	**(0.003)**
$P * C * \Delta r^2$	**0.703*****	2.050***	**0.824*****	0.922***	**0.363*****
	(0.014)	(0.125)	**(0.017)**	(0.023)	**(0.012)**
Constant	**-0.004**	-0.456***	**-0.294*****	0.181***	**0.553*****
	(0.008)	(0.011)	**(0.009)**	(0.016)	**(0.012)**
Observations	**5,287**	1,055	**1,373**	1,414	**1,475**
Adjusted R^2	**0.984**	0.973	**0.994**	0.992	**0.989**
Note:					*p**p***p<0.01

The TSLS estimation results confirmed the reliability of the duration metric. However, we also observed inaccuracies and volatility in convexity measurements. Some values even turned negative again, while others were very close to the desired 0.5. Specifically, in the case of Czech bonds, we observed a value of 0.544 in 2022, a year marked by volatility due to significant changes in policy rates, and a value of 0.647 in 2023, a period characterized by stable but relatively high rates.

We have thoroughly described the numerical outcomes of our estimation models. The last subchapter of the results section will focus on evaluating the hypotheses proposed earlier in the thesis.

6.5. Adjusted model estimations

The previous extensive results analysis focused on how the duration metric performed over the past years, adhering to the theoretical framework. In this subchapter, we present two further adjusted estimations that are more likely to be used by investors and high positioned risk managers in the real world and on portfolio analysis. For this analysis, we will examine Czech long-term government bonds.

Firstly, we will estimate Equation 13 with a modified computation of duration. Previously, we computed the duration using the time to maturity after the yield change to account for the elapsed time frame (three months, as we work with quarterly data). Now, we will compute it using the original time to maturity and subtract the average duration change within this period (for three months, values vary between 0.2 – 0.25). There is no general rule on how to adjust for the time change. The theoretical duration accounts for the yield changes at the moment. Estimating both ways will provide another robustness check on whether our expectations are correct, independent of the duration calculation approach. Equation 17 mathematically describes this pattern:

$$D_t = D_{t-1} - average\ \Delta D_t \tag{17}$$

Table 21 below presents the results of the estimation of Czech government long-term bonds using the aforementioned approach to duration calculation. We excluded the convexity part of the equation for simplicity since investors are likely to compute only duration. Based on the presented results, we can claim that the duration metric is a reliable predictor of price changes. The overall coefficient reports a value of -1.036, very close to the expected value of

-1. The yearly coefficients varied between -0.999 and -1.071, outcomes very similar to those in Table 5, with the most deviated coefficient in 2023.

Table 21 – Fixed effects estimation, Czech SSD's with adjusted duration computation

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-1.036***** **(0.012)**	-1.046*** (0.005)	**-0.999***** **(0.006)**	-1.029*** (0.025)	**-1.071***** **(0.020)**
Observations	**321**	58	**82**	84	**97**
Adjusted R^2	**0.961**	0.999	**0.997**	0.952	**0.874**
Note:					*p**p***p<0.01

The second adjusted estimation will alter the input of the yield change in our equations. So far, we worked with real-time government bond yield changes. We used this under the assumption that the risk managers and investors have precise information about the future shifts of the yield curve and thus can accurately predict the future. The next estimation gives them the benefit of the doubt by not requiring 100% accuracy, working instead with general expectations about the future. Therefore, instead of the precise yield change, we will use the average change of all yields during that period (assuming parallel shifts in the yield curve based on the general economic outlook).

In Table 22 below, such results are reported, again for Czech government long-term bonds. At first glance, the results are notably less accurate than we expected. The overall coefficient reports a value of -0.742, which is not precise based on our expectations, but it still implies behaviour consistent with the theoretical framework. In 2020, the reported coefficient perfectly mirrors our expectations, likely due to relatively stable market rates and low policy rates, leading to expected yields. However, the 2021 coefficient is almost half of the expected value (a year marked by significant changes in the yield curve's volume and shape, thus simple pararel shift is not reliable), highlighting the importance of precise yield change expectations. The next two years are at least partially stabilized, yet not very close to our desired value. Based on the adjusted R-squared, especially the 2021 model has very low predictive power, suggesting possible omitted variables and observation errors due to imprecise expectations or missing convexity. Still, all reported coefficients are statistically significant, suggesting that the relationship between the change in average yield change and the bond price change is not due to random chance.

Table 22 – Fixed effects estimation, Czech SSDs with average expectations on yield

	ΔP				
	2020-23	2020	**2021**	2022	**2023**
$P * D_{mod} * \Delta r$	**-0.742***** **(0.056)**	-1.000*** (0.040)	**-0.523***** **(0.099)**	-0.764*** (0.038)	**-1.238***** **(0.083)**
Observations	**321**	58	**82**	84	**97**
Adjusted R^2	**0.740**	0.916	**0.230**	0.828	**0.749**
Note:					$^{*}p^{**}p^{***}p<0.01$

In this subchapter, we have provided another variation of the model and confirmed our results hold even when adjusting for different duration computations or imperfect yield change predictions, even during times of interest rate volatility. This highlights the importance of prediction precision.

6.6. Hypotheses evaluation

The initial goal of this thesis was to examine the accuracy of duration metrics applied to government bonds from the Czech Republic, Germany, and the USA during periods of varying interest rates, specifically from 2020 to 2023. To address our research question, we proposed four hypotheses. Based on the presented results, we assess whether these hypotheses can be confirmed or rejected.

Hypothesis #1: Government bond duration is a relevant metric even in times of changing interest rates. We believe this hypothesis can be considered satisfied, as the results for all four years and different sets of bonds showed significant duration coefficients very close to the desired value of -1. The exception was the short-term bonds for the Czech Republic, which proved to be more sensitive to yield changes. This is likely due to their very short maturities, zero-coupons, and smaller sample sizes for the study.

Hypothesis #2: Government bond duration is a more precise measure when accounting for convexity. At the beginning of our analysis, we presented results both without and with convexity. In every case, convexity proved to be an important factor, with statistically significant coefficients that brought the duration coefficient closer to the desired value. Although the convexity coefficients were unevenly distributed and did not attain the desired value of 0.5 or similar in all cases, this hypothesis is still considered satisfied.

Hypothesis #3: The duration metric is not precise for short-term government bonds. Based on the presented results, we believe this hypothesis can be rejected. Duration is understood as an estimation of the period when the initial cash flow invested in a bond will be repaid. Since these bonds usually lack coupon payments and have maturities below one year, they are generally less sensitive to changes in yields. The duration of these bonds is very low and the price change is affected (based on the outcome) by something else rather than only interest rate changes. Therefore, the duration metric may tend to imprecise results, as it did in the case of the Czech Republic. However, for the USA or Germany, it proved to be still a quite reliable measure.

Hypothesis #4: The accuracy of the duration metric differs at different levels of interest rates. The conclusion to this claim is rather complex. During periods of very low or even zero policy rates, the duration metric held up well for the USA and the Czech Republic, with changes in prices reacting less sensitively. On the contrary, for Germany, this was the year of the highest deviation from the expected value. The highest sensitivity was observed during 2023, a period with the highest policy rates. However, once rates exceeded a certain level, approximately between 3% to 5%, depending on the country, the duration stabilized and was very close to the desired value of -1, even with higher rates and bond yields, except for the Czech Republic. Despite these irregularities, the duration coefficients were still sufficiently close to our expectations, and thus we believe this hypothesis is also supported.

In the next chapter, we will focus on the implications of these results. This chapter has presented extensive evidence of the duration coefficient values, and we further inspect patterns behind trends and developments across years and countries. We will propose possible reasons for the outcomes, link them to existing literature, and acknowledge the limitations of our research. Lastly, we will identify areas for further potential research.

7. Discussion

The influence of interest rate changes has been the central theme in our analysis. During periods of economic fluctuations, we observed varying bond yield values as a result of changing policy rates. This variability is well captured by the modified duration and convexity employed in our models (Kozlov, 2023). The studied bond markets highlight how economic conditions affect bond sensitivity over time. During years marked by significant economic turbulence, the duration coefficients in our models were generally higher compared to other years. This contrasts with the more stable sensitivity observed in 2020 and 2021, where the coefficients were closer to expected theoretical values, reflecting more predictable market conditions. However, some results suggested that even during a rather stable 2020, the coefficients can deviate more than expected. This can be explained by factors such as changes in supply and demand (increased demand for safe assets during the COVID-19 pandemic), risk aversion or coupon effects (bonds with different coupon structures can have varying sensitivities, potentially not fully captured by duration metric). The observed variations in coefficients underscore the dynamic nature of the bond markets and the influence of macroeconomic factors. Viceira (2012) demonstrates that bond price sensitivity is crucial for investors and policymakers, effectively captured through modified duration and convexity measures.

Different monetary and fiscal policies adopted by countries significantly influence their bond markets. For instance, Germany's strict budgetary discipline and the ECB's monetary policies historically contribute to a stable bond market with relatively low yields (Regan, 2024). In contrast, the Czech Republic's more flexible fiscal policies and early monetary interventions have yielded greater bond market volatility (Stoupos & Kiohos, 2022). In the United States, its large and diverse bond market was heavily influenced by the Federal Reserve's monetary policies. In addition to raising policy rates, the FED implemented a quantitative tightening, reducing its balance sheet size to reduce money in circulation and dampen inflation (Du et al., 2024). Followed by forward guidance to communicate its intentions to the public and more, these country-specific policy actions highlighted the importance of considering national contexts, studying them separately and therefore reporting different coefficients in each country.

In presenting our results, we have placed significant importance on categorizing our findings by bond maturities. Marsoem & Varirahartia (2022) have shown that bond maturity has a substantial effect on bond yields and price developments. Additionally, Buera & Nicolini (2004) base their study on government bonds and debt based on maturity segmentation, reinforcing the importance of this approach in our analysis. Beginning with long-term bonds, our study across all three countries demonstrates their superior precision in computing duration and measuring bond price sensitivity. Long-term bonds inherently possess longer durations compared to medium-term or short-term bonds, spreading their cash flows over an extended time horizon. This extended duration increases their sensitivity to interest rate fluctuations, allowing duration metrics to accurately capture these impacts. As duration represents the weighted average time to receive cash flows, longer periods enhance the significance of this average, providing a more accurate reflection of future cash flow present values. Moreover, long-term bonds significantly influence the shape and movements of the yield curve, serving as reliable predictors due to their yield changes reflecting market sentiments and expectations about future economic growth.

Nevertheless, we have demonstrated that for short-term bonds, duration and convexity can be relied upon as the measure of interest rate risk despite the abovementioned. These bonds are characterized by short durations and cash flow horizons, often with zero coupon structure, leading to less sensitivity to interest rate changes. Consequently, the duration coefficient for short-term bonds may be overestimated, frequently falling below the theoretical value of -1. This reduced sensitivity arises because their cash flows are concentrated in the near term (Campbell & Viceira, 2001). Moreover, the need for frequent rollover or reinvestment of short-term bonds as they approach maturity exposes investors to fluctuations in prevailing interest rates at the time of reinvestment. While duration measures the sensitivity of bond prices assuming reinvestment at current yield, the variability in reinvestment rates in practice can significantly alter the actual interest rate risk faced by investors (Luck & Schempp, 2014). Furthermore, short-term bonds may not reliably predict economic conditions, as their yields tend to react more to short-term market shocks rather than reflecting longer-term economic trends. However, in developed markets such as the US or Germany, the duration metric still can predict bond price changes accurately, as our results suggest.

Apart from the duration coefficients, we also included convexity in our analysis, as we believe in the non-linear relationship between changes in yield and bond prices. However, instead of the expected value of 0.5, we mostly obtained a value higher. One possible explanation might be the unusual market conditions. Although 2020 presented relatively stable results for the most part, it was already heavily impacted by the pandemic. Furthermore, during periods of significant interest rate fluctuations, the relationship between prices and yields can result in a more pronounced curvature (Gava & Lefebvre, 2020; Mondello, 2023). Lastly, we discussed the different approaches to computing modified duration and convexity measures. Ideally, these approaches should yield the same values, but other factors, such as semi-annual couponing, maturities in decimal years, or different sources of market data, can significantly alter the final results.

7.1. Limitations and potential research

Before concluding our research and its main statistical evidence, it is crucial to identify the limitations of our study and evaluate how these may have affected our results. Firstly, focusing on the robustness of the models used, we have adhered strictly to the theoretical equation structures proposed in Equations 13 and 14. However, price developments are influenced by various factors, including market behaviour, investor sentiment, and supply or demand conditions. Our proposed equations may omit other key variables, potentially leading to biased results. Moreover, the choice of functional form (linear and quadratic) can affect the estimated relationship. Despite these concerns, we believe our results provide robust evidence, supported by the high statistical significance of the coefficients, relatively low standard errors, and satisfactory values of adjusted R-squared.

Further, some might question our choice of estimation techniques. In the case of fixed effects, if its assumptions do not hold, the results may lack robustness. In the Methodology chapter, we thoroughly investigated each assumption and explained our approach to addressing any violation. Regarding the instrument validity in Two-Stage Least Squares, if the instruments are weak or not truly exogenous, the results from the TSLS estimation could be biased and inconsistent. Choosing appropriate instruments is indeed challenging; we followed the practice proposed by Bellemare & Wang (2019), using lagged price variables. While our methods might be considered basic by some, they are well-understood, and we ensured their proper application, fitness and assumptions. Techniques like the Generalized Methods of Moments (GMM) could also address issues like endogeneity, and the Bootstrap method could provide robust standard errors where the underlying assumptions of asymptotics theory do not hold.

Our study aimed to include periods with varying policy rates, as years prior were marked by relatively stable markets and smaller changes in bond yields. However, significant events such as the COVID-19 pandemic, Russia's invasion of Ukraine, and inflationary pressures affected the stability and robustness of the model estimates. We mitigated these effects, at least partially, by running sub-period analyses, which revealed different results over different time frames.

Concerning data structure, due to the unavailability of some data points, we were unable to work with the full sample as initially planned. After scanning the available sources and identifying missing observations, we proceeded as if we had a full sample. In the Data chapter, we identified some outliers that could potentially distort the analysis. These outliers did not arise from unusual market conditions but rather from higher-than-usual coupon bonds, which we included in our analysis due to their relevance. We thoroughly described our dataset and believe any inconsistencies would have been discovered prior to the estimations.

Another probable limitation of our study could be the omission of credit and default risk. Although government bonds are generally considered low-risk, variations in credit ratings and perceptions of fiscal stability can significantly affect market dynamics. According to Standard & Poor's ratings, the USA, Germany, and the Czech Republic have ratings AA+, AAA, and AA-, respectively, indicating very low default risk (Standard & Poor's Financial Services LLC, 2024). However, given the geopolitical instability, these ratings are subject to potential downgrades. By not incorporating other county-specific variables into our regressions, we may have overlooked important factors influencing price developments. However, our study includes a comprehensive analysis of each country individually, allowing for direct comparisons of results using fixed effects to account for country-specific effects. This approach enhances the internal validity of our findings by controlling for unobserved heterogeneity.

The research on government bond duration opens up numerous avenues for further study. One area of interest could be analyzing the economic recovery in the upcoming years and its effect on government bond markets. Additionally, investigating how advancements in financial technology (FinTech) and algorithmic trading affect bond pricing and the duration metric would be valuable. Central bank communication strategies and their impact on bond market dynamics are another crucial area, given their role in shaping market expectations and pricing.

As we have narrowed the research question to government bonds for the three studied countries in the last four years, there remains a vast space for further potential research. Expanding the scope to corporate bonds opens up a wide array of potential research topics. One such topic could be comparing durations between government and corporate bonds. This study could reveal how credit risk, liquidity, and other factors influence the sensitivity of corporate bonds. Additionally, it would be interesting to analyze the duration and convexity across different sectors, such as healthcare, finance, technology etc. A very popular research topic in recent years is ESG investing. With the growing importance of ESG factors, understanding how they influence corporate bonds can inform investment strategies. Other potential topics include comparing corporate bonds between emerging and developed markets, examining the impact of credit ratings, and corporate governance practices, and studying the effects of mergers and acquisition activities.

8. Conclusion

The purpose of this thesis was to determine whether the duration metric is a reliable measure of interest rate risk during 2020-2023 in the Czech Republic, Germany and the USA. We conducted separate regressions for different types of bonds in each country, incorporating convexity measures and an alternative model addressing potential endogeneity.

Our findings confirm that an increase in interest rate by one percentage point results in a price decrease by one percentage point for each year of the remaining modified duration. Additionally, incorporating convexity measures into regressions enhances the precision of duration metrics. The accuracy of the duration metric varied over the studied years, with the highest sensitivity observed in 2023 amid substantial policy rate increases. In the prior years, the duration was observed as more stable and closely approximated the expected -1 value. These findings highlight the asymmetric price movements and additional risks associated with interest rate changes.

For short-term government bonds, the minimal fluctuations in interest rates showed that the duration metric captures accurately interest rate risk in the USA and Germany, despite their short durations and absenting coupon payments. Yet, in the case of the Czech Republic, negligible yield changes do not have to capture the interest rate risk accurately, with a diminishing relevance of duration in this context. Therefore, applying the duration measure to short-term government bonds is suitable for measuring interest rate risk but considered with caution.

Future research could improve this analysis by including more countries and extending the study period to include more recent data, such as the initial decreases in policy rates observed in 2024. Additionally, adopting a more complex methodology and incorporating other factors that influence bond price changes could further validate our results. Research could also be extended to corporate bonds, which are heavily used in developed financial markets.

Overall, our results indicate that the duration metric is a reliable measure of interest rate risk for government bonds, during market instabilities and significant changes in bond yields, providing a valuable tool for investors and policymakers. However, for short-term government bonds and their low durations, the price change is most likely influenced by more factors than only interest rate changes.

List of References

Ambastha, M., Dor, A. B., Dynkin, L., Hyman, J., & Konstantinovsky, V. (2010). Empirical duration of corporate bonds and credit market segmentation. The Journal of Fixed Income, 20(1), 5-27.

Andritzky, M. J. R. (2012). Government bonds and their investors: What are the facts and do they matter?. International Monetary Fund.

Babbel, D. F., Merrill, C., & Panning, W. (1997). Default risk and the effective duration of bonds. Financial Analysts Journal, 35-44.

Boquist, J. A., Racette, G. A., & Schlarbaum, G. G. (1975). Duration and risk assessment for bonds and common stocks. The Journal of Finance, 30(5), 1360-1365.

Campbell, J. Y., & Viceira, L. M. (2001). Who should buy long-term bonds?. American Economic Review, 91(1), 99-127.

Chance, D. M. (1990). Default risk and the duration of zero coupon bonds. The journal of finance, 45(1), 265-274.

Chen, L., Lesmond, D. A., & Wei, J. (2007). Corporate yield spreads and bond liquidity. The journal of finance, 62(1), 119-149.

Czech National Bank, Interest rates on financial markets. (2024). Czech government bond yield, Rate in %, Percentages, Monthly [Data file]. Retrieved from: https://www.cnb.cz/arad/#/en/search/v%C3%BDnosov%C3%A1%20k%C5%99ivka

Czech National Bank, Monetary Policy. (2024). 2W repo rate [Data file]. Retrieved from: https://www.cnb.cz/en/monetary-policy/bank-board-decisions/

Czech Statistical Office, Inflation, Consumer Prices. (2024). Consumer price index according to ECOICOP [Data file]. Retrieved from: https://vdb.czso.cz/

DataCamp (2024). Bond Valuation and Analysis in Python. Retrieved from: https://campus.datacamp.com/courses/bond-valuation-and-analysis-in python/convexity?ex=5

de Jong, M. (2023). Global Bond Allocation Using Duration Times Spread. The Journal of Portfolio Management.

Di Asih, I. M., & Abdurakhman, A. (2021). Delta-Normal Value at Risk Using Exponential Duration with Convexity for Measuring Government Bond Risk. DLSU Business and Economics Review, 31, 72-80.

Di Giovanni, J., Kalemli-Özcan, Ş., Silva, A., & Yildirim, M. A. (2023). Pandemic-Era Inflation Drivers and Global Spillovers (No. w31887). National Bureau of Economic Research.

Domanski, D., Shin, H. S., & Sushko, V. (2017). The hunt for duration: not waving but drowning?. IMF Economic Review, 65, 113-153.

Du, W., Forbes, K., & Luzzetti, M. N. (2024). Quantitative Tightening Around the Globe: What Have We Learned? (No. w32321). National Bureau of Economic Research.

Ericsson, J., & Renault, O. (2006). Liquidity and credit risk. The Journal of Finance, 61(5), 2219-2250.

European Central Bank, Euro area yield curves. (2024). AAA rate bonds, Par yield [Data file]. Retrieved from: https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html

European Central Bank, FM – Financial market data (Other data). (2024). Main refinancing operations – Minimum bid rate-fixed rate (date of changes) – Level [Data file]. Retrieved from: https://data.ecb.europa.eu/data/datasets/FM

Fabozzi, F. J., & Fabozzi, F. A. (2021). Bond markets, analysis, and strategies. MIT Press.

Fabozzi, F. J., & Mann, S. V. (2011). Bond Analytics: Spot Rates, Forward Rates, Yield Spreads, and Valuation. The Theory and Practice of Investment Management: Asset Allocation, Valuation, Portfolio Construction, and Strategies, 489-534.

Federal Reserve Bank of New York, Effective Federal Funds Rate. (2024). Target Rate/Range [Data file]. Retrieved from: https://www.newyorkfed.org/markets/reference-rates/effr

Federal Statistical Office of Germany, Consumer Price Index for Germany. (2024). Change compared to the same month last year [Data file]. Retrieved from: https://www.destatis.de/EN/Home/_node.html

Fooladi, I. J., Roberts, G. S., & Skinner, F. (1997). Duration for bonds with default risk. Journal of Banking & Finance, 21(1), 1-16.

Galliani, C., Petrella, G., Resti, A., & Cazan, F. (2014). The liquidity of corporate and government bonds: Drivers and sensitivity to different market conditions. Joint Research Centre Technical Reports, 1, 38.

Gava, J., Lefebvre, W., & Turc, J. (2020). Beyond carry and momentum in government bonds. The Journal of Fixed Income, 29(4), 48-74.

Gomez-Cram, R., & Yaron, A. (2021). How important are inflation expectations for the nominal yield curve?. The Review of Financial Studies, 34(2), 985-1045.

Greene, W.H. (2012). Econometric Analysis. Prentice Hall.

Grieves, R., Marcus, A. J., & Woodhams, A. (2010). Delivery options and convexity in Treasury bond and note futures. Review of Financial Economics, 19(1), 1-7.

Guttmann, R., & Guttmann, R. (2022). The New Deflation: From Great Recession to Global Pandemic. Multi-Polar Capitalism: The End of the Dollar Standard, 117-167.

Hatchondo, J. C., & Martinez, L. (2009). Long-duration bonds and sovereign defaults. Journal of International Economics, 79(1), 117-125.

Heider, F., Saidi, F., & Schepens, G. (2021). Banks and negative interest rates. Annual Review of Financial Economics, 13, 201-218.

Hopewell, M. H., & Kaufman, G. G. (2017). Bond price volatility and term to maturity: A generalized respecification. In Bond Duration and Immunization (pp. 64-68). Routledge.

Hsiao, C. (2022). Analysis of panel data (No. 64). Cambridge university press.

Hyman, J., Dor, A. B., Dynkin, L., Horowitz, D., & Xu, Z. (2014). Coupon effects on corporate bonds: Pricing, empirical duration, and spread convexity. The Journal of Fixed Income, 24(3), 52-63.

Ingersoll, J. E., Skelton, J., & Weil, R. L. (1978). Duration forty years later. Journal of financial and quantitative analysis, 13(4), 627-650.

Ivanovski, Z., Draganov Stojanovskib, T., & Ivanovska, N. (2013). Interest rate risk of bond prices on Macedonian stock exchange-empirical test of the duration, modified duration and convexity and bonds valuation. Economic research-Ekonomska istraživanja, 26(3), 47-62.

Jacoby, G. (2003). A duration model for defaultable bonds. Journal of Financial Research, 26(1), 129-146.

Jin, J., Rivadeneyra, F., & Sierra, J. (2018). Government bond clienteles and yields. Advances in the Practice of Public Investment Management: Portfolio Modelling, Performance Attribution and Governance, 369-391.

Jorion, P. (2007). Financial risk manager handbook (Vol. 406). John Wiley & Sons.

Kahneman, D., & Tversky, A. (2013). Prospect theory: An analysis of decision under risk. In Handbook of the fundamentals of financial decision making: Part I (pp. 99-127).

Kozlov, R. (2023). The effect of interest rate changes on consumption: An age-structured approach. Economies, 11(1), 23.

Kraft, H., & Munk, C. (2007). Bond durations: Corporates vs. treasuries. Journal of Banking & Finance, 31(12), 3720-3741.

Leombroni, M., Vedolin, A., Venter, G., & Whelan, P. (2021). Central bank communication and the yield curve. Journal of Financial Economics, 141(3), 860-880.

Luck, S., & Schempp, P. (2014). Outside liquidity, rollover risk, and government bonds. MPI Collective Goods Preprint, (2014/14).

Macaulay, F. R. (1938). Some theoretical problems suggested by the movements of interest rates, bond yields and stock prices in the United States since 1856. NBER Books.

Mejstřík, M., Pečená, M., & Teplý, P. (2015). Bankovnictví v teorii a praxi/Banking in Theory and Practice. Charles University in Prague, Karolinum Press.

Merton, R. C. (1977). On the pricing of contingent claims and the Modigliani-Miller theorem. Journal of Financial Economics, 5(2), 241-249.

Mondello, E. (2023). Duration and Convexity. In Applied Fundamentals in Finance: Portfolio Management and Investments (pp. 413-447). Wiesbaden: Springer Fachmedien Wiesbaden.

Purnomo, L. I., Wijaya, M., & Pratama, E. (2022). Yield To Maturity Bond: Interest Rate, Inflation and Bond Rating–Empirical Research on Banking Sector. JRAK, 14(2), 167-174.

Rebonato, R., & Putyatin, V. (2018). The value of convexity: A theoretical and empirical investigation. Quantitative Finance, 18(1), 11-30.

Refinitv. (2024). Government Bond Data. In Refinitiv Eikon [Data file]. Retrieved from: https://eikon.thomsonreuters.com

Regan, A. (2024). Growth models and the comparative political economy of Europe. Comparative European Politics, 22(1), 1-4.

Sarkar, S., & Hong, G. (2004). Effective duration of callable corporate bonds: Theory and evidence. Journal of Banking & Finance, 28(3), 499-521.

STANDARD & POOR's (2024). Ratings Performance Insights: 2023 In Review. Available at: https://www.spglobal.com/_assets/documents/ratings/research/101591985.pdf

Schestag, R., Schuster, P., & Uhrig-Homburg, M. (2016). Measuring liquidity in bond markets. The Review of Financial Studies, 29(5), 1170-1219.

Schuknecht, L., von Hagen, J., & Bernoth, K. (2004). Sovereign risk premia in the European government bond market (No. 369).

Shiller, R. J. (2003). From efficient markets theory to behavioral finance. Journal of economic perspectives, 17(1), 83-104.

Sims, E., & Wu, J. C. (2021). Evaluating central banks' tool kit: Past, present, and future. Journal of Monetary Economics, 118, 135-160.

Singleton, K. J. (2000). Yield curve risk in Japanese government bond markets. International Review of Finance, 1(2), 97-121.

Stoupos, N., & Kiohos, A. (2022). Bond markets integration in the EU: New empirical evidence from the Eastern non-euro member-states. The North American Journal of Economics and Finance, 63, 101827.

U.S. Bureau of Labor Statistics, Consumer Price Index (CPI) Database. (2024). Consumer Price Index for All Urban Consumers (CPI-U) [Data file]. Retrieved from: https://www.bls.gov/cpi/data.htm

U.S. Departement of the Treasury, Daily Treasury Par Yield Curve rates. (2024). Daily Treasury Par Real Yield Curve Rates [Data file].. Retrieved from: https://home.treasury.gov/resource-center/data-chart-center/interest-rates/TextView?type=daily_treasury_yield_curve&field_tdr_date_value=2023

van Binsbergen, J. H., & Schwert, M. (2022). Duration-based valuation of corporate bonds. Jacobs Levy Equity Management Center for Quantitative Financial Research Paper.

Varirahartia, D., & Marsoem, B. S. (2022). Effect of Bonds Maturity Date, Interest Rates, Inflation, Exchange Rates and Foreign Exchange Reserves on Yield To Maturity of Government Bonds 2014-2020. Jurnal Syntax Admiration, 3(2), 373-387.

Viceira, L. M. (2012). Bond risk, bond return volatility, and the term structure of interest rates. International Journal of Forecasting, 28(1), 97-117.

Wang, Y., & Bellemare, M. F. (2019). Lagged variables as instruments. Working Paper, Department of Applied Economics, University of Minnesota.

Wooldridge, J. M. (2010). Econometric analysis of cross section and panel data. MIT press.

Wu, J. C., & Xia, F. D. (2020). Negative interest rate policy and the yield curve. Journal of Applied Econometrics, 35(6), 653-672.

Xie, Y. A., Liu, S., Wu, C., & Anderson, B. (2009). The effects of default and call risk on bond duration. Journal of Banking & Finance, 33(9), 1700-1708.

Zipf, R. (2003). Fixed income mathematics. Elsevier.