FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

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**Supervisor review of the Ph.D. thesis of *Sunčica Sakić***

The Ph.D. thesis "Numerical solution of degenerate parabolic problems" deals with the numerical solution of the Richards' equation by discontinuous Galerkin finite element methods. The Richards' equation is used to model flow in variably saturated porous media. The equation is a nonlinear parabolic PDE which can degenerate into a hyperbolic (*slow-diffusion* degeneracy) or elliptic problem (*fast-diffusion* degeneracy). This degeneracy results in challenges for the numerical simulation.

The main focus of the thesis is on the stability and *a priori* error analysis of a semi-discrete (time continuous) local discontinuous Galerkin (LDG) finite element formulation for the Richards' equation, allowing for the *fast-diffusion* degeneracy case. Numerical experiments are presented which validate the analytical results. Additionally, the numerical solution of the Richards' equation using a fully discrete $hp$-STDG (space-time DG) method is considered, including the practical implementation of solving the resulting nonlinear algebraic system.

The thesis is very interesting, contains novel and new results, and is completed to an excellent standard. It is well structured, first describing the equations and its derivation, then progression to analytical results and finally numerical experiments. Although it contains a few minor grammatical and spelling mistakes, the general level of English and explanation is excellent. This thesis has given rise to a journal article published in a top numerical analysis journal (Congreve, Dolejší, and Sakić. Error analysis for local discontinuous Galerkin semidiscretization of Richards' equation. *IMA J. Numer. Anal.*, 2024) and an accepted proceedings in an international conference (Sakić and Congreve. Numerical study of a discontinuous Galerkin method for a degenerate parabolic equation. *ENUMATH 2023*).

Since Sunčica Sakić started her Ph.D. in October 2020 I have found her to be dedicated, hard-working, and motivated student. She has shown good research capabilities, able to independently research and find solutions to the problems which arose in the analysis. She passed the exams of all courses in her study plan with very good (or better) grades. Additionally to the analytical results presented in the thesis she also wrote part of the code to support the simulations within our in-house Fortran code. She is the co-author of two journal articles and two conference proceedings, and has presented talks at several international conferences.

In general the thesis is excellent and I recommend to award the title of Ph.D. to Sunčica Sakić.

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