Abstract: In this thesis, we investigate various systems of strongly-coupled nonlinear partial and ordinary differential equations, which mainly originate from bio-science, both theoretically and numerically. For the main part of this work, systems of parabolic equation with cross-diffusion is considered. It is well-known that, the systems of these types usually suffer from low regularity due to the nature of the cross-diffusion term(s). Lack of regularity may also be caused due to the structure of the other equations present in the system. We address these difficulties and establish the existence of global classical solutions for different cross-diffusion systems. Next, we show that the analytical investigation may get very difficult or simply fail to solve or capture the behavior of the solutions of the considered systems and it is necessary to approximate the respective solutions by means of numerical methods. We show that the behavior of numerical solutions heavily depends on the effect of the cross-diffusion term(s), i.e., when these terms are dominant the standard numerical methods become unstable and the approximate solutions are usually polluted by spurious oscillations. We present high-resolution nonlinear finite element flux-corrected transport (FE-FCT) methods to overcome this problem. Then, we analyze the proposed schemes and address their solvability, positivity, and satisfaction of discrete maximum principle. The theoretical and numerical results are validated by several numerical experiments in various spatial dimensions.

In the last part of this work, we investigate the qualitative and quantitative behavior of a strongly-coupled nonlinear system of ordinary differential equations. We employ a nonstandard finite difference scheme to approximate the solutions of the system under consideration and address the questions of positivity, elementary stability and conservation.

Keywords: cross diffusion, existence of solutions, FE-FCT stabilization methods, positivity preservation