Supervisor report on the Ph.D. thesis by Shahin Heydari

Development and analysis of monotone numerical schemes

The thesis is devoted to the analysis and numerical solution of several mathematical models consisting of coupled partial and ordinary differential equations describing concentrations of various constituents of interest, mainly with applications in modelling of cancer invasion and spreading of diseases. The emphasis is on nonlinear models where an important role is played by cross-diffusion, which is a phenomenon where a gradient in the concentration of one quantity induces a flux of another quantity. Such models are challenging from the point of view of both the analysis and the numerical solution. In particular, it is necessary to guarantee that the approximate solutions are nonnegative, which is one of the main goals of the thesis.

The thesis consists of an introduction to the topic, a chapter on stabilized finite element methods for convection–diffusion problems, five published papers with commentaries, and conclusions with an outlook. The main discretization technique applied in these papers is the finite element method. In most cases a stabilization is necessary, for which algebraic flux correction is applied. The last paper is devoted to a nonstandard finite difference method. The thesis contains many new analytical results for both the continuous problems and their discretizations, and the theoretical findings are complemented by numerical studies. I would like emphasize that the mentioned papers were published in leading international mathematical journals including M2AN (Mathematical Modelling and Numerical Analysis), JCP (Journal of Computational Physics), and M3AS (Mathematical Models and Methods in Applied Sciences). This shows that the thesis clearly contributes to the further development in the considered areas.

The thesis is well structured, clearly written and easy to read. The respective literature is cited appropriately and the graphical layout is at a good level. The number of typos and various insufficiencies is adequate to the extent of the thesis. The thesis demonstrates that the author has very good knowledge in numerical analysis as well as programmer skills and that she is able to carry out independent scientific work. It should also be mentioned that the author has a good idea of further directions of the research.

During the Ph.D. study, Shahin Heydari has always presented herself as a talented hard-working student with a great ability and sense of responsibility. She mostly worked independently and devoted a lot of effort to the implementation of the various methods and to the design of the test problems and their assessment. She achieved many interesting numerical results which she presented not only in the mentioned papers but also at various international conferences. She has continued a cooperation with her colleagues from Iran on

numerical simulation in modelling of diseases. Moreover, she initiated a new collaboration with colleagues from the University of Hannover (group of Prof. Thomas Wick) where she spent six months. In addition, she obtained and successfully solved a project of the Grant Agency of the Charles University. She also showed her organisational abilities as she helped to organise several workshops. Moreover she was active in the Charles University Chapter of SIAM.

Altogether, in my opinion, the thesis is a very nice and interesting work of a high quality. Therefore, I recommend accepting the thesis for a defence and awarding Shahin Heydari the Ph.D. degree.

Prague, 16 Oktober 2024 doc. Mgr. Petr Knobloch, Dr., DSc.