

Title: Multigrid methods for large-scale problems: approximate coarsest-level solves and mixed precision computation

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Abstract: The development of new computational hardware components opens possibilities to solve larger and larger problems. It, however, also brings new challenges. In this thesis we study multigrid methods for solving large-scale systems of linear equations. The multigrid approach relies on having a hierarchy of problems, ranging from the smallest (coarsest-level) problem to the original (finest-level) problem. We focus on settings where even the problem on the coarsest-level is large and can be solved only approximately. Such hierarchies arise, for example, when solving problems on domains with complicated geometry or when computing in parallel. We present an approach for analyzing the effects of approximate coarsest-level solves on the convergence of the multigrid V-cycle scheme and derive new coarsest-level stopping criteria tailored to multigrid methods. The multigrid hierarchy can be also used to construct residual-based a posteriori error estimates. We present a new approximation of the term associated with the coarsest level, which results in effective and robust estimates. Finally, we present a new formulation of a mixed precision V-cycle method and provide its finite precision analysis. We apply the analysis to understand how to choose the finite precisions inside a V-cycle scheme with smoothing based on incomplete Cholesky factorization.

Keywords: multigrid, coarsest-level stopping criteria, multilevel residual-based error estimator, mixed precision, smoother based on incomplete Cholesky factorization