Report on Petr Vacek's doctoral thesis:

"Multigrid methods for large-scale problems: approximate coarsest-level solves and mixed precision computation".

The thesis contains several analyses of multigrid methods for the solution of large sparse symmetric positive definite linear systems. It is structured as a collection of articles. Chapter 1 contains an analysis of V-cycle multigrid methods used with an approximate coarsest-level solver; the aim is to determine the accuracy of the solver for which the resulting multigrid convergence is roughly the same as with an exact coarsest-level solve. Chapter 2 is concerned with multilevel residual-based error estimates for the scalar elliptic boundary value problems discretized on shape-regular meshes. The focus of the chapter is on accurately approximating the coarsest-level contribution to the residual-based error estimates in the situation where the size of the associated problem is still large. The last chapter contains a rounding-error analysis of V-cycle multigrid methods without post-smoothing. The emphasis of the analysis is on the smoothing step of the multigrid method, with the intention to perform this step in one or several reduced precisions.

As far as I can judge, the results in the thesis are new. Some of the results (like those from Chapters 2 and 3) build upon known contributions, but aim at extending them, address specific difficulties or seek new insights. Some others (like those from Chapter 1) adopt a novel approach to tackle a known problem. In particular, the content of the first chapter of the thesis is already published in *SIAM Journal on Scientific Computing*, a highly regarded journal. The content of the other chapters is also strong enough to be published in good peer-reviewed journals.

The results of the thesis are relevant for the numerical analysis community. Indeed, multigrid methods are state-of-the-art methods for the solution of linear systems arising from a discretization of elliptic boundary value problems. Therefore, pertinent analysis of these methods, as presented in the thesis, are useful and welcome. More specifically, the author rises several relevant questions in the thesis, gives theoretically justified answers to these questions, and illustrates these answers with numerical experiments.

The form of the thesis is appropriate. A collection of articles is a format used in the field and is justified for the present thesis. The various analyses present in the thesis are based on reasonable, often standard, assumptions and are structured in a logical and rather easy-to-follow fashion. I especially appreciate the effort of the author to make the presentation self-consistent. I also note the effort to illustrate the analysis with numerical results, and the effort to make the numerical results reproducible through publishing the associated software codes. The references to the literature are relevant and the cited sources are chosen appropriately.

Besides, to further improve the presentation, a list of comments was sent to the author.

All in all, the thesis contains enough novel and relevant results for a doctoral thesis. The thesis attests the ability of the author to conduct a creative scientific research. Based on the above, **I recommend the thesis for a defense.**

Artem Napov,
Brussels, 18/11/2024