



June 6, 2024

Opponent's Opinion: Habilitation Thesis of Dr. Lenka Slavikova

This letter constitutes my opinion on the Habilitation Thesis of Dr. Lenka Slaviková, entitled, “Operators of harmonic analysis, related function spaces, and applications.” In the US, this would be considered an “arm’s length” letter, since I have only met Slaviková once or twice (such as when she came to Madison several years ago to give a very nice seminar talk).

In brief, the thesis contains an impressive collection of nine original and high-quality results in harmonic analysis and related areas, and certainly meets the requirements of the habilitation thesis. As such, I strongly recommend that Dr. Slaviková be advanced in the habilitation process.

More generally, Dr. Slaviková has established herself as an international leader in the study of function spaces and Fourier multiplication operators, and her research productivity is comparable with what is expected for promotion to Associate Professor with tenure at a research university within the United States, an impressive achievement given the time elapsed since her Ph.D.

The TurnItIn analysis raises no concerns about originality of the thesis, as the matches were exclusively found in references, limited and appropriate use of a few technical terms or formulas, or in preprint versions of the papers constituting the thesis.

The remainder of this letter will contain a more detailed scientific discussion of the work contained within the habilitation thesis.

The habilitation thesis is comprised of work in four different areas of mathematical analysis: embeddings of fractional Orlicz-Sobolev spaces into rearrangement-invariant spaces,

Department of Mathematics

University of Wisconsin–Madison 480 Lincoln Dr, Madison, WI 53706
Phone: (608) 263-3054 Fax: (608) 263-8891 Web: <http://math.wisc.edu/>

boundedness and unboundedness of Fourier multiplication operators, boundedness of multilinear Fourier multiplication and singular integral operators, and norm variation bounds for multilinear ergodic averages.

The papers [A] and [B] from the thesis provide, for the first time, a complete characterization of embeddings of fractional order Orlicz-Sobolev spaces into Orlicz spaces and other rearrangement-invariant spaces. These are hard, technical results with immediate application to PDE. Indeed, the first of these, [A], though only published in 2021, has already been cited dozens of times by many different authors, largely with PDE applications, and [B], which concerns the supercritical case, has already been cited a number of times, despite only having been published in 2023. Slaviková also has a very nice paper in *Ann. Inst. H. Poincaré C Anal. Non Linéaire* from 2020 concerning other embeddings of Sobolev-type spaces into rearrangement invariant spaces, with interesting connections to geometric measure theory.

Dr. Slaviková has a number of important results on Fourier multiplication operators, of which the articles [C,D,E] are just examples. Precise conditions for boundedness of Fourier multiplication operators is known to be a very difficult problem, harder in its full generality than some of the most famous open problems in harmonic analysis. One of the best known multiplier theorems is due to Hörmander and Mihlin, and establishes sufficient conditions on certain derivative estimates on a multiplier for L^p boundedness. In [C], Grafakos–Slaviková have strengthened this classical result by reducing the order of decay required. This is an excellent result, sure to find significant application both in PDE and within harmonic analysis. (Proving such theorems requires a lot of delicate decompositions and analysis.) Another very nice result is a solo-authored paper of Slaviková showing that a natural endpoint version of the Hörmander multiplier theorem is false, establishing sharpness of Hörmander’s theorem.

Finally, there has been a lot of interest within the harmonic analysis community in various multilinear versions of classical theorems. As an example, boundedness of the Hilbert transform is a very old result, but boundedness of the bilinear Hilbert transform is much harder and thus much more recent, and very little is known about boundedness of the trilinear Hilbert transform. Slaviková and her collaborators have produced some very interesting results in this area, including the papers included in the habilitation thesis. In addition, several interesting open questions are mentioned in the thesis, and Slaviková is involved in some very productive collaborations, so it seems like she has excellent momentum for producing more great results in the future.

In short, Slaviková has had excellent research productivity since completing her postdoc.

She has produced very strong, original work in the areas of function spaces, multiplication operators, and multilinear singular and oscillatory integrals. The habilitation thesis is of very high quality, and I strongly recommend that the thesis be accepted and Dr. Slaviková be advanced in the habilitation process.

Sincerely,

Betsy Stovall
Mary Herman Rubinstein Professor of Letters and Science
Professor
Department of Mathematics
University of Wisconsin–Madison
stovall@math.wisc.edu
+1 (608) 263–2933