

Report on the Habilitation thesis

"Operators of harmonic analysis, related function spaces and applications"

by Lenka Slavíková

This thesis is largely concerned with various important properties of function spaces and the boundedness of operators of significance in harmonic analysis acting on them. It begins with a very helpful account of the principal topics to be discussed, dealing with such matters as fractional Sobolev spaces, Fourier multipliers and singular integral operators, including multilinear theory. Following this is a brief description of the nine papers ([A]-[I]) that constitute the heart of the thesis: of these, seven have already appeared, in excellent journals, one is to appear (again in a prestigious journal), and the remaining paper is in course of publication. Finally, each of these nine papers is given in full.

Paper [A] deals with fractional Orlicz-Sobolev spaces based on a Young function A with subcritical growth, in a certain sense, and in particular provides a complete characterisation of embeddings of such spaces in Orlicz spaces or rearrangement-invariant spaces. The results obtained are precisely analogous to those known for Sobolev embeddings of classical integer-order spaces. In [B] corresponding results are derived under the assumption that A has supercritical growth. These two papers are of exceptional interest and originality.

Attention now switches to multipliers. [C] considers the multiplier theorem of Calderón and Torchinski, stemming from the Hörmander multiplier theorem, and shows that the fractional Sobolev space appearing in it may be replaced by a fractional Lorentz-Sobolev space. In [D] it is shown that the Calderón-Torchinski theorem does not hold in a certain limiting case defined by a smoothness parameter s . The next three papers are largely concerned with multilinear matters. Paper [E] gives an improvement of the Marcinkiewicz multiplier theorem, obtained by replacing a Lebesgue space by a suitable Lorentz space, [F] provides a sharp criterion for the boundedness of certain bilinear Fourier multipliers, and [G] establishes the optimal open range of particular multilinear singular integral operators associated with kernels satisfying weaker conditions than those required by other authors in the past. The next paper studies the boundedness properties of a multilinear form that involves functions each of which can be identified with a corner of the m -dimensional unit cube. This accounts for the terminology "cubical structure" in the title of the paper. Finally, [I] derives a norm-variation estimate by reducing the problem to estimation of a six-linear Brascamp-Lieb form that, although it does not have cubical structure, can in certain cases be handled.

The topics considered are of central importance in analysis. My opinion is that the thesis presents results of exceptional depth and originality that are

notable contributions to this area. I feel that in producing it Lenka Slavíková has displayed creative ability of a high order, a remarkable technical command of sophisticated procedures, and excellent mathematical taste.

The work under review shows no sign of plagiarism.

I believe that the thesis meets the standard requirements for a Habilitation thesis: indeed I regard it as an absolutely outstanding piece of work and have no hesitation in recommending approval.

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