THE UNIVERSITY OF ILLINOIS AT CHICAGO UIC

External Examiner report for Tyomkyn Habilitation

The habilitation considers myriad problems and results in extremal combinatorics, including extremal graph and hypergraph theory and Ramsey theory. I will briefly summarize the results according to chapter, as well as my general impressions of them, and then provide a summary statement.

Chapter 2: The main result here is a partial solution to an important and well-known conjecture of Frankl and Füredi about the maximum possible lagrangian of an r-graph with m edges. They conjectured that this maximum is achieved by the hypergraph comprising the first m edges in the colex ordering and this is proved when m lies in a small interval surrounding integer binomial coefficients. Subsequently, others proved the conjecture for $r = 3$ and large n and disproved the conjecture for $r \geq 4$. The lagrangian is a fundamental concept in extremal hypergraph theory, intimately connected to the Turán function so this line of research is important.

Chapter 3: The results in this chapter are surrounding hypergraph Ramsey problems, where some induced structure is forbidden. There are three results, the first is an improvement of a bound of Conlon et. al. about finding complete 3-partite subgraphs in triple systems missing some induced subgraph, the second is an r-uniform extension of this using a variant of the stepping up lemma and the third is a Ramsey version of the Brown-Erdős-Sós problem. The first two results are actually consequences of a general stepping up technique. For triple systems we start with a graph triangle and then get K_4^3 and the proof boils down to a K_4^3-free construction previously done by Conlon et. al. but the stepping up ideas for larger uniformity seems somewhat novel. The last result about Ramsey Brown-Erdős-Sós are more recent with much less pedigree but it is nice that the result is probably sharp.

Chapter 4: This chapter studies the maximum number of induced copies of some graph in a larger graph. The main conjecture in this area is due to Pippenger and Golumbic about the number of induced cycles of length k in a large graph and the first nontrivial bound is proved here. Subsequently, a better bound has been proved by Kral and others. This conjecture is fundamental. The second contribution is to formulate the so-called edge-statistics conjecture about the number of induced k-vertex subgraph with ℓ edges in a large graph. Again, some nontrivial results are proved and the full conjecture has subsequently been proved independently by two different sets of authors. Both contributions in this chapter are about fundamental problems in extremal combinatorics and while the results were only partial, they spurred more advanced work towards the conjectures.

Chapter 5: Two different strands of research are studied, both connected to triangles. The first is a conjecture of Linial and Morgenstern (that is proved) which states that if the number of triangles and complements of triangles in a graph is roughly the same as in the random graph, then the graph contains all 4-vertex graphs as induced subgraphs. The proof is a nice application of standard methods in graph theory. The second problem studied is a special case of an old conjecture of Erdős about the number of edges in monochromatic edge-disjoint triangles in a 2-coloring of K_n. This is clearly at most $n^2/4$ by taking a complete bipartite graph in one color and the conjecture is that this is asymptotically optimal. The conjecture is open and Alon and Linial proposed the special case where one of the color classes in triangle free. This conjecture is proved using the Haxell-Rödl theorem about integer versus fractional packings in graphs. Further, a stability result is also proved using some general results about edit distance. These results are all very pleasing, settling a natural problem and giving more evidence for the original Erdős conjecture.

Chapter 6: A long and old line of research, starting with a result of Gyárfás concerns the size of monochromatic connected structures guaranteed in an edge-coloring of cliques. Here a variant of the question is considered where we seek the maximum number of edges (instead of vertices) in a largest monochromatic component. This problem is recent and the main result is to prove the 4-color version of a natural conjecture. The conjecture is that affine planes give the correct answer if they exist and otherwise a slight improvement is possible. The proofs involve some nice structural and extremal arguments. This variant considered is new with limited pedigree so it is hard to judge the significance or importance of the results.

Chapter 7: Here the theme is to consider proper edge-colorings and our goal is to find k vertex disjoint color-isomorphic copies of a graph F. The results show different behavior for the extremal function depending on the structure of F. The regimes are similar to classical Turán theory where the answer depends on whether F is bipartite and within that class, the case of trees is better understood. Like the previous chapter, this is a new problem and it remains to be seen how influential the results turn out to be.

Chapter 8: The final chapter concerns the weak saturation number of a hypergraph, which is a refinement of the saturation number first studied by Bollobás. Using algebraic techniques, some sharp results are proved for complete hypergraphs and complete partite hypergraphs. Some partite versions of the exterior algebra are used which I have not seen before. The final part of this chapter is to show that certain limits exist for weak saturation numbers. Results about approximate designs (Rödl's packing theorem) are used in the proof.

General Comments: Overall the results in the habilitation are quite substantial and varied. There is no single major result but rather many contributions in different directions. Chapters 2, 3 and 4 are about really fundamental problems in the field and some nontrivial progress is made in each case; moreover even further progress has been made by others later. The problems in Chapters 5-8 are less fundamental though quite natural. Here the techniques are more standard though there is novelty in various proofs. The output of the plagiarism check seems acceptable to me. All the work is published or in preprint form and the candidate is an author or coauthor on all the works in the dissertation. The profile/stature, research output and contribution to the field based on the candidate's cv all seem appropriate for promotion to Associate Professor. Overall, this habilitation represents many substantial results and I recommend promotion

One comment: The citation labeling/numbering should be done by last name (not first name) to make it easier to read.

Sincerely,

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