



Universidad de Huelva

Centro de Estudios Avanzados en  
Física, Matemáticas y Computación: CEAFCM  
Fac. CC. Experimentales  
Campus de El Carmen - Huelva 21071

CEAFMC and Integrated Sciences Department  
Prof. Francisco B. Pérez-Bernal,

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Prof. Zdenek Dolezal  
Vice-Dean of the Faculty of Mathematics and Physics  
Charles University  
Ke Karlovu 2027/3, 121 16 Prague  
Czech Republic

Dear Prof. Zdenek Dolezal

I have perused Dr. Pavel Stransky's Habilitation thesis and, in my opinion, the manuscript demonstrates that the candidate has a very solid career with several relevant contributions in the study of quantum many body systems in general, and, in particular, on different aspects of the so called *excited state quantum phase transitions* (ESQPTs). The manuscript includes a memory that explains in a clear way the basics, the candidate contribution, and current developments on ESQPTs, quantum chaos, and extensions to non-Hermitian Hamiltonians on quantum systems. Apart from the memory, the author includes as appendices a selected subset of his production with twelve articles. In this report, I will briefly discuss the memory and the twelve selected author contributions, followed by an analysis of the quality of the candidate's scientific production.

The memory revolves around three main aspects, ESQPTs, quantum chaos, and non-Hermitian extensions of Quantum Mechanics. It is very well-written, with very high quality figures, and a clear presentation of the subjects under discussion, despite its short length. The main theme is the study of ESQPTs, an extension of the well-known zero temperature ground state quantum phase transitions to the realm of excited states introduced around 2008. In most cases, ESQPTs are associated with a ground state quantum phase transition and ESQPTs manifest as nonanalyticities in the excited level density of the system or the level flow at a critical value of the excitation energy.

The candidate presents a clear conspectus on the concept of QPT and ESQPT in Chapter 2, showing applications to four models of interest: the quartic potential quantum cusp, the Creagh-Whelan model, the Lipkin-Meshkov-Glick (LMG) model, and the two-dimensional limit of the vibron model (2DVM). Each of the four models is an interesting example by itself and it introduces new and timely features. The quantum cusp model is very pedagogical, with a single effective degree of freedom, a clear classical limit, and, despite its apparent simplicity, plenty of applications. For example, this is probably the most frequently used model when discussing spontaneous symmetry breaking. The Creagh-Whelan model is less known but it complicates the previous one with the inclusion of a second effective degree of freedom that changes the order of the singularities in the excited state level density and flow. The LMG model has a long history since its inception in the 70s in the field of Nuclear Physics as a toy model to test approximate methods. Since then, it has been realized experimentally in different platforms and researchers from different branches of Physics have paid heed to this apparently simple model. Finally, the 2DVM, was introduced for the study of molecular bending vibrations and it is a model for the



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study of two-dimensional systems with a single effective degree of freedom. The 2DVM in its simplest formulation has a second order ground state quantum phase transition and an associated ESQPT. Recent contributions from the candidate have brought to the limelight and extension of this model, breaking the angular momentum conservation. In fact, the candidate discuss in the memory a groundbreaking aspect: the appearance of a second ESQPT, that is not explored to date, when considering the full system Hilbert space.

Chapter 3 contains a brief and neat introduction to classical chaos and the candidate manages not to get entrapped in the quicksands of its extension to quantum systems, a problem that is still open to discussion. The candidate moves in this terrain in a brilliant way, demonstrating a solid background on both quantum and classical techniques and he introduces as quantum chaos probes the quantum Lyapunov exponent and the *wiggleness*, defined as the square root of the relative variance for out-of-time-order correlators (OTOCs). The candidate includes results for the non-integrable Creagh-Whelan model and a non-integrable version of the 2DVM recently introduced by himself.

Finally, Chapter 4 brings the reader into the domain of non-Hermitian quantum mechanics, extending Hamiltonian control parameter values of the systems under study from the real line to the complex plane. Apparently this is a small step in the formalism, however it brings us into rarely strolled domains in quantum systems and it entails a giant leap in the richness of the results and also in the complexity of their interpretation. For example, the replacement of real spectra by complex ones allows for the study of exceptional points. The author, using this formalism together with some collaborators, has recently paved new ways of understanding ESQPTs with their application to systems where ESQPTs have been yet unexplored. This is the case of non-Hermitian superradiance on open systems or scattering in unbound systems.

All the papers included as appendices have been published in prestigious international research journals: Annals of Physics; The Physical Review A, C, and E; The Physical Review Letters; and Physics Letters A. All of them treat different aspects of the aforementioned three subjects and can be considered significant contributions and advances in the field. Appendices A, B, and C are a very well known series of three papers where a deep and detailed extension of the ESQPT concept to systems with more than one effective degree of freedom is presented. Appendix D is a very valued reference, where a systematical classification of ESQPTs is presented and it is considered now as a seminal reference in the field of ESQPTs. Appendix E explores the consequences of ESQPTs on thermodynamics. Appendices F and G study two interesting aspects of non-Hermitian systems: the appearance of exceptional points in the proximity of quantum phase transitions and the phenomenon of superradiance in open quantum systems. Appendices H, and I delve into quantum chaos, proposing a quantum Lyapunov exponent from the short-time exponential growth of an OTOC, evincing also that around unstable stationary points in integrable quantum systems the OTOC can also grow exponentially. Appendix L introduces a non-integrable version of the 2DVM via the breaking of the rotational invariance of the model Hamiltonian, explores its classical limit, and explores chaos in the quantum regime with the long time limit average value of the wiggleness for an OTOC. Finally, appendices J and K use non-Hermitian quantum mechanics to look into the role of ESQPTs in the continuum spectra of unbound quantum systems.



Apart from this selected set of papers, the candidate list of works is very complete. To date, the candidate has a total of 56 publications in Web of Science, with 941 citations, and his H-index is 17. The evolution with time of his publications and number of citations is the expected one for a solid researcher and it allows to foresee a promising consolidated career. A more detailed scrutiny of his publications confirms this conclusion. The category normalized citation impact for the candidate full career is 1.32 and his journal normalized citation impact is 1.25. Considering only the last ten years, the value of these two indexes are 1.61 and 1.65, respectively. In total, he has published 21 papers in first quartile, 2% of his papers in the top 1% journals and 20% of his papers in the top 10% journals based on citation by category, year, and document type. The same figures for the last ten years are 10 papers in first quartile out of 30 publications with 3% in the top 1% and 27% in the top 10%. The journal and category normalized citation impact factor (JNCI and CNCI) for the papers included as appendices in the memory are:

Appendix	JNCI	CNCI	Appendix	JNCI	CNCI	Appendix	JNCI	CNCI
A	4.07	3.51	B	3.38	2.33	C	2.02	2.91
D	3.07	1.76	E	0.71	0.41	F	0.99	1.02
G	0.19	0.19	H	2.89	7.23	I	9.44	8.23
J	0.18	0.39	K	0.32	0.27	L	-	-

Both indices highlight the highly above average impact on the category and the community of the works included in the appendices. The only contributions with lower indexes than expected are the papers included in App. G, J, and K. This can be explained considering that these papers deal with ESQPTs in almost unexplored frameworks: open and unbound quantum systems. Hence, the community working on ESQPTs would need some time to digest them and to explore the new ways that these three papers open. However, in my opinion, they are very valuable contributions that will probably increase their impact in the future.

Apart from the papers selected by the author, considering his full list of publications, I find of interest the fact that he has a considerable degree of internationalization in his works, with contributions worked in collaborations with researchers in Mexico, Israel, Italy, Switzerland, USA, or France. Anyways, in the near future, it would be an interesting venue to explore an increase in the international component in his works. The author has shown to be able to work and produce relevant contributions in fields other than the ones included in the thesis, as his works in Complex Systems demonstrate. Also it is important to mention that his full scientific production includes articles of relevance other than the ones included in the Appendices. I would like to emphasize his authorship in the first review published on ESQPTs, reference J. Phys. A Math. and Theor. **54** 133001 (2021), with JNCI and CNCI indexes values 3.73 and 4.29. The authorship in this key reference demonstrates the relevance of the author in this field.

The selected collection of articles in the appendices, and the full scientific works of the candidate, include original and very relevant results in the field of ESQPTs and other. They denote a solid mathematical background together with a clear writing: two features that do not coexist as often as it would be desirable. In fact, the present thesis writing is an excellent example of this rare combination. Apart from some unimportant clerical errors, the author has managed to write a



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concise guide to three complex aspects, connecting them and rigorously describing state-of-the-art results in such a way that the reader can enjoy the study of the document.

For the sake of completeness, as requested by the normative, I have also checked the output of the anti-plagiarism code *turnitin*. There are neither significant warnings nor signs of plagiarism in the submitted memory. Hence, all things considered, the submitted memory is a completely original piece of work, that includes in its appendices twelve papers that contain novel and very relevant developments in the study of quantum many-body systems and ESQPTs. The author has been an active researcher in these research areas and can be considered one of the key figures in the field.

Therefore, considering all that has been expressed above, Dr. Pavel Stransky's submitted manuscript meets largely the standard requirements for a Habilitation thesis. I strongly recommend further progresses in the candidate habilitation procedure.

Sincerely,

Prof. Francisco B. Pérez-Bernal  
CEAFMC Director