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2024 Dušan Stojanović

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Essays on Aggregate and Distributional Effects of Macroeconomic Policies

Dissertation Thesis

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Abstract

The first chapter examines distributional effects of quantitative easing of the ECB in the euro area. Previous theoretical models have investigated the dynamics of inequality measures through different access of households to financial/capital markets, neglecting the labor market differences. My contribution lies in considering segmented labor markets coming from capital-skill complementarity in production and asymmetric wage rigidities. In comparison with the models with only segmented financial markets, the introduction of segmented labor markets significantly mitigates the observed drop in total income inequality, while a rise in wealth inequality is largely amplified.

In the second chapter, I develop a model with high- and low-skilled workers and show the expansionary effects of government spending despite large training costs for new hires. When firms invest in training activity for new hires, production is disrupted as some experienced workers are diverted from production to training the new hires. In the heterogeneous agent framework, firms do not need to postpone hiring but have a choice to hire the cheaper type of workers regarding training costs. The output expansion occurs as the economy experiences an extensive hiring activity for low-skilled workers.

In the third chapter, we study the influence of changes in firms' entry, exit and borrowing on the propagation of tax shocks in the U.S. economy. We apply a proxy-SVAR model to isolate exogenous variations in tax changes. The model indicates that corporate income tax cuts increase capital accumulation, which relaxes collateral constraints and provides (existing and entering) firms with additional funds. These funds sustain initial tax stimulative effects on aggregate productivity and output growth.

Abstrakt

První kapitola zkoumá distribuční efekty kvantitativního uvolňování ECB v eurozóně. Předchozí teoretické modely zkoumaly dynamiku měření nerovnosti prostřednictvím různého přístupu domácností na finanční/kapitálové trhy, přičemž opomíjely rozdíly na trhu práce. Můj přínos spočívá v zohlednění segmentovaných trhů práce vycházejících z komplementarity kapitálu a dovedností ve výrobě a asymetrických mzdových rigidit. Ve srovnání s modely pouze se segmentovanými finančními trhy, zavedení segmentovaných trhů práce výrazně zmírňuje pozorovaný pokles celkové příjmové nerovnosti, zatímco růst majetkové nerovnosti je značně zesílen.

Ve druhé kapitole vyvíjím model s vysoce a nízko kvalifikovanými pracovníky a ukazuji expanzivní efekty vládních výdajů i přes vysoké náklady na školení nových zaměstnanců. Když firmy investují do školení nových zaměstnanců, produkce je narušena, protože někteří zkušení pracovníci jsou převedeni z výroby na školení nových zaměstnanců. V rámci modelu s heterogenními agenty nemusí firmy odkládat nábor, ale mají možnost najmout levnější typ pracovníků s ohledem na náklady na školení. K rozšíření produkce dochází, protože ekonomika zažívá rozsáhlou náborovou aktivitu pro nízko kvalifikované pracovníky.

Ve třetí kapitole zkoumáme vliv změn v vstupu, výstupu a půjčování firem na šíření daňových šoků v americké ekonomice. Používáme proxy-SVAR model, abychom izolovali exogenní variace ve změnách daní. Model ukazuje, že snížení daně z příjmu právnických osob zvyšuje akumulaci kapitálu, což uvolňuje zajišťovací omezení a poskytuje (existujícím a nově vstupujícím) firmám dodatečné prostředky. Tyto prostředky podporují počáteční stimulační účinky daní na celkovou produktivitu a růst výstupu.

Keywords: Quantitative easing, capital-skill complementarity, asymmetric wage rigidity, government spending, training costs, search and match frictions, financial frictions, firm entry and exit, borrowing.

Klíčová slova: Kvantitativní uvolňování, komplementarita kapitálu a dovedností, asymetrická rigidita mezd, vládní výdaje, náklady na školení, třecí nezaměstnanost, finanční třecí nezaměstnanost, vstup a výstup firem, půjčování.

Declaration

1. I hereby declare that I have compiled this thesis using the listed literature and resources only.
2. I hereby declare that my thesis has not been used to gain any other academic title.
3. I fully agree to my work being used for study and scientific purposes.

In Prague on
September 23, 2024

Dušan Stojanović

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Introduction

The three chapters include the investigation of aggregate and distributional effects of fiscal and unconventional monetary policy. They are all related to the broader topic of understanding the interaction between the effects of macroeconomic policies and segmented labor and financial market structures. In what follows, I will summarize the primary findings from these three chapters.

The first chapter studies how and to what extent quantitative easing of the ECB affects income and wealth of wealthy and poor households in the euro area. Previous theoretical models have investigated the dynamics of income and wealth inequality through financial market segmentation (only wealthy households have access to financial markets), neglecting labor market heterogeneity (distinct categories of workers). Although a setting with segmented financial markets may provide insight into wealth inequality and non-labor income inequality, this is not the case with labor (and thus total) income inequality. To be in line with the empirical evidence of Lenza and Slacalek (2018) on reduced labor income inequality, I also consider segmented labor markets coming from capital-skill complementarity in production and asymmetric real (nominal) wage rigidities. When only financial market segmentation is considered, the quantitative results indicate a drop in total income inequality that is diminished over time, while wealth inequality experiences a rise that gradually becomes weaker. The introduction of segmented labor markets significantly mitigates the observed drop in total income inequality, while a rise in wealth inequality is largely amplified. Given the possible broadening of the ECB's mandate towards distributional issues in the future, the analysis of segmented labor and financial markets can be more beneficial to the ECB as it provides a clearer picture of the inequality and aggregate effects than the analysis of only segmented financial markets.

The second chapter studies the real effects of government spending in the presence of large training costs for new hires. The main idea is that a fiscal stimulus induces changes in the composition of the labor force conditional on the extent of aggregate demand pressure. A period of high aggregate demand pressure is followed by a high value of forgone output as training activity causes production disruption. In this period firms decide to hire more low-skilled workers, who constitute a cheaper part of the labor force. When aggregate demand pressure is diminished, firms switch to hiring more high-skilled workers. However, the current literature considers only one type of workers, who tend to increase saving in government bonds to protect against poor employment prospects. In this case, the combination of weak employment prospects and the crowding-out effects of higher lump-sum taxes and government debt on private consumption and capital investment gives rise to recessionary effects. In contrast, I provide a model with a more realistic labor market structure and suggest that countercyclical government spending in the form of government consumption and especially government investment can be used to deal with recessions.

The third chapter explores the real effects of the tax cuts on aggregate TFP and output through changes in the composition of firms and collateral borrowing. Using the proxy SVAR model in the spirit of Mertens and Ravn (2013) we document that the tax cuts lead to a temporary rise in aggregate TFP and output because of an increase in firms' net entry. These expansionary effects become persistent only when firms are allowed to borrow external funds. The intuition is that a higher capital accumulation relaxes the collateral constraint, providing firms with additional funds to sustain previously increased aggregate TFP and output growth. We quantify the importance of corporate borrowing in transmitting the tax effects by following the empirical work of Wong (2015) in constructing a counterfactual economy where borrowing is not allowed to firms.

1 Quantitative Easing in the Euro Area: Implications for Income and Wealth Inequality

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1.1 Introduction

Following the outbreak of the global financial crisis of 2007-2008, the euro area (EA) experienced a severe liquidity shortage, while at the same time the conventional monetary policy of the European Central Bank (ECB) was constrained by the zero-lower bound (ZLB). To support price stability and the real economy as a whole, the ECB implemented unconventional monetary policy such as quantitative easing (QE).[1] In addition to aggregate effects, Ampudia et al. (2018) and Lenza and Slacalek (2018) empirically show that QE generates distributional effects in the EA economy: (1) labor and total income inequality are reduced significantly and (2) wealth inequality[2] is decreased to a lesser extent. Using only the portfolio rebalancing channel, the previous model economies cannot capture the empirical evidence on labor income inequality[3], and thus only partially shed light on total income inequality and related wealth inequality. This paper contributes to the literature by incorporating the earnings heterogeneity channel that distinguishes labor income sources between the wealthy

[1]The QE program of the ECB is defined as the Asset Purchase Program (APP). In January 2015, the ECB announced the introduction of the APP, but started its implementation in March 2015. The APP includes the combined purchases of public and private sector securities. Initially, total APP purchases amounted to as much as 60 billion euros a month until the end of September 2016. This paper focuses on the Public Sector Purchase Program (PSPP), the largest part of the APP that includes only the purchases of public sector securities (50 billion euros) - sovereign bonds from euro-area governments. The Governing Council of the ECB expanded the initial purchases within the APP on multiple occasions so that in March 2016 the amount of monthly purchases was increased to 80 billion euros. A detailed discussion of the APP is provided in Gambetti and Musso (2017).

[2]The result about increasing wealth inequality is common to theoretical models that abstract from housing wealth in studying QE implications (see e.g., Hohberger et al., 2020).

[3]The empirical evidence on labor income inequlity of Lenza and Slacalek (2018) is shown in Appendix 1.8.C.

and the poor. Accordingly, the combination of these two channels is used to examine the extent to which income and wealth of poor and wealthy households are affected by QE over different time horizons, i.e. short, medium and long run.

This study develops a model that is characterized by the two types of household heterogeneity within a New Keynesian framework: financial (capital) and labor market segmentation. *Financial (capital) market segmentation* makes a distinction between wealthy and poor households in the sense that only wealthy households have access to financial/capital markets. This segmentation is related to the portfolio rebalancing channel, according to which households' rebalancing of their asset portfolio induces aggregate and distributional effects on the economy. Specifically, the QE policy implies that the central bank purchases long-term government bonds, and thus reduces its amount relative to short-term government bonds in the portfolio of households. In response to QE, households rebalance their asset portfolio as they are assumed to have a preference for holding a certain mix of assets with different maturities. In addition, the model economy includes the portfolio adjustment costs that make the assets with different maturities imperfect substitutes so that changes in the relative supply of long-term bonds affect the term spread and then the real economy through general equilibrium forces.

Labor market heterogeneity refers to the existence of two distinct categories of workers: high-skilled workers and low-skilled workers. This segmentation is considered as labor income is an important component of total disposable income and as such plays an important role in driving income inequality (see e.g., Ampudia et al., 2018 and Lenza and Slacalek, 2018). The segmented labor market is associated with the earnings heterogeneity channel, which transmits its effects through capital intensive production and asymmetric real wage rigidities. To provide a clearer picture of the different role of high/low skilled workers in the production process, there exists capital-skill complementarity in the production process in the spirit of Krusell et al. (2000), which results in capital being more complementary with high-skilled labor.[4] Additionally, asymmetric real wage rigidities are introduced to acknowledge the

[4]Despite a rise in capital in response to QE, a slow capital accumulation (due to capital adjustment costs)

markedly sluggish adjustment of real wages, which is a characteristic of the euro area labour market documented among others by Kollmann et al. (2016). As a robustness check, the case of asymmetric nominal wage rigidities is also analysed.[5]

The novelty of this study lies in considering the interaction of labor and financial market segmentation, which leads to the separation of the euro area population into two distinct groups: wealthy households (70%) and poor households (30%). Wealthy households have access to financial/capital markets and provide high-skilled labor services. Poor households do not have access to financial/capital markets and supply low-skilled labor services. Accordingly, 30 per cent of the total population does not participate in financial and capital markets and has attained at most post-secondary education. However, this setting is in contrast to Hohberger et al. (2020) and Tsiaras (2023), who consider only financial market segmentation. In their studies, the heterogeneity in households' labor income is neglected, which in turn provides a rather limited insight into the dynamics of total income inequality. The same conclusion applies to the dynamics of wealth inequality due to the close relationship between total income and wealth inequality, a finding that is also reported by Bilbiie et al. (2022b) but for conventional monetary policy.

The main quantitative results of this study are as follows. Purchasing long-term government bonds from wealthy households, the ECB reduces the term spread. In response to a lower term spread and to restore the duration of their portfolio, the wealthy increase investment in other long-term assets, such as physical capital, and redirect resources from short-term government bonds to consumption. A higher level of investment and consumption increases aggregate demand pressure, which stimulates higher employment and wages of both types of households. A larger upward real wage rigidity for poor households implies a rise in the wage premium, and

induces a smaller increase in the demand for complementary high-skilled labor compared to low-skilled labor. This is in line with Bilbiie et al. (2022a), who indicate that low-skilled workers are characterised with a more cyclical labor demand as they are more readily available for increasing production at the time of an aggregate demand expansion.

[5]The comparison between the two sticky wage settings is presented in Appendix A.6.

also in unskilled employment inequality as their labor supply is more sensitive to the change in labor income and a rise in capital structures stimulates labor demand for the unskilled. Given that the rise in employment of the poor is larger than the rise in wages of the wealthy, there is a drop in skilled labor income inequality. In addition, capital-skill complementarity (CSC) amplifies the drop in the said inequality. This is because the labor supply of the wealthy is more responsive than the capital stock, which leads to their lower marginal productivity and thus a lower wage premium. With higher wages under CSC relative to CD economy, the poor can enjoy a larger consumption, stimulating further aggregate demand and employment. However, in the medium/long run, CSC refers to increasing labor income inequality.

The results of this study also indicate a fall in non-labor income inequality. In addition to paying higher net lump-sum taxes after QE, wealthy households have losses on profit income and interest income on holding short- and long-term government bonds. However, the presence of real wage rigidity largely limits a drop in profit income, leading to a rise in the non-labor income of wealthy households. There are two important implications of higher non-labor income of the wealthy. First, a drop in total income inequality is mitigated compared to the economies with flexible wages, i.e. the economy with segmented labor and financial markets and the economy with only a segmented financial market. Second, wealth inequality rises as more resources are available for the accumulation of larger amount of assets. In addition, the shape of consumption inequality dynamics closely follows that of total income inequality. Specifically, the consumption of poor households exhibits a higher response than that of the wealthy in the short-run as the poor spend a much larger fraction of an increase in their income on consumption goods. However, this trend of consumption inequality reverses in favor of the wealthy in the medium/long run.

1.2 Related Literature

This paper relies on two strands of literature. The first highlights the importance of the portfolio rebalancing channel in studying the effects of QE. In theoretical models, the identification of this channel is mostly based on financial friction in the form of transaction costs that investors pay when they face portfolio changes. Transaction costs are associated with the assumption of imperfect substitutability of assets with different maturities, which allows central bank purchases of assets to affect the real economy. Andrés et al. (2004) are the first to introduce such financial friction in the standard DSGE model to make short- and long-term bonds imperfect substitutes. Similar to Andrés et al. (2004), Chen et al. (2012) introduce segmentation and transaction costs in bond markets to show the stimulative effects of the Federal Reserve LSAP program on GDP growth and inflation. Harrison (2012) uses a representative agent NK model amended with portfolio adjustment costs to indicate that QE scales up the aggregate demand and inflation in the UK. In addition to portfolio adjustment costs, Falagiarda (2014) introduces a secondary market for bond trading to indicate that QE2 in the US and the first phase of the APF in the UK exert upward pressure on output and inflation, with the effects more pronounced in the UK. What is common to all these papers is their focus on the aggregate effects of QE in the representative NK framework, while this study focuses on the distributional effects of QE in the (tractable) heterogeneous NK setting.

Hohberger et al. (2020) use the portfolio rebalancing channel to compare the distributional implications of expansionary conventional and unconventional monetary policy (QE) for the EA. The results of their estimated open-economy DSGE model indicate a fall in income and a rise in wealth inequality between the wealthy and the poor in the short and medium term, but the persistent inequality effects are largely absent in the long term. However, by means of the portfolio rebalancing channel, Hohberger et al. (2020) could account for household heterogeneity only in terms of financial income. As stated by Ampudia et al. (2018), household labor income in the EA is an important component of total income, which goes in favor of considering labor market heterogeneity and corresponding inequality in labor income. Although Cui and Sterk (2021) and Sims et al. (2022) study the distributional implications of QE in the US in the presence of household heterogeneity, they neglect labor market segmentation. A similar analysis for the EA can be found in Tsiaras (2023).

To acknowledge household heterogeneity in labor income, and thus to provide a clearer picture of the distributional effects of QE in the EA, this paper also considers a second strand of literature that focuses on the earnings heterogeneity channel. In this regard, Dolado et al. (2021) distinguish between high-skilled and low-skilled workers by introducing capital-skill complementarity (CSC) and asymmetric search and matching frictions within a New Keynesian model for the US. They show that expansionary conventional monetary policy leads to increasing income inequality between high- and low-skilled workers. Unlike Dolado et al. (2021), the present study introduces the earnings heterogeneity channel (EHC) through CSC and asymmetric wage rigidities such that EHC coexists with the portfolio rebalancing channel. This interaction is in line with Sakkas and Varthalitis (2021), who indicate that households' savings and income in the EA can be associated with their skills and educational attainment. That is, we could consider the joint heterogeneity of households where the wealthy are treated as high-skilled and the poor as low-skilled.

As regards the distributional effects of QE in the EA, the empirical studies that provide support for including the portfolio rebalancing channel are, among others, Krishnamurthy et al. (2018), Urbschat and Watzka (2020). In addition, Albertazzi et al. (2021) show that portfolio rebalancing is particularly distinct to vulnerable European economies such as Ireland, Greece, Spain, Italy, Cyprus, Portugal, and Slovenia. Ampudia et al. (2018) and Lenza and Slacalek (2018) provide empirical evidence that motivates the present study to incorporate the earnings heterogeneity channel. Coibion et al. (2017) refer to several factors that the earnings heterogeneity channel includes: unemployment risk, asymmetric wage rigidities, different complementarity with physical capital across the agents' skill sets, and different household-specific characteristics that underlie households' labor supply. Recent paper by Donggyu (2021) introduces the earnings heterogeneity channel on the basis of an idiosyncratic productivity shock and unemployment risk to examine the inequality effects of QE in the US. In contrast to Donggyu (2021), the current study focuses on CSC in production and asymmetric wage rigidities under the earnings heterogeneity channel.

This chapter is organized as follows. Section 1.3 describes the model economy. Section 1.4 explains the transmission channels of QE. Section 1.5 refers to the calibration, while Section 1.6 indicates the simulation results of QE. Section 1.7 concludes.

1.3 Model Economy

This study considers a closed-economy model whose demand side is characterized by two different types of infinitely-lived representative households[6]: a fixed fraction of wealthy households indexed by $w \in (0, 1)$ and poor households indexed by $p \in (0, 1)$. Wealthy households have access to financial markets and provide skilled labor services. Poor households do not participate in financial markets and supply unskilled labor services. Hence, the model incorporates two important sources of households heterogeneity: labor market services (high- and low-skilled workers) and access to financial markets (Ricardian and non-Ricardian households). On the production side, perfectly competitive intermediate goods producers rent capital and the two types of labor services from the households to produce a homogeneous intermediate output. In addition, capital-skill complementarity is incorporated in the production function à la Krusell et al. (2000) to capture the different roles of high- and low-skilled workers in the production at the time of increased capital stock due to QE. Intermediate output is then differentiated by monopolistically competitive final-goods producers. The final output is used for consumption, investment, and government expenditure.

The model also features nominal and real frictions to ensure that the main variables of interest respond smoothly to an exogenous QE shock. These frictions are sticky prices, sticky wages, quadratic costs for changes in the capital stock and portfolio adjustment costs. The government conducts fiscal and monetary policy. Specifically, the fiscal authority follows the passive fiscal policy rule so that the lump-sum taxes/transfers respond to the deviation in the

[6] In the present model, households are different between types (poor and wealthy). However, as idiosyncratic income risk is absent within types, there is a representative household within each type. Given that the focus of this paper is on the comparison of the two types of households, a less rich setting of heterogeneity than the Aiyagari-incomplete type model is used.

value of short- and long-term debt from their respective steady state. The monetary authority implements monetary policy at the exogenous ZLB and purchases long-term government bonds from wealthy households. To motivate the non-neutrality of the QE policy, the model includes the imperfect substitutability between assets of different maturities (short-term and long-term government bonds) by means of portfolio adjustment costs.

1.3.1 Households

1.3.1.1 Wealthy Households

Wealthy households maximize their expected lifetime utility, which is a separably additive function of consumption $c_{w,t}$, real money holdings m_t and labor supply $n_{w,t}$:

$$\max_{c_{w,t},n_{w,t},m_t,b_t^s,b_t^{l,h},i_{\varsigma,t},k_{\varsigma,t}} \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c} (c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} + \frac{\varphi_m}{1-\chi} (m_\tau)^{1-\chi} - \varphi_{n,w} \frac{(n_{w,\tau})^{1+\eta}}{1+\eta} \Big\},$$

subject to the real budget constraint in every period t:

$$c_{w,t} + q_t b_t^s + q_{L,t} b_t^{l,h} \left(1 + \frac{\phi_b}{2}\left(\kappa \frac{b_t^s}{b_t^{l,h}} - 1\right)^2\right) + t_{w,t} + i_{s,t} + i_{e,t} + m_t \leq w_{w,t} n_{w,t} + \frac{b_{t-1}^s}{\pi_t} +$$

$$+ (1 + \varrho q_{L,t}) \frac{b_{t-1}^{l,h}}{\pi_t} + r_{s,t}^k k_{s,t-1} + r_{e,t}^k k_{e,t-1} + \frac{m_{t-1}}{\pi_t} + tr_{w,t} + \frac{\Pi_t^{int}}{s_w} + \frac{\Pi_t^r}{s_w},$$

where $\mathbb{E}_t$ is the expectation operator conditional on information in period t, $\beta \in (0,1)$ is the subjective discount factor, $c_{w,t}(C_{w,t})$ is the time-t individual level of consumption (aggregate consumption), σ_c is the inverse of the intertemporal elasticity of substitution, $h < 1$ is the parameter for external habit formation in consumption, s_w is the population share of the wealthy, $\chi > 0$ is the inverse of the elasticity of real money balances, $\eta > 0$ is the inverse Frisch elasticity of labour supply, $\varphi_m > 0$ and $\varphi_{n,w} > 0$ are the relative utility weights on real money holdings and labor supply, respectively.

Total resources of wealthy households include real labor income $w_{w,t}n_{w,t}$, real payoff on previous period short-term government bonds $\frac{b_{t-1}^s}{\pi_t}$ and long-term government bonds $\frac{b_{t-1}^{l,h}}{\pi_t}$

(where $\pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation rate), rental income on capital stock $r^k_{s,t}k_{s,t-1} + r^k_{e,t}k_{e,t-1}$, real money holdings m_{t-1}, real transfers from the government $tr_{w,t}$, and real profits in the form of dividends $\Pi^{int}_t + \Pi^r_t$ from ownership of intermediate and final goods firms. These total resources can be used for purchasing consumption goods $c_{w,t}$, investment in short-term government bonds b^s_t and long-term government bonds $b^{l,h}_t$, and for paying real lump-sum taxes $t_{w,t}$ to the government. The wealthy also invest in (structure and equipment) physical assets:

$$k_{\varsigma,t} = (1-\delta_\varsigma)k_{\varsigma,t-1} - S\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}}\Big)k_{\varsigma,t} + i_{\varsigma,t}, \quad \text{for} \quad \varsigma \in \{s, e\},$$

subject to quadratic capital adjustment costs defined as in Hayashi (1982):

$$S\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}}\Big) = \frac{\phi_k}{2}\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\right)^2,$$

where ϕ_k is the capital adjustment cost and $S(\cdot)$ is the capital adjustment cost function that satisfies the following properties: $S' \geq 0, S'' \geq 0$ and $S(1) = 0$.

To solve the maximization problem of wealthy household, the Lagrangian function is set up:

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\Bigg\{\frac{1}{1-\sigma_c}(c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} + \frac{\varphi_m}{1-\chi}(m_\tau)^{1-\chi} - \varphi_{n,w}\frac{(n_{w,\tau})^{1+\eta}}{1+\eta} - \lambda_{w,\tau}\Bigg(c_{w,\tau} + q_\tau b^s_\tau + \\ &+ q_{L,\tau}b^{l,h}_\tau\left(1 + \frac{\phi_b}{2}(\kappa\frac{b^s_\tau}{b^{l,h}_\tau} - 1)^2\right) + t_{w,\tau} + (k_{s,\tau} - (1-\delta_s)k_{s,\tau-1}) + (k_{e,\tau} - (1-\delta_e)k_{e,\tau-1}) + \\ &+ m_\tau - w_{w,\tau}n_{w,\tau} - \frac{b^s_{\tau-1}}{\pi_\tau} - (1+\varrho q_{L,\tau})\frac{b^{l,h}_{\tau-1}}{\pi_\tau} + \frac{\phi_k}{2}\Big(\frac{k_{s,\tau}}{k_{s,\tau-1}} - 1\Big)^2 k_{s,\tau} + \frac{\phi_k}{2}\Big(\frac{k_{e,\tau}}{k_{e,\tau-1}} - 1\Big)^2 k_{e,\tau} - \\ &- r^k_{s,\tau}k_{s,\tau-1} - r^k_{e,\tau}k_{e,\tau-1} - \frac{m_{\tau-1}}{\pi_\tau} - tr_{w,\tau} - \frac{\Pi^{int}_t}{s_w} - \frac{\Pi^r_t}{s_w}\Bigg)\Bigg\}\end{aligned}$$

Taking the FOCs, we have the following optimality conditions:

$$[c_{w,t}]: \quad \lambda_{w,t} = \frac{1}{(c_{w,t} - hC_{w,t-1})^{\sigma_c}} \tag{1.1}$$

$$[n_{w,t}]: \quad \lambda_{w,t}w_{w,t} = \varphi_{n,w}(n_{w,t})^\eta \tag{1.2}$$

$$[m_t]: \quad \varphi_m m_t^{-\chi} + \mathbb{E}_t \beta \frac{\lambda_{w,t+1}}{\pi_{t+1}} = \lambda_{w,t} \tag{1.3}$$

$$[b_t^s]: \quad \mathbb{E}_t \beta \left(\frac{\lambda_{w,t+1}}{\pi_{t+1}} \right) = q_t \lambda_{w,t} + q_{L,t} \lambda_{w,t} \phi_b \Big(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \Big) \kappa \tag{1.4}$$

$$[b_t^{l,h}]: \quad \mathbb{E}_t \beta \left(\frac{\lambda_{w,t+1}}{\pi_{t+1}} (1 + \varrho q_{L,t+1}) \right) = q_{L,t} \lambda_{w,t} + q_{L,t} \lambda_{w,t} \frac{\phi_b}{2} \Big(k \frac{b_t^s}{b_t^{l,h}} - 1 \Big)^2 - q_{L,t} \lambda_{w,t} \phi_b \Big(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \Big) \kappa \frac{b_t^s}{b_t^{l,h}} \tag{1.5}$$

$$\begin{aligned} [k_{\varsigma,t}]: \quad & \lambda_{w,t} \left(1 + \frac{\phi_k}{2} \Big(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \Big)^2 + \phi_k \Big(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \Big) \frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} \right) = \\ & = \mathbb{E}_t \beta \lambda_{w,t+1} \left((1 - \delta_\varsigma) + r^k_{\varsigma,t+1} + \phi_k \Big(\frac{k_{\varsigma,t+1}}{k_{\varsigma,t}} - 1 \Big) \Big(\frac{k_{\varsigma,t+1}}{k_{\varsigma,t}} \Big)^2 \right), \quad \text{for} \quad \varsigma \in \{s, e\} \end{aligned} \tag{1.6}$$

At the beginning of period t, the portfolio of the wealthy includes nominal (one-period) short-term risk-less government bonds b_{t-1}^s and (perpetual) long-term government bonds $b_{t-1}^{l,h}$. One-period bonds issued in period t are purchased at the real price $q_t = \frac{1}{R_t}$ and deliver the payoff one in period $t+1$, where R_t is a one-period nominal risk-free interest rate that is controlled by the central bank. As in Woodford (2001), long-term government bonds are modeled as perpetual nominal bonds that pay a nominal coupon starting at one unit in the first period after issuance and decaying over time geometrically at the rate $\varrho \in [0, 1]$. The real price of long-term government bonds issued in period t is given by $q_{L,t} = \frac{1}{R_t^L - \varrho}$, where R_t^L is the gross yield-to-maturity on a perpetual bond in period t and ϱ is the coupon decay factor. The duration (maturity) of long-term bonds is $d_t = \frac{R_t^L}{R_t^L - \varrho}$, where ϱ is used to match the average duration of long-term government bonds.

Wealthy households have a preference or target $\kappa = \frac{b^{l,h}}{b^s}$ for holding a mix of short-term and long-term government bonds. Deviation from this target value triggers portfolio adjustment cost $\phi_b > 0$, which makes two assets of different maturities imperfect substitutes, opening the space for the portfolio rebalancing channel to function.

1.3.1.2 Poor Households

Poor households maximize their lifetime utility, which is a separably additive function of consumption $c_{p,t}$ and labor supply $n_{p,t}$:

$$\max_{c_{p,t},n_{p,t}} \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c} (c_{p,\tau} - hC_{p,\tau-1})^{1-\sigma_c} - \varphi_{n,p} \frac{(n_{p,\tau})^{1+\eta}}{1+\eta} \Big\}$$

subject to the real budget constraint in every period t:

$$c_{p,t} + t_{p,t} \leq w_{p,t} n_{p,t} + tr_{p,t}$$

The Lagrangian function associated with the maximization problem of a poor household is:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{1}{1-\sigma_c} (c_{p,\tau} - hC_{p,\tau-1})^{1-\sigma_c} - \varphi_{n,p} \frac{(n_{p,\tau})^{1+\eta}}{1+\eta} - \lambda_{p,\tau} \Big(c_{p,\tau} + t_{p,\tau} - w_{p,\tau} n_{p,\tau} - tr_{p,\tau} \Big) \right)$$

Taking the FOCs, we have the following optimality conditions:

$$[c_{p,t}]: \quad \lambda_{p,t} = \frac{1}{(c_{p,t} - hC_{p,t-1})^{\sigma_c}} \tag{1.7}$$

$$[n_{p,t}]: \quad \lambda_{p,t} w_{p,t} = \varphi_{n,p} (n_{p,t})^{\eta} \tag{1.8}$$

Total income of the poor includes real labor income $w_{p,t} n_{p,t}$ from supplying unskilled labor services to intermediate goods firms and real transfers $tr_{p,t}$ received from the government. The poor spend their disposable income on consumption goods $c_{p,t}$ and on paying real lump-sum taxes $t_{p,t}$. Following Kaplan et al. (2014), poor households in the present model fit the definition of hand-to-mouth households because they hold no liquid and illiquid wealth, and as such spend all of their disposable income every period. As hand-to-mouth households have larger marginal propensity to consume than the other type of households, they are expected to be more sensitive to small and temporary changes in income.

1.3.2 Producers

1.3.2.1 Intermediate (Wholesale) Goods Producers

There is a continuum of measure one of perfectly competitive firms that take prices $P_{int,t}$ as given and produce a homogeneous good $Y_{int,t} = Y_t$. To produce output, firms use the aggregate stock of structure capital $K_{s,t-1}$ and equipment capital $K_{e,t-1}$, aggregate skilled labor from wealthy households $N_{w,t}$, and aggregate unskilled labor from poor households, $N_{p,t}$. In the spirit of Krusell et al. (2000), the production function is given in the form of a nested CES composite of factor inputs:

$$Y_{int,t} = F(K_{s,t-1}, K_{e,t-1}, N_{w.t}, N_{p,t}) = AK^{\iota}_{s,t-1}\Big[m(N_{p,t})^{\sigma}+(1-m)\Big(\rho(K_{e,t-1})^{\nu}+(1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}}\Big]^{\frac{1-\iota}{\sigma}} \tag{1.9}$$

where $A > 0$ stands for aggregate productivity, $m, \rho < 1$ determine the income shares of unskilled labor, equipment capital and skilled labor. The parameter ι indicates the income share of structure capital. Two parameters $\sigma, \nu \leq 1$ govern factor inputs elasticities. The elasticity of substitution between equipment capital and skilled labor is defined as $\varepsilon_1 = \frac{1}{1-\nu}$, while the elasticity of substitution between equipment capital and unskilled labor and between skilled and uskilled labor is defined as $\varepsilon_2 = \frac{1}{1-\sigma}$.

Intermediate goods producers seek to maximize their nominal profits, which are distributed as dividends to wealthy households, subject to the production function given by the equation (1.9):

$$P_t\Pi^{int}_t = P_{int,t}Y_{int,t} - W_{w,t}N_{w,t} - W_{p,t}N_{p,t} - R^k_{s,t}K_{s,t-1} - R^k_{e,t}K_{e,t-1},$$

while the real profit of the intermediate goods firms is expressed as:

$$\Pi^{int}_t = \frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r^k_{s,t}K_{s,t-1} - r^k_{e,t}K_{e,t-1},$$

where $x_t = \frac{P_t}{P_{int,t}}$ is the markup of the price of the final consumption good over the price of the intermediate good, while $\frac{1}{x_t}$ is the real marginal cost for retailers or the real price of the intermediate goods.

Taking the first order conditions of the real profit function with respect to capital and (skilled and unskilled) labor inputs, we have the following demands for capital and labor:

$$[K_{s,t-1}] : r^k_{s,t} \equiv \frac{1}{x_t} F^s_{k,t} = \frac{1}{x_t} A \cdot \iota \cdot K^{\iota-1}_{s,t-1} \Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}} \tag{1.10}$$

$$[K_{e,t-1}] : r^k_{e,t} \equiv \frac{1}{x_t} F^e_{k,t} = \frac{1}{x_t} A K^{\iota}_{s,t-1}(1-\iota) \Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}-1}$$
$$(1-m)\rho \cdot \Big(\rho(K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1} (K_{e,t-1})^{\nu-1} \tag{1.11}$$

$$[N_{w,t}] : w_{w,t} \equiv \frac{1}{x_t} F^w_{n,t} = \frac{1}{x_t} A K^{\iota}_{s,t-1}(1-\iota) \Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}-1}$$
$$(1-m)(1-\rho)\Big(\rho(K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1} (N_{w,t})^{\nu-1} \tag{1.12}$$

$$[N_{p,t}] : w_{p,t} \equiv \frac{1}{x_t} F^p_{n,t} = \frac{1}{x_t} A K^{\iota}_{s,t-1}(1-\iota) \Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}-1} m(N_{p,t})^{\sigma-1} \tag{1.13}$$

The optimal demand for labor and capital inputs equates real prices (wage and rental rate) to their marginal products times the real marginal cost.

Combining equations (1.12) and (1.13), the so-called skill premium can be expressed as function of labor input ratios:

$$\frac{w_{w,t}}{w_{p,t}} \equiv \frac{F^w_{n,t}}{F^p_{n,t}} = \frac{(1-m)}{m}(1-\rho)\left(\rho\left(\frac{K_{e,t-1}}{N_{w,t}}\right)^\nu + (1-\rho)\right)^{\frac{\sigma}{\nu}-1} \left(\frac{N_{p,t}}{N_{w,t}}\right)^{1-\sigma} \tag{1.14}$$

As shown in Krusell et al. (2000), capital-skill complementarity in the production function is present if $1 > \sigma > \nu$. This implies that $\varepsilon_2 > \varepsilon_1$. The skill premium increases with a rise in the equipment capital stock $\partial\Big(\frac{F^w_{n,t}}{F^p_{n,t}}\Big)\Big/\partial K_{e,t-1} > 0$, keeping all the other factors constant. However, the skill premium decreases in the skilled to unskilled labor ratio, $\partial\Big(\frac{F^w_{n,t}}{F^p_{n,t}}\Big)\Big/\partial\Big(\frac{N_{w,t}}{N_{p,t}}\Big) < 0$, under the assumption that all other factors remain unchanged.

To evaluate the quantitative importance of CSC in driving the QE distributional effects, my model also considers an alternative benchmark economy where CSC is not present. This model is characterized with a standard CD structure:

$$Y_t = AK_{t-1}^{\theta}\Big(\varkappa N_{w,t}^{\gamma} + (1-\varkappa)N_{p,t}^{\gamma}\Big)^{\frac{1-\theta}{\gamma}}$$

There are two types of CD economy: (1) with capital and two types of labor that are imperfect substitutes; (2) with capital and two types of labor that are perfect substitutes, with parameters $\varkappa = 0.5$ and $\gamma = 1$. The second type of CD economy features only the portfolio rebalancing channel, while the EHC is still present in the first type of CD economy due to the asymmetric real wage rigidities. In addition, the changes in the capital stock do not affect changes in the skill premium in the first type of CD economy. That is, there is only the relative quantity effect while the capital-skill complementarity effect is not present:

$$\frac{w_{w,t}}{w_{p,t}} \equiv \frac{F_{n,t}^{w}}{F_{n,t}^{p}} = \frac{\varkappa}{1-\varkappa}\left(\frac{N_{p,t}}{N_{w,t}}\right)^{1-\gamma}$$

As in Kina et al. (2020), the calibration procedure for the first type of CD economy is the same as the CSC economy except for the two internally calibrated parameters. The first parameter is A, which is calibrated to make output Y equivalent in the two economies, while the second parameter $\varkappa$ is chosen to have the same skill premium in the two economies. The same calibration procedure is used for both CSC and CD economies to guarantee that any differences in QE effects (skill premium) between the two economies cannot be attributed to their initial conditions.

1.3.2.2 Final (Retail) Goods Producers

There exists a continuum $j \in [0,1]$ of monopolistically competitive retail firms. Each firm buys an amount $Y_t(j)$ of the homogeneous intermediate good $Y_{int,t}$, and produces a variety of the final good $Y_t^f(j)$ which is an imperfect substitute for varieties produced by other final goods firms. The technology used in the production process is linear, $Y_t^f(j) = Y_t(j)$ (see e.g., Dolado et al., 2021). These differentiated products are then aggregated into a homogeneous final good Y_t^f by the following CES aggregator:

$$Y_t^f = \left[\int_0^1 Y_t^f(j)^{\frac{\epsilon}{\epsilon-1}} dj \right]^{\frac{\epsilon}{\epsilon-1}} = \left[\int_0^1 Y_t(j)^{\frac{\epsilon}{\epsilon-1}} dj \right]^{\frac{\epsilon}{\epsilon-1}} = Y_t$$

where $\epsilon > 1$ is the exogenous elasticity of substitution between the different types of goods, Y_t^f stands for final goods, and Y_t refers to intermediate goods. Final good could be used for consumption, investment, and government expenditure.

Retail firms purchase intermediate goods from wholesale producers at the wholesale price $P_{int,t}$, which is equal to the nominal marginal cost $mc_{int,t}^n$ in the intermediate goods sector. The fact that wholesale producers are perfectly competitive implies that $P_{int,t} = mc_{int,t}^n$. Purchased intermediate goods are differentiated by the retailers at no cost, so that the nominal marginal cost of producing final goods coincides with that of wholesale goods. Then, each retail firm sells its unique variety at a retail mark-up over the wholesale price in a monopolistically competitive market. Although retailers have monopolistic power by setting the price for their own products $P_t(j)$, as in Dolado et al. (2021) they take aggregate price P_t and the price of intermediate good $P_{int,t}$ as given.

The retail sector plays the role of introducing the nominal price rigidity into the economy as it has to pay quadratic price adjustment costs when changing prices. Price stickiness is important for ensuring the real effects of monetary policy on the economy. To motivate price stickiness, the Rotemberg (1982) price adjustment costs model is used. This means that final goods firms maximize their current and expected discounted profits subject to quadratic price adjustment costs measured in terms of the final good. Specifically, each retailer indexed by j pays an increasing and convex cost measured in terms of Y_t when the size of its price increases, $P_t(j)/P_{t-1}(j)$, deviates from the steady state inflation rate π:

$$\frac{\phi_p}{2} \left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 Y_t \tag{1.15}$$

where $\phi_p \geq 0$ measures the degree of price stickiness. Higher values of ϕ_p indicate greater price stickiness, while $\phi_p = 0$ implies perfectly flexible prices of final goods.

Given the equation (1.15), each final good firm j chooses $P_t(j)$ to maximize the present discounted value of real profits for its owners (wealthy households):

$$\max_{P_t(j)} \ \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \Pi_t^r(j) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{\lambda_{w,\tau}}{\lambda_{w,t}} \left(\left(\frac{P_\tau(j)}{P_\tau} - \frac{P_{int,\tau}}{P_\tau} \right) Y_t(j) - \frac{\phi_p}{2} \left(\frac{P_\tau(j)}{\pi P_{\tau-1}(j)} - 1 \right)^2 Y_\tau \right)$$

subject to the price-elastic demand of households[7]

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t$$

where $\Lambda_{t,\tau}$ is the stochastic discount factor in period t for real payoffs in period τ, $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate ($\pi = 1$) and $mc_t^r = \frac{P_{int,t}}{P_t}$ is the real marginal cost of producing an additional unit of output (or the Lagrange multiplier from the cost minimization problem of the intermediate firm producer[8]).

By substituting the constraint related to demand of households for final goods into the objective function, we have:

$$\max_{P_t(j)} \ \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \Pi_t^r(j) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{\lambda_{w,\tau}}{\lambda_{w,t}} \left(\left(\frac{P_\tau(j)}{P_\tau} - \frac{P_{int,\tau}}{P_\tau} \right) \left(\frac{P_\tau(j)}{P_\tau} \right)^{-\epsilon} Y_\tau - \frac{\phi_p}{2} \left(\frac{P_\tau(j)}{\pi P_{\tau-1}(j)} - 1 \right)^2 Y_\tau \right)$$

and solving the resulting profit maximization problem with respect to $P_t(j)$ yields

$$\begin{aligned}[P_t(j)]:\ (1-\epsilon) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - (-\epsilon) \left(\frac{P_{int,t}}{P_t} \right) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon-1} \frac{Y_t}{P_t} - \phi_p \left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right) \frac{Y_t}{\pi P_{t-1}(j)} \\ + \mathbb{E}_t \beta \frac{\lambda_{w,t+1}}{\lambda_{w,t}} \phi_p \left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1 \right) \frac{P_{t+1}(j) Y_{t+1}}{\pi P_t(j)^2} = 0\end{aligned}$$

Since $mc_t^r = P_{int,t}/P_t$ and $Y_t(j) = Y_t$ are identical for all final goods firms, every firm sets the same price. Combining that result with $P_t = (\int_0^1 P_t(j)^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$ indicates that $P_t^*(j) = P_t^*$. In a symmetric equilibrium, the first-order condition for the retailers' problem becomes:

$$(1-\epsilon) + \epsilon \frac{P_{int,t}}{P_t} - \phi_p \left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right) \frac{P_t}{\pi P_{t-1}(j)} + \mathbb{E}_t \Lambda_{t,t+1} \phi_p \left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1 \right) \frac{P_{t+1}(j)}{\pi P_t(j)} \frac{Y_{t+1}}{Y_t} = 0$$

[7]Derivation of the price-elastic demand of households and the aggregate price level is provided in Appendix A.1 and Appendix A.2.

[8]Derivation of the real marginal cost is provided in Appendix A.3.

$$\Leftrightarrow \quad (1-\epsilon)+\epsilon mc_t^r-\phi_p\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}+\mathbb{E}_t\beta\frac{\lambda_{w,t+1}}{\lambda_{w,t}}\phi_p\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}\frac{Y_{t+1}}{Y_t}=0$$

If the cost of price adjustment is $\phi_p = 0$, i.e. when prices are fully flexible, the above equation reduces to the standard markup rule: $P_t = \frac{\epsilon}{\epsilon-1} mc_t^n$, where prices are set as a markup over nominal marginal costs. When $\phi_p > 0$, changes in marginal costs translate only gradually into changes in prices.

Rearranging terms and log-linearizing the above equation around a symmetric steady state, we obtain the expression known as the log-linearized New Keynesian Phillips Curve

$$\widetilde{\pi}_t=\frac{(\epsilon-1)}{\phi_p}\widetilde{mc}_t^r+\beta\mathbb{E}_t\widetilde{\pi}_{t+1}$$

As for the aggregate real profit that the continuum of unit mass retailers makes, the symmetric equilibrium ($P_t(j) = P_t,\ Y_t(j) = Y_t$ for $\forall j$) yields:

$$\Pi_t^r=\int_0^1\Pi_t^r(j)dj=\int_0^1\left(\frac{P_t(j)}{P_t}Y_t(j)-mc_t^r\cdot Y_t(j)-\frac{\phi_p}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)}-1\right)^2Y_t\right)dj$$

$$\Leftrightarrow \quad \Pi_t^r=\left(1-mc_t^r-\frac{\phi_p}{2}\left(\frac{\pi_t}{\pi}-1\right)^2\right)Y_t$$

1.3.3 Monetary and Fiscal Policies

The central bank monetary policy sets the short term nominal interest rate following the standard Taylor rule, which includes an interest rate smoothing component and a potential reaction to the deviations of inflation and output from their respective steady states:

$$\frac{R_t}{R}=\left(\frac{R_{t-1}}{R}\right)^{\theta_r}\left[\left(\frac{\pi_t}{\pi}\right)^{\theta_\pi}\left(\frac{Y_t}{Y}\right)^{\theta_y}\right]^{1-\theta_r}exp(\epsilon_t^r) \tag{1.16}$$

where R is the steady-state value of the (gross) nominal policy rate, $0 \leq \theta_r \leq 1$ is the parameter associated with interest rate smoothing, $\theta_\pi > 0$ and $\theta_y > 0$ measure the interest rate response to inflation and output, respectively. The monetary policy shock ϵ_t^r is an *i.i.d.*

with zero mean and standard deviation σ_R, $ln(\epsilon_t^r) = \rho_r ln(\epsilon_{t-1}^r) + \nu_t^r$.

The central bank also performs the asset purchases that have been previously issued by the government. Following Hohberger et al. (2019), QE policy is simulated as an AR(2) process to provide a hump-shape path of the central bank holdings of long-term government bonds:

$$lnB_t^{l,cb} = (\phi_{cb1} + \phi_{cb2})lnB_{t-1}^{l,cb} - (\phi_{cb1}\phi_{cb2})lnB_{t-2}^{l,cb} + \epsilon_t^{l,cb}$$

The specification of QE as an AR(2) process is important for capturing the initial purchase of long-term government bonds by the ECB in 2015q1, followed by further extension of the central bank holdings for three years, and a gradual exit from QE.

Total government debt includes short-term (B_t^s) and long-term (B_t^l) government bonds:

$$B_t = qB_t^s + q_L B_t^l,$$

where B_t^l is further decomposed into long-term bonds held by the central bank ($B_t^{l,cb}$) and by the household sector ($B_t^{l,h}$):

$$B_t^l = B_t^{l,cb} + B_t^{l,h} = f_t^l \cdot B_t^l + (1 - f_t^l) \cdot B_t^l$$

When the central bank conducts the QE program, it purchases long-term government bonds from the private sector, which in turn increases the amount of long-term bonds in the asset side of its balance sheet. The liability side of the central bank's balance sheet also increases as the central bank pays for the purchased bonds by the newly created money provided to the private sector, $(M_t - M_{t-1}/\pi_t)$.

The real operational profit of the central bank is:

$$\Pi_t^{cb} = M_t - \frac{M_{t-1}}{\pi_t} - \left(q_{L,t}B_t^{l,cb} - (1 + \varrho q_{L,t})\frac{B_{t-1}^{l,cb}}{\pi_t} \right)$$

As for the fiscal policy, in each period the fiscal authority purchases the final consumption good, G_t, issues government bonds to refinance its outstanding debt, B_t^s and B_t^l, distributes lump-sum transfers TR_t and raises lump-sum taxes T_t.

The consolidated government budget constraint (in aggregate real terms) is:

$$T_t+q_tB_t^s+q_{L,t}B_t^l+M_t-\frac{M_{t-1}}{\pi_t}-\left(q_{L,t}B_t^{l,cb}-(1+\varrho q_{L,t})\frac{B_{t-1}^{l,cb}}{\pi_t}\right)=\frac{B_{t-1}^s}{\pi_t}+(1+\varrho q_{L,t})\frac{B_{t-1}^l}{\pi_t}+G_t+TR_t \tag{1.17}$$

The real government spending G_t follows a serially correlated process

$$G_t=(Y\Gamma)^{1-\phi_g}(G_{t-1})^{\phi_g}exp(\epsilon_t^g),$$

where $\Gamma=G/Y$ is the steady state share of government consumption in output.

Similarly to G_t, lump-sum transfers TR_t are assumed to follow a serially correlated process:

$$TR_t=TR^{(1-\phi_{tr})}TR_{t-1}^{\phi_{tr}}exp(\epsilon_t^{tr})$$

Lump-sum taxes T_t are adjusted as a result of discrepancies in the value of long- and short-term government bonds from their steady-state. The passive fiscal policy rule that the lump-sum taxes follow can be written as:

$$T_t=\Phi\left(\frac{q_{L,t-1}B_{t-1}^l+q_{t-1}B_{t-1}^s}{q_LB^l+qB^s}\right)^{\rho_1}$$

The rationale behind the passive fiscal policy rule is to prevent the emergence of inflation as a fiscal phenomenon and the explosive path of government debt. The parameter Φ makes the fiscal rule an identity in steady state (see e.g., Chen et al., 2012), while the parameter $\rho_1>0$ determines the response of taxes to total government debt.

1.3.4 Aggregate Variables and Market Clearing

The aggregate per-capita quantity of any household specific variable $x_t(i)$ is given by

$$x_t=\int_0^1 x_t(i)di=s_w\cdot x_{w,t}+s_p\cdot x_{p,t}$$

as households within each of the two types (i.e. wealthy and poor) are identical.

The aggregate resource constraint or the goods market clearing condition[9] is given by:

$$Y_t = C_t + I_t + G_t + \sum_{\varsigma \in \{s,e\}} s_w \frac{\phi_k}{2} \left(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \right)^2 k_{\varsigma,t} + \frac{\phi_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 Y_t - q_{L,t} s_w b_t^{l,h} \frac{\phi_b}{2} \left(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \right)^2$$

This condition implies that final output is used for consumption, investment, government expenditures and covering adjustment costs.

1.4 Transmission Channels of QE

This section explains two important channels for the transmission of QE effects to the real economy and to the inequality measures: the portfolio rebalancing channel and the earnings heterogeneity channel.

The portfolio rebalancing channel of QE can be illustrated with the analysis of the term spread, which is the difference between the long-term and short-term interest rates. We combine the log-linearised first-order conditions for short-term and long-term bond holdings ($\widetilde{b}_t^s$ and $\widetilde{b}_t^{l,h}$) of wealthy households to express the gross yield-to-maturity on a perpetual bond:

$$\widetilde{R}_t^L = \frac{\varrho}{R^L} \mathbb{E}_t \widetilde{R}_{t+1}^L + \frac{R^L - \varrho}{R^L} \left(\widetilde{R}_t - \phi_b (1 + \frac{\pi}{\beta} q_L \kappa)(\widetilde{b}_t^s - \widetilde{b}_t^{l,h}) \right) \tag{1.18}$$

where $\frac{R^L - \varrho}{R^L} > 0$ and $(1 + \frac{\pi}{\beta} q_L \kappa) > 0$.

Iterating on (1.18), we obtain the expression for long-term yields as the sum of (current and) expected future short-term interest rates and changes in relative bond holdings:

$$\widetilde{R}_t^L = \left(\frac{R^L - \varrho}{R^L} \right) \mathbb{E}_t \sum_{s=0}^{\infty} \left(\frac{\varrho}{R^L} \right)^s \left(\widetilde{R}_{t+s} - \phi_b (1 + \frac{\pi}{\beta} q_L \kappa)(\widetilde{b}_{t+s}^s - \widetilde{b}_{t+s}^{l,h}) \right)$$

It follows that the term premium depends on changes in relative bond holdings:

$$\widetilde{R}_t^L - \left(\frac{R^L - \varrho}{R^L} \right) \mathbb{E}_t \sum_{s=0}^{\infty} \left(\frac{\varrho}{R^L} \right)^s \widetilde{R}_{t+s} = \left(\frac{R^L - \varrho}{R^L} \right) \phi_b (1 + \frac{\pi}{\beta} q_L \kappa) \mathbb{E}_t \sum_{s=0}^{\infty} \left(\frac{\varrho}{R^L} \right)^s \left(\widetilde{b}_{t+s}^{l,h} - \widetilde{b}_{t+s}^s \right) \tag{1.19}$$

[9] Derivation of the goods market clearing is provided in Appendix A.5.

The term spread is a positive function of long-term bonds held by wealthy households, but a negative function of short-term bonds. Accordingly, by purchasing the long-term bonds of households (QE), the central bank induces a fall in the long-term yield relative to the short-term yield. The term spread experiences a fall, which is actually the way to stimulate households to hold a relatively larger amount of short-term bonds in their portfolio (the preferred habitat theory). In equation (1.19), the transaction costs parameter ϕ_b controls for the influence of the changes in the relative size of bond holdings with different maturities on the term spread. Higher parameter ϕ_b means that households are less motivated to equalise returns through arbitrage behaviour. Although QE makes the short-term bonds more attractive as R_t^L reduces relative to R_t, the parameter ϕ_b discourages households from equalising returns via reallocation of portfolio funds to short-term bonds. In this way, larger portfolio adjustment costs ϕ_b refer to lower substitutability between assets of different maturities and thus a stronger response of the term spread. However, if the transaction costs are absent $\phi_b = 0$, the portfolio rebalancing channel cannot be identified. In this case, the central bank long-term bond purchases do not affect the term spread (and the real economy) because long-term and short-term bonds become perfect substitutes. The term spread remains unchanged when households compensate for the smaller supply of long-term bonds in their portfolio by purchasing short-term bonds in the same amount.

To show the relationship between the term spread and the real economy, we combine the log-linearised first-order condition for consumption of wealthy households and the term spread equation (1.18):

$$\widetilde{\lambda}_{w,t} = \left(\frac{q_L\kappa}{q_L\kappa + q}\right)\left(1 + \frac{\beta q}{\pi q_L \kappa}\right)\widetilde{R}_t + \left(\frac{q_L\kappa}{q_L\kappa + q}\right)\left(\frac{R^L}{R^L - \varrho}\right)\left(\widetilde{R}_t^L - \frac{R^L - \varrho}{R^L}\widetilde{R}_t\right)$$
$$+ \left(\frac{q_L\kappa}{q_L\kappa + q}\right)\left(\frac{\beta\varrho}{\pi}\mathbb{E}_t\widetilde{q}_{L,t+1} + \left(1 + \frac{\beta}{\pi}\frac{1}{q_L\kappa}\right)(\mathbb{E}_t\widetilde{\lambda}_{w,t+1} - \mathbb{E}_t\widetilde{\pi}_{t+1})\right)$$

The above expression indicates that a fall in the term spread, which is triggered by the QE program, leads to a higher consumption of the wealthy and, through the general equilibrium forces, to a higher consumption of the poor. This result comes from:

$$\frac{\partial \widetilde{\lambda}_{w,t}}{\partial \left(\widetilde{R}_t^L - \frac{R^L-\varrho}{R^L}\widetilde{R}_t\right)} = \frac{q_L\kappa}{q_L\kappa + q}\left(\frac{R^L}{R^L-\varrho}\right) > 0,$$

$$\text{and} \qquad \frac{\partial \widetilde{\lambda}_{w,t}}{\partial \widetilde{c}_{w,t}} = -\frac{\sigma_c c_w}{c_w - hC_w} < 0$$

In response to a higher consumption, aggregate demand experiences a rise. Given that the two different types of workers respond differently regarding their respective consumption (due to different income sources), there is a change in consumption inequality. Similarly, the other inequality measures, such as total income and wealth inequality, also experience a change in response to the changes in the real economy induced by QE. The derived expressions related to the inequality measures can be found in Appendix A.9.3.

Considering the portfolio rebalancing channel, previous studies have accounted for household heterogeneity in terms of financial income. However, in these models, the labor market is structured so that different types of workers work the same number of hours and receive the same wage. Consequently, these models cannot explain differences in labor income between different household types. Ampudia et al. (2018) emphasize that labor income is an important component of total income in the EA. To address labor market heterogeneity and the associated labor income inequality, this paper introduces *the earnings heterogeneity channel*. Following Dolado et al. (2021), the present study distinguishes between the roles of high-skilled and low-skilled workers in the production process, but within the context of implemented QE by the ECB. In addition to CSC, the earnings heterogeneity channel includes asymmetric wage rigidity, which is distinctive to the labor market in the EA. Given the interaction between labor and financial markets, wage rigidity affects both labor and non-labor income inequality (wage is a cost part of the wealthy's profits).

To analyze the skill premium dynamics, we log-linearize the equation (1.14):

$$\widetilde{w}_{w,t}-\widetilde{w}_{p,t} = (\sigma-\nu)\rho\left(\frac{K_e}{N_w}\right)^{\nu}\left(\rho\left(\frac{K_e}{N_w}\right)^{\nu}+(1-\rho)\right)^{-1}(\widetilde{K}_{e,t-1}-\widetilde{N}_{w,t})+(\sigma-1)(\widetilde{N}_{w,t}-\widetilde{N}_{p,t}) \quad (1.20)$$

The growth rate of the skill premium is decomposed into two parts. Holding the first component fixed, the second component $(\sigma - 1)(\widetilde{N}_{w,t} - \widetilde{N}_{p,t})$ indicates that the faster growth rate of the relative supply of skilled labor under $\sigma < 1$ reduces the skill premium. Krusell et al. (2000) calls this part the "relative quantity effect". Taking as given the second component, the first component $(\sigma - \nu)\rho\left(\frac{K_e}{N_w}\right)^{\nu}\left(\rho\left(\frac{K_e}{N_w}\right)^{\nu} + (1-\rho)\right)^{-1}(\widetilde{K}_{e,t-1} - \widetilde{N}_{w,t})$ indicates that the faster growth rate of equipment capital than that of skilled labor under $\sigma > \nu$ increases the skill premium. This second component is called the "capital-skill complementarity effect". The trade-off between those two effects determines the dynamics of the skill premium.

Under higher wage rigidity for low-skilled workers, the wage premium increases. If the employment of high-skilled workers is more responsive than equipment capital (due to the need to compensate for lower non-labor income after QE), the CSC effect diminishes. For the wage premium to rise, the relative quantity effect must be stronger than the CSC effect. The section on quantitative results indicates that QE stimulates a rise in both equipment and structures capital in the CSC economy. Importantly, these two components of total capital have different implications for the labor demand. Equipment capital is more complementary with high-skilled labor, while structures capital is more complementary with low-skilled labor. Apart from the labor demand effects, the labor supply of low-skilled workers is more responsive due to less available resources to protect against income fluctuations. Given the labor demand and labor supply responses, the employment of low-skilled workers may increase more, leading to a larger rise in the relative quantity effect relative to the CSC effect.

1.5 Calibration

In Table 1.1, the calibrated values of structural parameters of the model are summarized. The model is calibrated at a quarterly frequency for the period 2000-2014, representing the period before the implementation of the QE program. The group of exogenous parameters is either set to the values consistent with literature or has a data counterpart. Other parameters are set to match the key macroeconomic ratios of the EA-19 economy.

Households are different in terms of their access to financial/capital markets and their labor services offered to the labor market. Following Sakkas and Varthalitis (2021), population shares are set to $s_p = 0.3$ and $s_w = 0.7$ so that 30 percent of the total population in the EA-19 does not participate in capital and financial markets and provides low-skilled labor services. A similar treatment of wealthy and poor households in the US can be found in Bhattarai et al. (2022) and Bilbiie et al. (2022a).

The subjective discount factor, $\beta = 0.9995$, is set to match a net annualised money-market interest rate of 2.21 percent (or a quarterly gross money-market rate of around $R = 1 + \frac{2.21}{4 \cdot 100} = 1.0055$). The coefficient of relative risk aversion of consumption σ_c is set to 1, giving the log utility function in consumption. The Frisch elasticity of labor supply is set to 1. The parameters for labor disutility, $\varphi_{n,w}$ and $\varphi_{n,p}$, are calibrated to obtain the average of skilled and unskilled hours worked per week of $0.247 (= 41.5h/168h)$, and to acknowledge that wealthy households work 8.27% more than poor households in the steady state. The elasticity of utility with respect to real money holdings $\chi = 3.42$ is borrowed from Neiss and Pappa (2005), who estimate this value on the basis of UK data. The choice of $\chi = 3.42$ implies an interest elasticity of money demand of $-1/\chi = -0.29$. The preference parameter for real money holdings in the utility function φ_m is chosen to obtain the steady state real money-to-consumption ratio of 1.905 per quarter. As in Coenen et al. (2008), the money-to-consumption ratio is computed as a ratio of monetary aggregate held by the household sector M1 and nominal consumption expenditure for the period 2000–2014.

The steady state gross inflation rate is set to 1.005, which is in line with the mandate of the ECB (2% annualised inflation). The elasticity of substitution among differentiated retail goods ϵ is set to 6 as in Gerali et al. (2010), which refers to the gross price markup of 20% over marginal cost ($\mu^p = \frac{\epsilon}{\epsilon - 1} = 1.2$). The Rotemberg adjustment cost parameter is set to 59.0259 so that the slope of the Phillips curve in the model corresponds to that in a Calvo staggered price-setting model with four quarters of an average price rigidity. In the Calvo (1983) model, the percent of reoptimizing firms or the average time for which firms set the new prices is $1-\theta$. This implies that the average frequency of price changes is $\frac{1}{1-\theta}$, leading to the value of the Rotemberg (1982) parameter:

$$\phi_p = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\beta\theta)} = \frac{5 \cdot 0.75}{(1-0.75)(1-0.75 \cdot 0.9945)} = 59.0259$$

The capital adjustment costs parameter ϕ_k is set to 5.28 so that the elasticity of the investment to capital ratio with respect to Tobin's q is 13.33 (see Matheron, 2018). For the sake of simplicity, the same parameter value ϕ_k is chosen for both types of capital.[10]

The steady-state level of the technological process A is normalized to 1 for the CSC economy. The depreciation rate of equipment capital δ_e and structures δ_s are used from Krusell et al. (2000). We also use the estimates of the key substitution parameters $\sigma = 0.401$ and $\nu = -0.495$ from Krusell et al. (2000). The choice of $\sigma = 0.401$ implies the elasticity of substitution between equipment capital (or skilled labor) and unskilled labor of $1/(1-\sigma) = 1.67$, while $\nu = -0.495$ implies the elasticity of substitution between equipment capital and skilled labor of $1/(1-\nu) = 0.67$. Thus, the skilled households are more complementary with equipment capital in the production than the unskilled households. That is, the production function exhibits capital–skill complementarity. The parameters corresponding to income shares $m = 0.2977$, $\rho = 0.5685$ and $\iota = 0.1679$ are simultaneously calibrated to match a skill (wage) premium of 1.55 and a labor income share of 65 percent, and the share of equipment capital in total capital of 1/3. The calibrated values of parameters in the production function are in line with those estimated or calibrated in the related literature. Table 1.2 and Table 1.3 present calibrated values in CD1 and CD2 economies for the parameters: A, θ, $\varkappa$, η, δ_k.

To be in line with the average historical EA data, we set government spending to output ratio at 18%, while government debt to output ratio is set to 2.96 or at 74% of annual output. Similar to Albonico and Tirelli (2020), transfers to non-Ricardian households are calibrated to obtain a steady-state consumption ratio between the two groups of households (c_p/c_w) around 0.8. The steady-state difference between aggregate transfers and lump-sum taxes

[10] Derivation for the the elasticity of the investment to capital ratio with respect to Tobin's q is provided in Appendix A.8.

to output ratios (net government transfers/taxes) is then calculated as a residual from the steady state government budget constraint.

The parameters of the fiscal and monetary policy rules are calibrated following Coenen et al. (2008). Specifically, fiscal policy responses to both short-term and long-term debt are set to 0.1. In addition, interest rate sensitivity to inflation gap and output gap are set to 2 and 0.10, respectively. The interest rate smoothing parameter is chosen very close to one as in Falagiarda (2014) to indicate the presence of the ZLB under which the monetary policy (short-term) interest rate is restricted to respond to fluctuations in inflation and output. Cui and Sterk (2021) also assume the ZLB by pegging the nominal interest rate at $R_t = R$ in the model version with QE.

The steady state values of the key variables related to the ECB asset purchase program are summarised in Table 1.4. Data for short-term and long-term bonds outstanding relative to annual GDP is taken from Eurostat Government Finance Statistics. The ECB provides data for 'Securities held for monetary policy purposes - ILM' that serve as a measure for long-term bonds held by the ECB. The amount of long-term bonds held by wealthy households is the difference between total long-term bond supply and long-term government bonds of the EBC. The parameter ϱ is set to match the average duration of 25 quarters long-term government debt, $d = \frac{1}{1-\frac{\beta}{\pi}\varrho} = 25$.

Table 1.1: Parameter values in the baseline CSC economy without real wage rigidity

Notation	Description	Value	Source
Households			
β	Subjective discount factor	0.9995	Calibration
χ	Elasticity of money demand	3.42	Neiss and Pappa (2005)
η	Elasticity of labor supply	1	Convention
σ_c	Coefficient of relative risk aversion	1	Convention
$\varphi_{n,w}$	Relative utility weight on labor-wealthy	16.917	Calibration
$\varphi_{n,p}$	Relative utility weight on labor-poor	14.771	Calibration
s_w	Population share of the wealthy	0.7	Sakkas and Varthalitis (2021)
s_p	Population share of the poor	0.3	Sakkas and Varthalitis (2021)
Intermediate goods firms			
A	Scale parameter	1	Convention
δ_s	Structure capital depreciation rate	0.014	Krusell et al. (2000)
δ_e	Equipment capital depreciation rate	0.031	Krusell et al. (2000)
ι	Structure capital income share	0.1679	Calibration
m	Low-skilled labor income share	0.2977	Calibration
ρ	Equipment capital income share	0.5685	Calibration
σ	Elasticity of subs between K_e and N_p	0.401	Krusell et al. (2000)
ν	Elasticity of subs between K_e and N_w	-0.495	Krusell et al. (2000)
ϕ_k	Capital adjustment cost	5.28	Matheron (2018)
Final goods firms			
ϕ_p	Price adjustment cost	59.0259	Gerali et al. (2010)
ϵ	Elasticity of substitution between retail goods	6	Gerali et al. (2010)
Fiscal and monetary policy			
ρ_1	Fiscal policy response to debt	0.1	Coenen et al. (2008)
θ_π	Monetary policy response to inflation	2	Coenen et al. (2008)
θ_y	Monetary policy response to output	0.1	Coenen et al. (2008)
θ_r	Monetary policy inertia	0.997	Falagiarda (2014)
Π	Gross inflation rate	1.005	Convention
Autoregressive parameters			
ϕ_g	Governement spending	0.9	Coenen et al. (2008)
ϕ_{tr}	Lump-sum transfers	0.9	Coenen et al. (2008)
Standard deviation			
σ_g	Government spending shock	0.18	Coenen et al. (2008)
σ_{tr}	Lump-sum transfers	0.195	Coenen et al. (2008)
σ_r	Monetary policy shock	0.1	Hohberger et al. (2019)

Table 1.2: Parameter values for the CD1 economy with real wage rigidity

Notation	Description	Value	Source
A	Total factor productivity	0.9962	Target output of CSC
θ	Income share of capital	0.35	Data
$\varkappa$	Income share of high-skilled labor	0.7494	Target $\frac{w_w}{w_p} = 1.55$
γ	Elasticity of subs between N_w and N_p	0.2908	Katz and Murphy (1992)
δ_k	Depreciation rate of capital	0.025	Data

Table 1.3: Parameter values for the CD2 economy with no real wage rigidity

Notation	Description	Value	Source
A	Total factor productivity	1	Convention
θ	Income share of capital	0.35	Data
$\varkappa$	Income share of high-skilled labor	0.5	Convention
γ	Elasticity of subs between N_w and N_p	1	Convention
δ_k	Depreciation rate of capital	0.025	Data

1.6 Results

The ECB started with the implementation of the QE program in March 2015. Figure 1.1 shows the impulse responses of selected endogenous variables after a one-standard-deviation QE shock in the Euro Area. Following Hohberger et al. (2019), QE shock is simulated as an AR(2) process so that the initial purchase is followed by a further accumulation of long-term assets by the ECB for another 12 quarters, after which a gradual exit takes place. This paper examines the distributional effects of QE by means of two channels: the portfolio rebalancing channel and the earnings heterogeneity channel.

The portfolio rebalancing channel establishes the relationship between the central bank QE and the whole economy through changes in investors' portfolios. Given that only wealthy

Table 1.4: Calibration in the analysis of asset purchase policy, CSC and CD economies

Notation	Description	Value
$B/4Y = (qB^s + q_L B^l)/4Y$	Total debt to GDP ratio	0.740
$qB^s/4Y$	Total short-term debt to GDP ratio	0.063
$q_L B^l/4Y = q_L(B^{l,h} + B^{l,cb})/4Y$	Total long-term debt to GDP ratio	0.677
$q_L B^{l,h}/4Y$	LT debt held by households	0.622
$q_L B^{l,cb}/4Y$	LT debt held by the central bank	0.055
$f^l = B^{l,cb}/B^l$	Fraction of LT debt by CB in total LT debt	0.0818
ϕ_b	Portfolio adjustment cost parameter	0.0015
$\sigma_{l,cb}$	Magnitude of the asset purchases	0.01
ϕ_{cb1}	Persistence of the asset purchases	0.89
ϕ_{cb2}	Persistence of the asset purchases	0.97
ϱ	Bonds payoff decay factor	0.9653

households have access to financial markets, QE starts having the effects through their portfolio. Specifically, by purchasing long-term government bonds, the central bank expands its balance sheet (the asset side of the balance sheet) and increases liquidity provision to the wealthy (the liability side of the balance sheet). As the wealthy receive central bank reserves (short-term assets) in exchange for long-term government bonds, QE changes the portfolio duration of the wealthy. According to the equation (1.19), a lower supply of $b_t^{l,h}$ relative to b_t^s implies an increase in price $q_{L,t}$ and a reduction in R_t^L. Given that the short-term interest rate is constrained at the (exogenous) ZLB, a smaller R_t^L causes the term-spread to decline. In response to a lower term-spread and to restore the portfolio duration, wealthy households increase investment in other long-term assets (e.g., physical capital), reduce savings in short-term bonds and increase current consumption. Cui and Sterk (2021) highlight the importance of both household consumption and investment in transmitting the QE effects to the real economy. They also show that the increase in investment demand is driven by the need of investors to replace government bonds (direct channel) and by the rise in goods demand (indirect equilibrium channel).

Table 1.5: Selected steady-state values in the CSC economy

Notation	Description	Value
c_p	Consumption of the poor	0.2415
c_w	Consumption of the wealthy	0.3019
n_p	Labor of the poor	0.2335
n_w	Labor of the wealthy	0.2528
w_p	Wage of the poor	0.8329
w_w	Wage of the wealthy	1.2909
r_s^k	Real return to structures	0.0145
r_e^k	Real return to equipment	0.0315
Y	Total output	0.5294

Table 1.6: Selected Steady-state ratios in the CSC economy

Notation	Description	Value
C/Y	Consumption as a share of GDP	0.536
K_s/Y	Structure capital as a share of GDP	9.628
K_e/Y	Equipment capital as a share of GDP	4.814
I_s/Y	Structure investment as a share of GDP	0.135
I_e/Y	Equipment investment as a share of GDP	0.149
G/Y	Government expenditure to GDP ratio	0.180
$B/4Y$	Total debt to GDP ratio	0.740
$T/Y - TR/Y$	Net lump sum tax as a share of GDP	0.178
M/C_w	Money-to-consumption ratio	1.905
w_w/w_p	Skill premium	1.55
li_share	Labor income share	0.65

By stimulating aggregate demand, QE has positive effects on the real economy and inflation.[11] To produce a larger amount of final goods, retail firms increase their demand for intermediate goods as inputs in production, which in turn causes a rise in the relative price of intermediate goods mc_t^r. A higher mc_t^r is associated with a higher demand of intermediate goods firms for capital and labor. An increase in investment and employment leads to higher wages and rental rate on capital. However, high- and low-skilled workers do not enjoy the same rise in wages and employment, an observation that can be explained by *the earnings heterogeneity channel*. Figure 1.1 shows a fall in unskilled employment inequality and in the skill-premium in the short run. The employment of high-skilled workers is more pronounced for two reasons. First, capital-skill complementarity implies a larger demand for skilled labor on the back of increased capital stock. Second, high-skilled workers increase their labor supply to compensate the loss in non-labor income induced by negative profits[12] and lower interest payments on long-term government bonds. As stated by Angelopoulos et al. (2014), a higher real return to capital can also stimulate the skilled to collect larger labor income resources that will be used for capital accumulation. There can be overshooting in labor supply ($\widetilde{K}_{t-1} - \widetilde{N}_{w,t} < 0$) due to a slower adjustment of capital, which results in decreasing CSC effects and the skill premium.[13] However, in the medium/long run, the relative supply of skilled labor decreases while the complementarity between capital stock and skilled labor increases. Both factors give rise to an increasing skill premium and decreasing relative skilled labor income in the medium/long run.

[11] Boeckx et al. (2017), among others, estimate that an exogenous expansion of the ECB's balance sheet has significant stimulative effects on the economic activity and inflation in the EA. In Appendix A.7. we show the QE multipliers on impact and cumulated over different time horizons.

[12] The countercyclical markups or negative profits are a standard feature of the model economies with only sticky prices but absent sticky wages.

[13] To prove that the skill-premium decreases in the short-run, we could use the equation (20). The first component of the skill-premium decreases as $(\sigma - \nu) > 0$ and $\rho\left(\frac{K}{N_w}\right)^{\nu}\left(\rho\left(\frac{K}{N_w}\right)^{\nu} + (1-\rho)\right)^{-1} > 0$ and $(\widetilde{K}_{t-1} - \widetilde{N}_{w,t}) < 0$. The second component also decreases as $(\sigma - 1) < 0$ and $(\widetilde{N}_{w,t} - \widetilde{N}_{p,t}) > 0$.

According to Christoffel et al. (2009), the labor market in the EA is highly rigid in many aspects. This particularly applies to wages that are less prone to instantaneous changes, and as such have a substantial degree of rigidity. The authors argue that the collective wage bargaining process lies behind the sluggish adjustment of wages. To be in line with the empirical findings of Lenza and Slacalek (2018) regarding the skill premium in favor of high-skilled labor and higher employment growth for low-skilled workers after QE, this paper incorporates real wage rigidity[14] as a second component of labor market segmentation. Following Blanchard and Galí (2007), ad-hoc real wage rigidity is introduced such that the slow adjustment of real wages is a result of (unmodelled) distortions instead of preferences in labor markets. For the same wage setting, Kollmann et al. (2016) provide the estimated value of 0.97 (0.96) for real wage rigidity in the EA (US) over the period 1999q1–2014q4. The current study uses the value of 0.97 for the wage of poor households, while the value of 0.8 applies to wealthy households.[15] Wealthy households face lower labor market friction in the form of (upward) real wage rigidity as they are a more valuable labor source for intermediate goods firms and as such enjoy larger (implicitly assumed) bargaining power in the wage determination.

Figure 1.2 reports the dynamic responses of selected variables when CSC and CD1 production functions are interacted with asymmetric real wage rigidity. Compared to the case with flexible wages, both types of workers increase their labor supply to smooth their level of consumption. However, poor households work harder relative to their richer counterparts as they do not have wealth to be protected against the changes in disposable income. Labor

[14]The equation for ad-hoc real wage rigidity is more elaborated in Appendix A.6.1. Interestingly, only asymmetric real wage rigidity is in line with empirical evidence, while symmetric real wage rigidity generates the same qualitative results as the flexible wage setting. In Appendix A.6.2, we also analyse the impulse responses of variables in Calvo-type nominal wage rigidity setting.

[15]The value of the real wage rigidity parameter of 0.8 for wealthy households corresponds approximately to the average of values used by Dolado et al. (2021) and Komatsu (2022). Specifically, Dolado et al. (2021) indicate 33% while Komatsu (2022) refers to 10% lower real wage rigidity for wealthy households. As for nominal wage rigidity, we follow Bilbiie et al. (2022a) who empirically estimate the degree of nominal wage rigidity of 0.70 for skilled and 0.86 for unskilled workers.

demand for the unskilled also increases due to a higher investment in capital structures. Given this stimulus for the employment of the poor, there is a rise in unskilled labor inequality $\widetilde{N}_{p,t} - \widetilde{N}_{w,t} > 0$, which outweighs a fall in capital to skill labor inequality $\widetilde{K}_t - \widetilde{N}_{w,t} < 0$, referring to the stronger relative quantity effects than the CSC effects. The presence of capital-skill complementarity in the economy where labor supply is more responsive than the capital stock implies a mitigated rise in the skill premium, which stimulates poor households to supply even more labor services, pushing up the inequality $\widetilde{N}_{p,t} - \widetilde{N}_{w,t} > 0$. As for the skilled labor income inequality, it experiences a fall up to seven quarters since the beginning of QE, which is consistent with the empirical evidence of Lenza and Slacalek (2018). Although in the short-run poor households are winners regarding total labor income, the model predicts a reversing and less pronounced trend of labor income inequality in favour of wealthy households in the medium and long run.

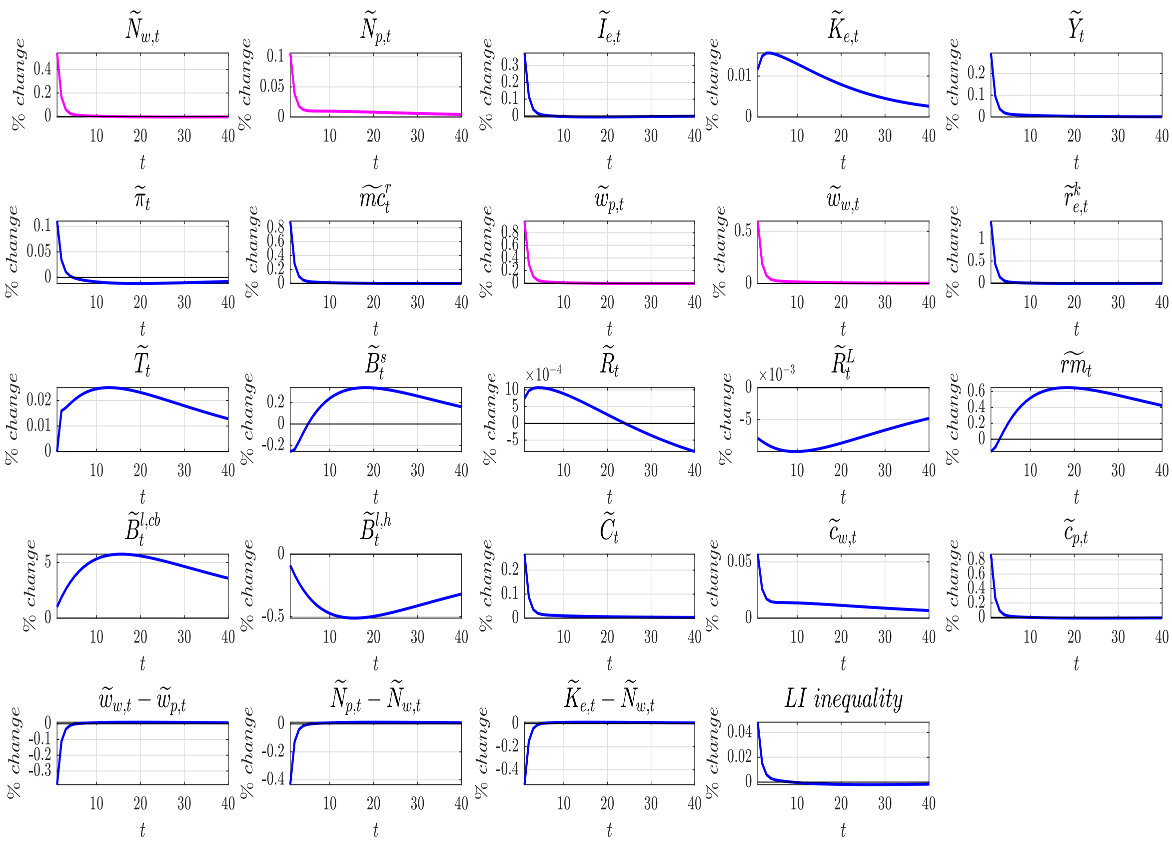

Figure 1.1: IRFs for the quantitative easing shock: CSC (the case of flexible wages)

In Figure 1.3, the economies with segmented labor and financial markets are compared to the economy with only segmented financial market (blue solid line) in terms of four inequality measures. The presence of real wage rigidity induces a drop in labor income inequality (see purple solid and red dashed lines), which becomes mitigated over time. Total income inequality experiences a fall, which is the most pronounced for the economy with only segmented financial market. Similar dynamics can be observed for consumption inequality. Although there is a rise in wealth inequality for all types of economies that persistently remains above the baseline, the segmented labor market generates a larger increase in wealth inequality. As a measure of wealth inequality, we use any increase in the value of asset holdings of wealthy households as a poorer part of the population is excluded from financial/capital markets. Given that the total income of wealthy households can be important for the dynamics of wealth inequality, we next examine the components of the total income.

Figure 1.4 shows that labor and non-labor income go in opposite directions except for the economy CD2+NRW, where poor households enjoy an increase in both components of total income. Generally, households tend to benefit from the rise in labor income, while non-labor income declines. A drop in non-labor income of wealthy households is noticeably mitigated in the economies with real wage rigidity (see green dashed lines for CSC+RW and CD1+RW) due to its counteracting effects on declining profit income. The total income of the wealthy becomes higher, which allows a larger accumulation of assets and thus causes greater wealth inequality. In Figure 1.5, we observe higher investment in capital and a larger amount of real money holdings in the economies CSC+RW and CD1+RW. The presence of real wage rigidity motivates the wealthy to increase capital investment, which enables higher rental income and compensates for lower wage payments, and increases real money holdings due to lower inflation.

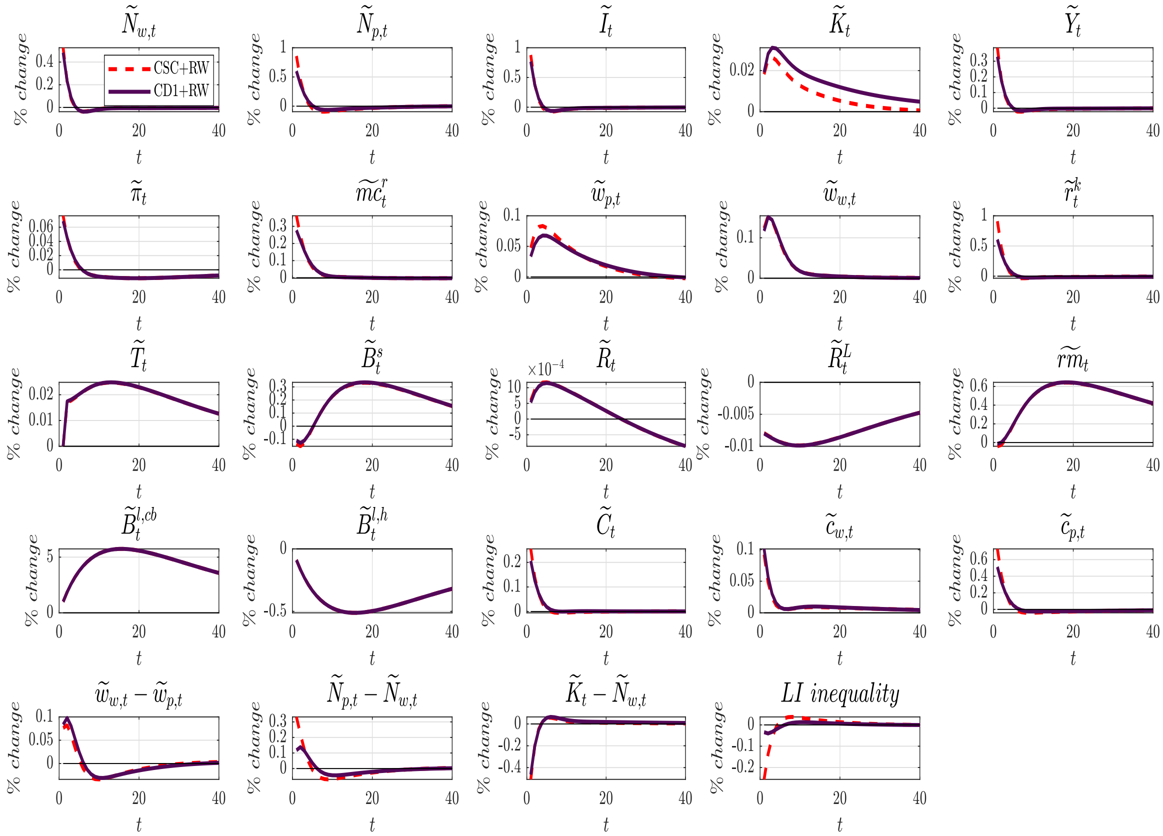

Figure 1.2: IRFs for the quantitative easing shock: CSC vs CD1 (the case of rigid wages)

Notes: For the CSC economy, the variables $\widetilde{I}_t$, $\widetilde{k}_t$, $\widetilde{K}_t$ - $\widetilde{N}_{w,t}$ and $\widetilde{r}_t^k$ stand for equipment investment, equipment capital, equipment to skilled labor ratio and equipment rental rate, respectively. In Appendix 1.8.B, we compare the impulse responses of equipment and structures investment and capital for the CSC economy.

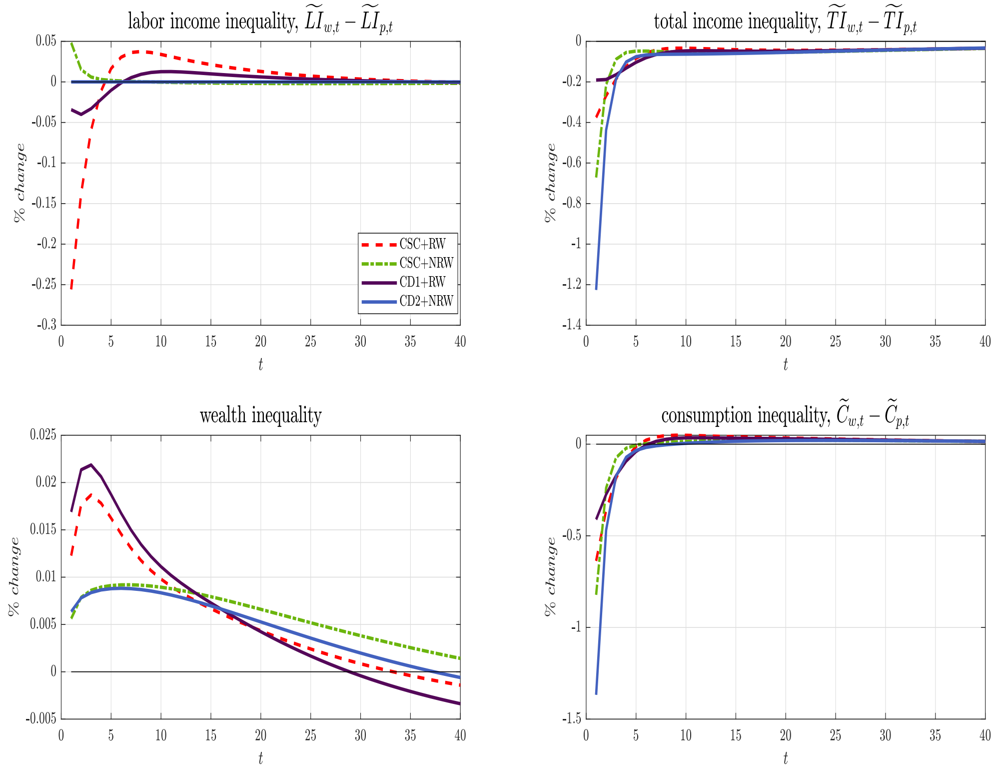

Figure 1.3: Inequality measures: The comparison of CSC and CD economies

Notes: Blue color indicates the portfolio rebalancing channel, while the other colors refer to the interaction of the earnings heterogeneity channel and the portfolio rebalancing channel.

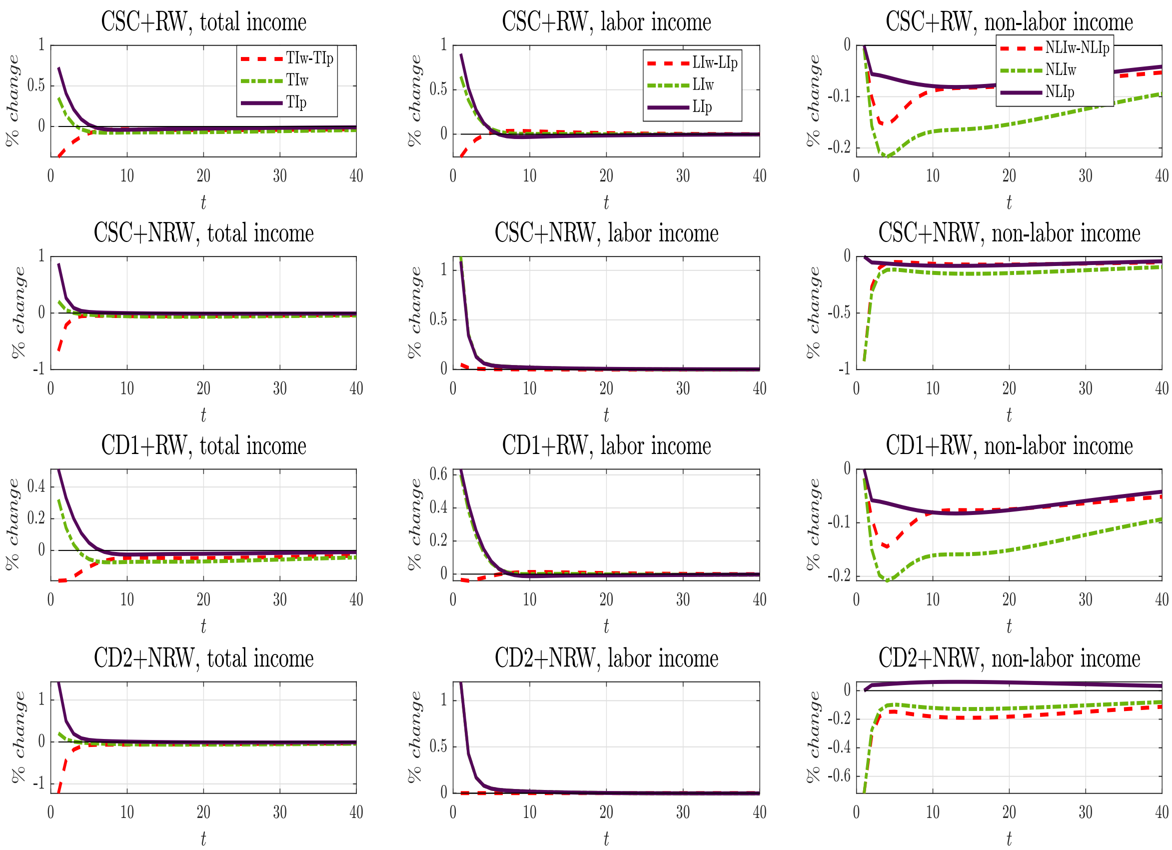

Figure 1.4: Total income components: The comparison of CSC and CD economies

Notes: The economy CD2+NRW includes the portfolio rebalancing channel, while the other economies refer to the interaction of the earnings heterogeneity channel and the portfolio rebalancing channel.

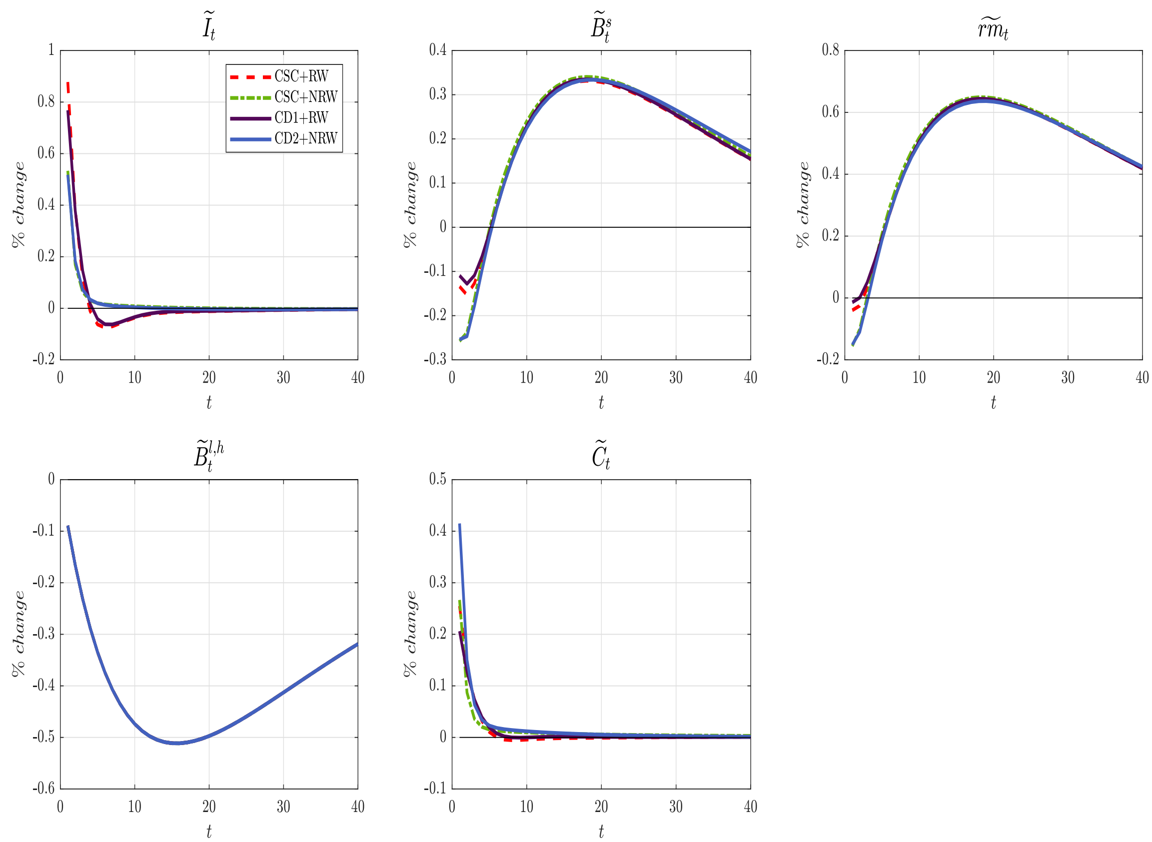

Figure 1.5: Investment sources: The comparison of CSC and CD economies

Notes: Blue color indicates the portfolio rebalancing channel, while the other colors refer to the interaction of the earnings heterogeneity channel and the portfolio rebalancing channel. $\widetilde{I}_t$ is the sum of investment in structure and equipment capital, $I \cdot \widetilde{I}_t = I_s \cdot \widetilde{I}_{s,t} + I_e \cdot \widetilde{I}_{e,t}$.

1.7 Conclusion

In response to the global financial crisis of 2007-2008, the ECB implemented the QE program by injecting central bank reserves into the economic system in exchange for purchased long-term government securities. The main objective of the QE program is to bring the euro area back to its potential in periods when the traditional monetary policy instrument (the short-term policy interest rate) is unavailable due to the zero lower bound. Although QE may be successful in achieving its main goal, there might be side effects of QE such that a certain fraction of the EA population benefits more from QE than the rest of the population. Given that the QE effects may go in opposite directions along different household heterogeneity dimensions, the overall distributional effects of QE could be better examined within a framework that includes joint household heterogeneity.

To have a clearer picture of the inequality effects of QE, this study considers a framework with two dimensions of household heterogeneity. First, we introduce financial market segmentation that separates the EA population of households into two distinct groups on the basis of different access to financial/capital markets. Additionally, labor market segmentation is considered in the form of capital-skill complementarity in the production process and asymmetric real/nominal wage rigidities. This segmentation implies that differently skilled workers work a different number of hours and receive different wages. Compared to the model economy with only financial market segmentation, the results indicate that the interaction of labor and financial market segmentation significantly mitigates a decrease in total income inequality and amplifies a rise in wealth inequality. Casiraghi et al. (2018) state that in the future the ECB will broaden its mandate, focusing on both price stability and the distributional effects of QE. Accordingly, this paper suggests that the ECB could benefit more from the analysis of labor and financial market segmentation as it provides a clearer picture of the inequality effects than the analysis with only financial market segmentation.

1.8 Appendix

1.8.A Model Derivation

A.1 The price-elastic demand of households

Final goods Y_t^f are expressed as the CES aggregate production function according to the equation called the "aggregate output index":

$$Y_t^f = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution among final or intermediate goods due to a linear technology in differentiation process, $Y_t^f(j) = Y_t(j)$.

A demand curve for final goods of each retailer can be derived by referring to the profit maximization problem of retail firms:

$$\max_{Y_t(j)} \quad \int_0^1 P_t(j)Y_t(j)dj - \int_0^1 P_{int,t}Y_t(j)dj$$

Given that the CES aggregate production function makes exact aggregation difficult, Iacoviello (2005) suggests a linear aggregator of the form $Y_t^f = \int_0^1 Y_t(j)dj = Y_t$ within a local region of the steady state.

$$\max_{Y_t(j)} \quad \int_0^1 P_t(j)Y_t(j)dj - P_{int,t}Y_t$$

$$s.t. \;\; Y_t^f = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

We set the Lagrangian function to solve the maximization problem:

$$\mathcal{L} = \int_0^1 P_t(j)Y_t(j)dj - P_{int,t}Y_t - \lambda_t^p \left[\left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} - Y_t^f\right]$$

Taking the FOC with respect to $Y_t(j)$ gives:

$$\int_0^1 P_t(j)dj - \lambda_t^p\left[\frac{\epsilon}{\epsilon-1}\left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}-1}\frac{\epsilon-1}{\epsilon}\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}-1}dj\right] = 0$$

$$\int_0^1 P_t(j)dj - \lambda_t^p(Y_t^f)^{\frac{1}{\epsilon}}\int_0^1 Y_t(j)^{-\frac{1}{\epsilon}}dj = 0$$

$$P_t(j) - \lambda_t^p(Y_t^f)^{\frac{1}{\epsilon}}Y_t(j)^{-\frac{1}{\epsilon}} = 0$$

$$Y_t(j)^{\frac{1}{\epsilon}} = \lambda_t^p\frac{(Y_t^f)^{\frac{1}{\epsilon}}}{P_t(j)} \tag{1.21}$$

$$\Leftrightarrow Y_t(j)^{\frac{\epsilon-1}{\epsilon}} = \left(\lambda_t^p\frac{(Y_t^f)^{\frac{1}{\epsilon}}}{P_t(j)}\right)^{\epsilon-1}$$

$$\Leftrightarrow Y_t^f = \left(\int_0^1 \frac{(\lambda_t^p)^{\epsilon-1}(Y_t^f)^{\frac{\epsilon-1}{\epsilon}}}{P_t(j)^{\epsilon-1}}dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$\Leftrightarrow \lambda_t^p = \left(\int_0^1 P_t(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}} \equiv P_t$$

Plugging P_t into equation (1.21) gives the expression that refers to a downward sloping demand function of each retailer:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t^f$$

A.2 The aggregate price level

As in Dolado et al., 2021, due to differentiation, retailers have pricing power and thus can set the price for their products $P_t(j)$ but take the aggregate price level P_t as given. To derive the aggregate price index, we express the nominal value of output as follows:

$$P_tY_t = \int_0^1 P_t(j)Y_t(j)dj$$

Plugging in the demand for each variety $Y_t(j)$ yields:

$$P_tY_t = \int_0^1 P_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t dj$$

Pulling out the integral things that are independent of j:

$$P_tY_t = P_t^{\epsilon}Y_t \int_0^1 P_t(j)^{1-\epsilon}dj$$

Simplifying, we obtain an expression for the aggregate price level:

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}}$$

A.3 Marginal costs for intermediate goods firms

The (nominal) cost minimization problem of intermediate goods firms for the case of having one type of capital in production:

$$\min_{N_{w,t},N_{p,t},K_{t-1}} \quad TC(Y_{i,t}) = W_{w,t}N_{w,t} + W_{p,t}N_{p,t} + R_t^kK_{t-1}$$

subject to the production technology:

$$A\left[m(N_{p,t})^{\sigma} + (1-m)\Big(\rho(K_{t-1})^{\nu} + (1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}}\right]^{\frac{1}{\sigma}} \geq Y_{i,t}$$

The Lagrangian function related to the cost minimization problem:

$$\begin{aligned}\mathcal{L} &= W_{w,t}N_{w,t} + W_{p,t}N_{p,t} + R_t^kK_{t-1} \\ &- \lambda_t\left(A\left[m(N_{p,t})^{\sigma} + (1-m)\Big(\rho(K_{t-1})^{\nu} + (1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}}\right]^{\frac{1}{\sigma}} - Y_{i,t}\right)\end{aligned}$$

where λ_t is the Lagrange multiplier from the cost minimization problem. The Lagrange parameter related to the technological constraint is the shadow price of change in the ratio of the use of capital and labor services. This means that the Lagrange parameter measures the nominal marginal cost, $\lambda_t = mc_t^n$.

The first order conditions of the minimization problem:

$$R_t^k = \lambda_t F_{k,t} = \lambda_t A\Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}}\Big]^{\frac{1}{\sigma}-1}(1-m)\rho\cdot$$
$$\cdot\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1}(K_{t-1})^{\nu-1}$$

$$W_{w,t} = \lambda_t F_{n,t}^w = \lambda_t A\Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}}\Big]^{\frac{1}{\sigma}-1}\cdot$$
$$\cdot(1-m)(1-\rho)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1}(N_{w,t})^{\nu-1}$$

$$W_{p,t} = \lambda_t F_{n,t}^p = \lambda_t A\Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}}\Big]^{\frac{1}{\sigma}-1}m(N_{p,t})^{\sigma-1}$$

The first order conditions of the minimization problem can be rewritten as:

$$R_t^k = \lambda_t A^\sigma Y_{i,t}^{1-\sigma}(1-m)\rho\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1}(K_{t-1})^{\nu-1}$$
$$W_{w,t} = \lambda_t A^\sigma Y_{i,t}^{1-\sigma}(1-m)(1-\rho)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1}(N_{w,t})^{\nu-1}$$
$$W_{p,t} = \lambda_t A^\sigma Y_{i,t}^{1-\sigma} m(N_{p,t})^{\sigma-1}$$

where the substitution is expressed as

$$A^\sigma Y_{i,t}^{1-\sigma} = A\left[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}}\right]^{\frac{1-\sigma}{\sigma}}$$

We express $N_{p,t}$ from the optimality condition related to the labor supply of the poor:

$$N_{p,t} = \left(\frac{W_{p,t}}{A^\sigma \lambda_t Y_t^{1-\sigma} m}\right)^{\frac{1}{\sigma-1}}$$

and combine the optimality conditions for K_{t-1} and $N_{w,t}$:

$$\frac{K_{t-1}}{N_{w,t}} = \left(\frac{R_t^k(1-\rho)}{W_{w,t}\rho}\right)^{\frac{1}{\nu-1}}$$

so that the second part of the RHS in the production function becomes:

$$\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}} \Leftrightarrow K_{t-1}^\sigma\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^\nu\right)^{\frac{\sigma}{\nu}}$$

Now we have the following in the production function

$$\left(\frac{Y_{i,t}}{A}\right)^{\sigma} = m\left(\frac{W_{p,t}}{A^{\sigma}\lambda_t Y_t^{1-\sigma} m}\right)^{\frac{\sigma}{\sigma-1}} + (1-m)K_{t-1}^{\sigma}\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma}{\nu}}$$

Next, we express K_{t-1}^{σ} from its optimality condition

$$K_{t-1}^{\sigma} = \frac{R_t^k K_{t-1}}{A^{\sigma}\lambda_t Y_t^{1-\sigma}(1-m)\rho\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma-\nu}{\nu}}}$$

to obtain

$$\left(\frac{Y_{i,t}}{A}\right)^{\sigma} = m\left(\frac{W_{p,t}}{A^{\sigma}\lambda_t Y_t^{1-\sigma} m}\right)^{\frac{\sigma}{\sigma-1}} +$$

$$+ (1-m)\frac{R_t^k K_{t-1}}{A^{\sigma}\lambda_t Y_t^{1-\sigma}(1-m)\rho\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma-\nu}{\nu}}}\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma}{\nu}}$$

In the above expression, we plug in K_{t-1}/Y_t:

$$\left(\frac{K_{t-1}}{Y_t}\right) = \left(A^{\sigma}(R_t^k)^{-1}\lambda_t(1-m)\rho\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma-\nu}{\nu}}\right)^{\frac{1}{1-\sigma}}$$

which yields

$$\left(\frac{Y_t}{A}\right)^{\sigma} = \frac{A^{\frac{\sigma^2}{1-\sigma}}Y_t^{\sigma}}{\lambda_t^{\frac{\sigma}{\sigma-1}}}\left(m^{\frac{1}{1-\sigma}}W_{p,t}^{\frac{\sigma}{\sigma-1}} + (1-m)^{\frac{1}{1-\sigma}}\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma}{1-\sigma}\frac{1-\nu}{\nu}}\left(\frac{\rho}{R_t^k}\right)^{\frac{\sigma}{1-\sigma}}\right)$$

Given the optimal allocation, the nominal marginal costs for intermediate goods firms for the case of having one type of capital in production are:

$$\lambda_t = \frac{1}{A}\left(m^{\frac{1}{1-\sigma}}W_{p,t}^{\frac{\sigma}{\sigma-1}} + (1-m)^{\frac{1}{1-\sigma}}\left(\rho^{\frac{1}{1-\nu}}(R_t^k)^{\frac{\nu}{\nu-1}} + (1-\rho)^{\frac{1}{1-\nu}}(W_{w,t})^{\frac{\nu}{\nu-1}}\right)^{\frac{\sigma}{1-\sigma}\frac{1-\nu}{\nu}}\right)^{\frac{\sigma-1}{\sigma}}$$

The marginal cost represents the cost, relative to each production factor, of producing an additional unit of the intermediate goods. All intermediate goods firms have the same marginal costs as they share the same technology and have the same prices of the production factors.

A.4 Long-Term Bond prices

If long-term government bonds are treated as perpetuities, we can keep track of the stock of total long-term government bonds rather than individual issues. In addition, we obtain the information about total payments that investors can receive in period t by purchasing perpetuities issued s periods ago, $b_{t-s}^{l,h}$. In this regard, the budget constraint of wealthy households where the focus is on the nominal long-term government bonds is:

$$q_{L,t}b_t^{l,h} + \ldots = \frac{1}{\pi_t}\sum_{s=1}^{\infty}\varrho^{s-1}b_{t-s}^{l,h} + \ldots$$

Following Niestroj et al. (2013), we define $\mathcal{B}_{t-1}^{l,h}$, the stock of long-term bonds in period t, as the sum of all nominal payments accumulated on past bond purchases in period t:

$$\mathcal{B}_{t-1}^{l,h} = \sum_{s=1}^{\infty}\varrho^{s-1}b_{t-s}^{l,h}$$

while corresponding $\mathcal{B}_t^{l,h}$ is defined as:

$$\mathcal{B}_t^{l,h} = \sum_{s=1}^{\infty}\varrho^{s-1}b_{t+1-s}^{l,h}$$

Given the definition of $\mathcal{B}_t^{l,h}$:

$$\mathcal{B}_t^{l,h} = \sum_{s=1}^{\infty}\varrho^{s-1}b_{t+1-s}^{l,h} = b_t^{l,h} + \sum_{s=2}^{\infty}\varrho^{s-1}b_{t+1-s}^{l,h} = b_t^{l,h} + \sum_{s=1}^{\infty}\varrho^{(s+1)-1}b_{t+1-(s+1)}^{l,h} = b_t^{l,h} + \varrho\sum_{s=1}^{\infty}\varrho^{s-1}b_{t-s}^{l,h},$$

we can relate the above two terms as follows:

$$\mathcal{B}_t^{l,h} = b_t^{l,h} + \varrho\mathcal{B}_{t-1}^{l,h}$$

and the budget constraint becomes:

$$q_{L,t}(\mathcal{B}_t^{l,h} - \varrho\mathcal{B}_{t-1}^{l,h}) + \ldots = \frac{1}{\pi_t}\mathcal{B}_{t-1}^{l,h} + \ldots$$

$$q_{L,t}\mathcal{B}_t^{l,h} + \ldots = \frac{1}{\pi_t}(1 + \varrho q_{L,t})\mathcal{B}_{t-1}^{l,h} + \ldots$$

The LHS term can be written as:

$$q_{t,t}^L\mathcal{B}_{t,t}^{l,h} = q_{t,t}^L(\varrho\mathcal{B}_{t,t-1}^{l,h} + b_{t,t}^{l,h}) = q_{t,t}^L b_t^{l,h} + q_{t,t}^L\varrho\sum_{s=1}^{\infty}\varrho^{s-1}b_{t,t-s}^{l,h} = q_{t,t}^L b_{t,t}^{l,h} + \sum_{s=1}^{\infty}q_{t,t-s}^L b_{t,t-s}^{l,h}$$

where in the last part of the above expression we use $q^L_{t,t-s} = \varrho^s q^L_{t,t}$.

The RHS term can be written as:

$$\frac{1}{\pi_{t,t}}(1+\varrho q^L_{t,t})\mathcal{B}^{l,h}_{t-1,t-1} = \frac{1}{\pi_{t,t}}\left(1+\varrho\frac{q^L_{t,t-s}}{\varrho^s}\right)\mathcal{B}^{l,h}_{t-1,t-1} =$$

$$= \frac{1}{\pi_{t,t}}\sum_{s=1}^{\infty}(1+\varrho^{1-s}q^L_{t,t-s})\varrho^{s-1}b^{l,h}_{t-1,t-s} = \frac{1}{\pi_{t,t}}\sum_{s=1}^{\infty}(\varrho^{s-1}+q^L_{t,t-s})b^{l,h}_{t-1,t-s}$$

where we use $\mathcal{B}^{l,h}_{t-1,t-1} = \sum\limits_{s=1}^{\infty}\varrho^{s-1}b^{l,h}_{t-1,t-s}$.

The budget constraint of the wealthy becomes:

$$q^L_{t,t}b^{l,h}_{t,t} + \sum_{s=1}^{\infty}q^L_{t,t-s}b^{l,h}_{t,t-s} + \ldots = \frac{1}{\pi_{t,t}}\sum_{s=1}^{\infty}(\varrho^{s-1}+q^L_{t,t-s})b^{l,h}_{t-1,t-s} + \ldots$$

Following Niestroj et al. (2013), assume that nominal debt in period $t \geq 0$ is $\sum\limits_{s=1}^{\infty}q^L_{t,t-s}b^{l,h}_{t,t-s} = 0$. This assumption is used to prove that the term on the RHS of the budget constraint related to the nominal long-term government bonds can be transformed into $(1+\varrho q_{L,t})b^{l,h}_{t-1}$:

$$\sum_{s=1}^{\infty}(\varrho^{s-1}+q^L_{t,t-s})b^{l,h}_{t-1,t-s} = (1+q^L_{t,t-1})b^{l,h}_{t-1,t-1} + \sum_{s=2}^{\infty}(\varrho^{s-1}+q^L_{t,t-s})b^{l,h}_{t-1,t-s} =$$

$$= (1+q^L_{t,t-1})b^{l,h}_{t-1,t-1} + \sum_{s=1}^{\infty}(\varrho^{s}+q^L_{t,t-s-1})b^{l,h}_{t-1,t-s-1} =$$

$$= (1+q^L_{t,t-1})b^{l,h}_{t-1,t-1} + \sum_{s=1}^{\infty}\left(\frac{\varrho^s q^L_{t,t-s-1}}{q^L_{t-1,t-s-1}}\right)q^L_{t-1,t-s-1}b^{l,h}_{t-1,t-s-1} =$$

$$= (1+q^L_{t,t-1})b^{l,h}_{t-1,t-1} + \left(\frac{\varrho q^L_{t,t}+1}{q^L_{t-1,t-1}}\right)\sum_{s=1}^{\infty}q^L_{t-1,t-s-1}b^{l,h}_{t-1,t-s-1} = (1+\varrho q^L_{t,t})b^{l,h}_{t-1,t-1}$$

Given the above expression for the payments on long-term government bonds, the budget constraint of the wealthy is written in a more convenient recursive way. Long-term government bonds are treated as perpetuities that pay coupon payments of 1, ϱ, ϱ^2,... in periods

$t+1$, $t+2$, $t+3$,..., respectively. This assumption implies that a payoff of one unit from holding a bond issued s periods ago is equivalent to a payoff of ϱ^s from holding a bond issued today. As in Carlstrom et al. (2017), $q_{L,t}$ is the new issue price that summarizes the prices at all maturities, while $\varrho q_{L,t}$ is the time-t price of the perpetuity issued in period $t-1$.

A.5 The aggregate resource constraint

If the budget constraint of households and government are satisfied, and the market clearing condition holds for $n-1$ markets, then Walras's law implies that the $n-th$ (goods) market will also be in equilibrium.

1. The real budget constraint of wealthy household:

$$s_w\Bigg(c_{w,t}+q_t b_t^s+q_{L,t}b_t^{l,h}\left(1+\frac{\phi_b}{2}\Big(\kappa\frac{b_t^s}{b_t^{l,h}}-1\Big)^2\right)+t_{w,t}+\sum_{\varsigma\in\{s,e\}}(k_{\varsigma,t}-(1-\delta_\varsigma)k_{\varsigma,t-1})+m_t=w_{w,t}n_{w,t}+$$

$$+\frac{b_{t-1}^s}{\pi_t}+(1+\varrho q_{L,t})\frac{b_{t-1}^{l,h}}{\pi_t}-\sum_{\varsigma\in\{s,e\}}\frac{\phi_k}{2}\Big(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}}-1\Big)^2 k_{\varsigma,t}+\sum_{\varsigma\in\{s,e\}}r^k_{\varsigma,t}k_{\varsigma,t-1}+\frac{m_{t-1}}{\pi_t}+tr_{w,t}+\frac{\Pi_t^{int}}{s_w}+\frac{\Pi_t^r}{s_w}\Bigg)$$

Real profits are distributed as dividends to wealthy household:

$$\Pi_t^{int}=\frac{Y_{int,t}}{x_t}-w_{w,t}N_{w,t}-w_{p,t}N_{p,t}-r^k_{s,t}K_{s,t-1}-r^k_{e,t}K_{e,t-1},$$

$$\Pi_t^r=\left(1-\frac{1}{x_t}-\frac{\phi_p}{2}(\frac{\pi_t}{\pi}-1)^2\right)Y_t,$$

$$Y_t=Y_{int,t}$$

2. The real budget constraint of poor household:

$$s_p\Big(c_{p,t}+t_{p,t}=w_{p,t}n_{p,t}+tr_{p,t}\Big)$$

3. The consolidated government budget constraint (in aggregate real terms):

$$T_t + q_t B_t^s + q_{L,t} B_t^l + M_t - \frac{M_{t-1}}{\pi_t} - \left(q_{L,t} B_t^{l,cb} - (1+\varrho q_{L,t}) \frac{B_{t-1}^{l,cb}}{\pi_t} \right) = \frac{B_{t-1}^s}{\pi_t} + (1+\varrho q_{L,t}) \frac{B_{t-1}^l}{\pi_t} + G_t + TR_t$$

The distribution of lump-sum taxes is:

$$T_t = s_w t_{w,t} + s_p t_{p,t}$$

The distribution of lump-sum transfers is:

$$TR_t = s_w tr_{w,t} + s_p tr_{p,t}$$

To derive the aggregate resource constraint, we start with the government budget constraint and express the distribution of lump-sum taxes:5

$$T_t \equiv s_w t_{w,t} + s_p t_{p,t} = \frac{B_{t-1}^s}{\pi_t} + (1+\varrho q_{L,t}) \frac{B_{t-1}^l}{\pi_t} + G_t + TR_t - q_t B_t^s - q_{L,t} B_t^l - M_t + \frac{M_{t-1}}{\pi_t} + \left(q_{L,t} B_t^{l,cb} - (1+\varrho q_{L,t}) \frac{B_{t-1}^{l,cb}}{\pi_t} \right)$$

Then, we express the lump-sum taxes from the household budget constraints and substitute them into the government budget constraint to obtain:

$$s_w w_{w,t} n_{w,t} + s_w \frac{b_{t-1}^s}{\pi_t} + s_w (1+\varrho q_{L,t}) \frac{b_{t-1}^{l,h}}{\pi_t} - \sum_{\varsigma \in \{s,e\}} s_w \frac{\phi_k}{2} \Big(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \Big)^2 k_{\varsigma,t} + \sum_{\varsigma \in \{s,e\}} s_w r_{\varsigma,t}^k k_{\varsigma,t-1} +$$

$$+ s_w \frac{m_{t-1}}{\pi_t} + s_w tr_{w,t} + \Pi_t^{int} + \Pi_t^r - s_w c_{w,t} - s_w q_t b_t^s - s_w q_{L,t} b_t^{l,h} \left(1 + \frac{\phi_b}{2} \Big(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \Big)^2 \right) -$$

$$- \sum_{\varsigma \in \{s,e\}} s_w (k_{\varsigma,t} - (1-\delta_\varsigma) k_{\varsigma,t-1}) - s_w m_t + s_p w_{p,t} n_{p,t} + s_p tr_{p,t} - s_p c_{p,t} =$$

$$\frac{B_{t-1}^s}{\pi_t} + (1+\varrho q_{L,t}) \frac{B_{t-1}^l}{\pi_t} + G_t + TR_t - q_t B_t^s - q_{L,t} B_t^l - M_t + \frac{M_{t-1}}{\pi_t} + \left(q_{L,t} B_t^{l,cb} - (1+\varrho q_{L,t}) \frac{B_{t-1}^{l,cb}}{\pi_t} \right)$$

Aggregating terms in the previous expression and given the market clearing conditions, we obtain the expression for the aggregate resource constraint.

The following market clearing conditions are satisfied

in the labour market:

$$N_{w,t} = s_w n_{w,t}, \quad N_{p,t} = s_p n_{p,t}$$

in the capital market:

$$K_{s,t} = s_w k_{s,t}, \quad K_{e,t} = s_w k_{e,t}$$

in the bond market:

$$B_t = B_t^s + B_t^l = B_t^s + B_t^{l,h} + B_t^{l,cb}$$

and in the money market:

$$M_t = s_w m_t$$

The aggregate resource constraint or the goods market clearing is:

$$Y_t = C_t + I_t + G_t + \sum_{\varsigma \in \{s,e\}} s_w \frac{\phi_k}{2} \left(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \right)^2 k_{\varsigma,t} + \frac{\phi_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 Y_t - q_{L,t} s_w b_t^{l,h} \frac{\phi_b}{2} \left(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \right)^2$$

A.6 Sticky wage settings

To better understand the role of wage rigidity in transmitting the effects of QE, this study considers both Calvo-type nominal wage rigidity in the spirit of Galí et al. (2008) and ad-hoc real wage rigidity as in Blanchard and Galí (2007). These two standard wage settings are extended to incorporate asymmetric wage rigidities for differently skilled workers.

A6.1 Ad-hoc real wage rigidity

As in Blanchard and Galí (2007), the real wage rigidity equation indicates that the current period real rigid wage is a function of the previous period real rigid wage and the household's marginal rate of substitution between consumption and leisure:

$$w_{k,t} = w_{k,t-1}^{\rho_w^k} \left(\varphi_{n,w} \cdot n_{k,t}^{\eta} \cdot c_{k,t} \right)^{1-\rho_w^k}$$

where ρ_w^k can be interpreted as an index of real wage rigidity for skill level $k \in \{w, p\}$ and $mrs_{k,t} = w_{k,t} = \varphi_{n,w} \cdot n_{k,t}^{\eta} \cdot c_{k,t}$ is the household's marginal rate of substitution between consumption and leisure for the case of $h = 0$ and $\sigma_c = 1$. This is a modified intertemporal optimality condition related to household's labor supply.

From the firm's side, we have the expression for the labor demand (*the market/contract wage*) as a function of the real marginal cost and marginal product of labor:

$$w_{k,t} = mc_t^r \cdot F_{n,t}^k$$

From the consumer-worker's side, we have labor supply relation (*the desired wage*):

$$w_{k,t} = mrs_{k,t} = \varphi_{n,w} \cdot n_{k,t}^{\eta} \cdot c_{k,t}, \quad \text{for } \sigma_c = 1 \text{ and } h = 0.$$

The log-linearized ad-hoc real wage rigidity for wealthy and poor households are as follows:

$$\widetilde{w}_{w,t} = \rho_w^w \cdot \widetilde{w}_{w,t-1} + (1 - \rho_w^w) \cdot \widetilde{mrs}_{w,t}, \qquad \widetilde{mrs}_{w,t} = \widetilde{w}_{w,t} = \eta \widetilde{n}_{w,t} + \widetilde{c}_{w,t}$$

$$\widetilde{w}_{p,t} = \rho_w^p \cdot \widetilde{w}_{p,t-1} + (1 - \rho_w^p) \cdot \widetilde{mrs}_{p,t}, \qquad \widetilde{mrs}_{p,t} = \widetilde{w}_{p,t} = \eta \widetilde{n}_{p,t} + \widetilde{c}_{p,t}$$

In the steady state we have

$$w_w = mrs_w, \qquad w_p = mrs_p$$

This paper focuses on the comparison of inequality measures between asymmetric real wage rigidity and flexible (symmetric) real wages. Although the response of inequality measures is somewhat dampened with the symmetric real wage setting, CSC and CD economies indicate the same qualitative results for flexible and symmetric real wage frameworks. The introduction of symmetric wage rigidity makes the poor work harder, but strong income effects have an influence on the wealthy to work even more than the poor.

The case of asymmetric real wage rigidity refers to

$$\rho_w^w = 0.8 \text{ and } \rho_w^p = 0.97,$$

while the case of flexible real wages indicates

$$\rho_w^w = 0 \text{ and } \rho_w^p = 0.$$

A.6.2 Calvo nominal wage rigidity

Labor demand from intermediate goods firms

The production function of intermediate goods firms is:

$$Y_{int,t} = F(K_{t-1}, N_{w.t}, N_{p,t}) = A\Big[m(N_{p,t})^{\sigma} + (1-m)\Big(\rho(K_{t-1})^{\nu} + (1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}}\Big]^{\frac{1}{\sigma}}$$

The aggregate skilled and unskilled labor inputs are:

$$N_{w,t} = s_w \cdot n_{w,t}, \qquad N_{p,t} = s_p \cdot n_{p,t},$$

where an index of (skilled and unskilled) labor input at the level of household is specified as the quantity of a continuum of different labor types $z \in [0,1]$ in period t:

$$n_{w,t} = \left(\int_0^1 n_{w,t}(z)^{\frac{\epsilon_w - 1}{\epsilon_w}} dz\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \qquad n_{p,t} = \left(\int_0^1 n_{p,t}(z)^{\frac{\epsilon_w - 1}{\epsilon_w}} dz\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

The parameter ϵ_w measures the elasticity of substitution among labor varieties. Following Galí et al. (2008), workers of each type-z have monopolistic power in the labor market and set nominal wages, which firms take as given. The demand for each type of differentiated labor services $z \in [0,1]$ is given by:

$$n_{w,t}(z) = \left(\frac{W_{w,t}(z)}{W_{w,t}}\right)^{-\epsilon_w} n_{w,t}, \qquad n_{p,t}(z) = \left(\frac{W_{p,t}(z)}{W_{p,t}}\right)^{-\epsilon_w} n_{p,t},$$

where $W_{w,t}(z)$ is nominal wage paid to the labor type-z, and $W_{w,t}$ is an aggregate wage index specified as:

$$W_{w,t} = \left(\int_0^1 W_{w,t}(z)^{1-\epsilon_w} dz\right)^{\frac{1}{1-\epsilon_w}}, \qquad W_{p,t} = \left(\int_0^1 W_{p,t}(z)^{1-\epsilon_w} dz\right)^{\frac{1}{1-\epsilon_w}}$$

Given that in equilibrium a fraction of nominal wages θ_w cannot be adjusted optimally, we have:

$$W_{w,t} = \left[\theta_w W_{w,t-1}^{1-\epsilon_w} + (1-\theta_w)(W_{w,t}^{*})^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}, \quad W_{p,t} = \left[\theta_w W_{p,t-1}^{1-\epsilon_w} + (1-\theta_w)(W_{p,t}^{*})^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

The above equation is log-linearised around the zero (wage) inflation steady state to yield

$$\widetilde{W}_{w,t} = \theta_w \widetilde{W}_{w,t-1} + (1-\theta_w)\widetilde{W}_{w,t}^{*}, \qquad \widetilde{W}_{p,t} = \theta_w \widetilde{W}_{t-1} + (1-\theta_w)\widetilde{W}_{p,t}^{*}$$

Labor supply from households

In this section, a detailed derivation of the wage Phillips curve is provided for only wealthy households as the same derivation applies to poor households.

The wealthy of type $z \in [0,1]$ choose $W^*_{w,t}$ to maximize their utility subject to the resource constraint and the labor demand for each of their variety:

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U(c_{w,t+k}(z), m_{t+k}(z), n_{w,t+k}(z))$$

$$s.t. \;\; P_{t+k} c_{w,t+k}(z) + \dots \leq W^*_{w,t} n_{w,t+k}(z) + \dots$$

$$n_{w,t+k}(z) = \left(\frac{W^*_{w,t}}{W_{w,t+k}}\right)^{-\epsilon_w} n_{w,t+k}$$

The Lagrangian function associated with the maximization problem of wealthy household is:

$$\mathcal{L} = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \Big(U(c_{w,t+k}(z), m_{t+k}(z), n_{w,t+k}(z)) - \\ - \lambda_{w,t+k}(z) \Big(P_{t+k} c_{w,t+k}(z) - W^*_{w,t} (\frac{W^*_{w,t}}{W_{w,t+k}})^{-\epsilon_w} n_{w,t+k} \Big) \Big)$$

The optimality condition related to consumption is:

$$[c_{w,t+k}(z)] : \quad U_{c,t+k} = \lambda_{w,t+k}(z) P_{t+k}$$

The optimality condition related to nominal wage is:

$$[W^*_{w,t}] : \quad E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \Big(U_{n,t+k}(-\epsilon_w) \Big(\frac{W^*_{w,t}}{W_{w,t+k}}\Big)^{-\epsilon_w - 1} \frac{n_{w,t+k}}{W_{w,t+k}} + \\ + \lambda_{w,t+k}(z)(1-\epsilon_w) \Big(\frac{W^*_{w,t}}{W_{w,t+k}}\Big)^{-\epsilon_w} n_{w,t+k} \Big) = 0$$

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(U_{c,t+k} n_{w,t+k}(z) \Big(\frac{W^*_{w,t}}{P_{t+k}} - \mu_w MRS_{t+k}(z) \Big) \right) = 0 \tag{1.22}$$

where $\mu_w = \frac{\epsilon_w}{\epsilon_w - 1}$ is the wage markup, and $MRS_{t+k}(z) = -\frac{U_{n,t+k}}{U_{c,t+k}}$ is the marginal rate of substitution between consumption and labor. The equation (22) in the zero inflation steady state is

$$\frac{W^*_w}{P} = \frac{W_w}{P} = \mu_w MRS$$

When the equation (22) is log-linearised around the zero inflation steady state, we have:

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k n_w U_c \left(\frac{W^*_w}{P} - \mu_w MRS \right) (\widetilde{n}_{w,t+k} + \widetilde{U}_{c,t+k}) +$$

$$+ E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k n_w U_c \left(\frac{W_w}{P} (\widetilde{W}^*_{w,t} - \widetilde{P}_{t+k}) - \mu_w MRS \widetilde{MRS}_{t+k}(z) \right) = 0$$

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\widetilde{W}^*_{w,t} - \widetilde{P}_{t+k} - \widetilde{MRS}_{t+k}(z) \right) = 0$$

The above expression gives wage setting rule

$$\widetilde{W}^*_{w,t} = (1 - \beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\widetilde{P}_{t+k} + \widetilde{MRS}_{t+k}(z) \right)$$

In the next step, we derive the expression for $\widetilde{MRS}_{t+k}(z)$ in wage setting rule.

The utility function of wealthy household (assuming that habit in consumption is $h = 0$):

$$U(n_{w,t}(z), m_t(z), n_{w,t}(z)) = log(c_{w,t}(z)) + \frac{\varphi_m}{1-\chi} (m_t(z))^{1-\chi} - \varphi_{n,w} \frac{(n_{w,t}(z))^{1+\eta}}{1+\eta}$$

As in equilibrium $c_{w,t}(z) = c_{w,t}$ due to the complete markets assumption, the marginal rate of substitution is:

$$MRS_t(z) = -\frac{U_{n,t}}{U_{c,t}} = \varphi_{n,w} \cdot n^{\eta}_{w,t}(z) \cdot c_{w,t}$$

The log-linear approximation of the above expression gives

$$\widetilde{MRS}_t(z) = \eta \widetilde{n}_{w,t}(z) + \widetilde{c}_{w,t} \tag{1.23}$$

The average MRS is:

$$\widetilde{MRS}_t = \eta \widetilde{n}_{w,t} + \widetilde{c}_{w,t}$$

When equation (23) is rewritten as a function of the average MRS:

$$\widetilde{MRS}_t(z) = \eta\widetilde{n}_{w,t}(z) + \widetilde{c}_{w,t} = \eta\widetilde{n}_{w,t}(z) + \widetilde{MRS}_t - \eta\widetilde{n}_{w,t} = \widetilde{MRS}_t + \eta(\widetilde{n}_{w,t}(z) - \widetilde{n}_{w,t})$$

Labor demand for wealthy households

$$n_{w,t}(z) = \left(\frac{W_{w,t}(z)}{W_{w,t}}\right)^{-\epsilon_w} n_{w,t}, \qquad \widetilde{n}_{w,t}(z) - \widetilde{n}_{w,t} = -\epsilon_w(\widetilde{W}^*_{w,t} - \widetilde{W}_{w,t})$$

Substituting the log-linearised labor demand in $\widetilde{MRS}_t(z)$ gives:

$$\widetilde{MRS}_t(z) = \widetilde{MRS}_t - \eta\epsilon_w(\widetilde{W}^*_{w,t} - \widetilde{W}_{w,t}),$$

and then substituting the household's z marginal rate of substitution in wage setting rule gives:

$$\widetilde{W}^*_{w,t} = (1-\beta\theta_w)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left(\widetilde{P}_{t+k} + \widetilde{MRS}_{t+k} - \eta\epsilon_w(\widetilde{W}^*_{w,t} - \widetilde{W}_{w,t+k})\right)$$

$$\widetilde{W}^*_{w,t} = \left(\frac{1-\beta\theta_w}{1+\eta\epsilon_w}\right)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left((1+\eta\epsilon_w)\widetilde{W}_{w,t+k} - (\widetilde{W}_{w,t+k} - \widetilde{P}_{t+k} - \widetilde{MRS}_{t+k})\right)$$

Defining the gross average wage markup as the ratio between the real wage and the average marginal rate of substitution and log-linearising it, we obtain:

$$\mu_{w,t} = \frac{W_{w,t}}{P_t}\frac{1}{MRS_t}, \qquad \widetilde{\mu}_{w,t} = (\widetilde{W}_{w,t} - \widetilde{P}_t) - \widetilde{MRS}_t,$$

where $\widetilde{w}_{w,t} = \widetilde{W}_{w,t} - \widetilde{P}_t$ is the real average wage.

Substituting the gross average wage markup in the optimal wage equation yields:

$$\widetilde{W}^*_{w,t} = \left(\frac{1-\beta\theta_w}{1+\eta\epsilon_w}\right)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left((1+\eta\epsilon_w)\widetilde{W}_{w,t+k} - \widetilde{\mu}_{w,t+k}\right)$$

Moving it one period ahead

$$\widetilde{W}^*_{w,t+1} = \left(\frac{1-\beta\theta_w}{1+\eta\epsilon_w}\right)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left((1+\eta\epsilon_w)\widetilde{W}_{w,t+k+1} - \widetilde{\mu}_{w,t+k+1}\right),$$

and striping out a one-period expression from the sum

$$\widetilde{W}^*_{w,t} = \frac{1-\beta\theta_w}{1+\eta\epsilon_w}((1+\eta\epsilon_w)\widetilde{W}_{w,t} - \widetilde{\mu}_{w,t}) + \left(\frac{1-\beta\theta_w}{1+\eta\epsilon_w}\right)E_t\sum_{k=1}^{\infty}(\beta\theta_w)^k\left((1+\eta\epsilon_w)\widetilde{W}_{w,t+k} - \widetilde{\mu}_{w,t+k}\right)$$

For $j = k - 1$, we have:

$$\widetilde{W}^*_{w,t} = \frac{1-\beta\theta_w}{1+\eta\epsilon_w}((1+\eta\epsilon_w)\widetilde{W}_{w,t} - \widetilde{\mu}_{w,t}) + \left(\frac{1-\beta\theta_w}{1+\eta\epsilon_w}\right) E_t \sum_{j=0}^{\infty} (\beta\theta_w)^{j+1} \left((1+\eta\epsilon_w)\widetilde{W}_{w,t+j+1} - \widetilde{\mu}_{w,t+j+1}\right)$$

$$\widetilde{W}^*_{w,t} = \frac{1-\beta\theta_w}{1+\eta\epsilon_w}((1+\eta\epsilon_w)\widetilde{W}_{w,t} - \widetilde{\mu}_{w,t}) + \left(\frac{1-\beta\theta_w}{1+\eta\epsilon_w}\right) (\beta\theta_w) E_t \sum_{j=0}^{\infty} (\beta\theta_w)^{j} \left((1+\eta\epsilon_w)\widetilde{W}_{w,t+j+1} - \widetilde{\mu}_{w,t+j+1}\right)$$

$$\widetilde{W}^*_{w,t} = \frac{1-\beta\theta_w}{1+\eta\epsilon_w}((1+\eta\epsilon_w)\widetilde{W}_{w,t} - \widetilde{\mu}_{w,t}) + \beta\theta_w E_t \widetilde{W}^*_{w,t+1}$$

The desired wage for wealthy households is given by

$$\widetilde{W}^*_{w,t} = \beta\theta_w E_t \widetilde{W}^*_{w,t+1} + (1-\beta\theta_w)(\widetilde{W}_{w,t} - \frac{1}{1+\eta\epsilon_w}\widetilde{\mu}_{w,t})$$

If this desired wage is put in the average wage equation (from labor demand condition)

$$\widetilde{W}_{w,t} = \theta_w \widetilde{W}_{w,t-1} + (1-\theta_w)\Big(\beta\theta_w E_t \widetilde{W}^*_{w,t+1} + (1-\beta\theta_w)(\widetilde{W}_{w,t} - \frac{1}{1+\eta\epsilon_w}\widetilde{\mu}_{w,t})\Big)$$

$$\widetilde{W}_{w,t} - \widetilde{W}_{w,t-1} = (1-\theta_w)\beta(E_t \widetilde{W}^*_{w,t+1} - \widetilde{W}_{w,t}) - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\eta\epsilon_w)}\widetilde{\mu}_{w,t}$$

Given that $\widetilde{W}_{w,t+1} = \theta_w \widetilde{W}_{w,t} + (1-\theta_w)\widetilde{W}^*_{w,t+1}$, we have

$$\widetilde{W}_{w,t} - \widetilde{W}_{w,t-1} = (1-\theta_w)\beta\left(\frac{1}{1-\theta_w} E_t(\widetilde{W}_{w,t+1} - \theta_w \widetilde{W}_{w,t}) - \widetilde{W}_{w,t}\right) - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\eta\epsilon_w)}\widetilde{\mu}_{w,t}$$

$$\widetilde{W}_{w,t} - \widetilde{W}_{w,t-1} = \beta\left(E_t \widetilde{W}_{w,t+1} - \widetilde{W}_{w,t}\right) - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\eta\epsilon_w)}\widetilde{\mu}_{w,t}$$

$$\widetilde{\pi}^w_{w,t} = \beta E_t \widetilde{\pi}^w_{w,t+1} - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\eta\epsilon_w)}\widetilde{\mu}_{w,t}$$

This is the nominal New Keynesian Wage Phillips Curve, where the inflation of nominal average wages is given by the growth rate in nominal wage.

The problem of choosing the optimal wage for households of skill type $k \in \{w, p\}$ is given by a wage Phillips curve, the definition of the wage markup and the dynamics of the real wage:

$$\widetilde{\pi}^k_{w,t} = \beta E_t \widetilde{\pi}^k_{w,t+1} - \frac{(1-\theta_k)(1-\beta\theta_k)}{\theta_k(1+\eta\epsilon_w)}\widetilde{\mu}_{k,t}$$

$$\widetilde{\mu}_{k,t} = \widetilde{w}_{k,t} - \widetilde{mrs}_{k,t}$$

$$\widetilde{mrs}_{k,t} = \eta\widetilde{n}_{k,t} - \widetilde{\lambda}_{k,t}$$

$$\widetilde{\pi}^k_{w,t} = \widetilde{w}_{k,t} - \widetilde{w}_{k,t-1} + \widetilde{\pi}_t$$

Figure 1.6 displays the impulse responses of selected variables to the QE shock for the model economies with Calvo-type nominal wage rigidity. These models generate qualitatively similar responses of variables compared to the models with ad-hoc real wage rigidity shown in Figure 1.2 in the main text. The exception is the response of real wages, which initially decline or experience a much less pronounced rise in the setting with nominal wage rigidity. Intuitively, when nominal wages are sticky, a rise in inflation induces real wages to decline. This is in contrast to sticky real wages, which protect the real value of wages against higher inflation. As for the quantitative responses of variables, nominal wage rigidity noticeably generates a higher level of persistence. Lower wage costs lead to higher real profits, which provide additional funds for the wealthy to invest in capital. This production input is the main source of generating persistent effects on the real economy. The same conclusion about qualitative and quantitative differences between the two sticky wage settings applies to the dynamics of inequality measures in Figure 1.7 and Figure 1.3 in the main text.

A.7 QE multipliers

In addition to the distributional effects of QE, monetary authorities care about the aggregate effects when designing QE programs. To quantify the role of the labor market differential (i.e. capital-skill complementarity in production and asymmetric wage rigidities) in shaping the responses of output and the main aggregate demand components to the QE shock, we calculate the QE multipliers on impact and cumulated over different time horizons. Table 1.7 illustrates five important findings. First, the QE multipliers become lower over time in all model specifications under consideration. Second, the economies with nominal wage rigidity have higher impact and cumulative QE multipliers compared to the models with real wage rigidity. Third, the economies CSC+RW and CSC+NomWR are characterized with the largest output impact multipliers of QE, while they are the smallest for the economies with either only capital-skill complementarity or asymmetric wage rigidities. Forth, consumption is the main driver of the output growth in the economy CD2+NRW, while investment plays that role in the economies CSC+RW and CSC+NomWR. Fifth, the introduction of rigid wages makes the impact investment multipliers larger in the range of 44.65-112.12%, while it leads to lower consumption multipliers by 4.21-49.95% compared to the economies with flexible wages.

Given that the models are log-linearised around the steady state, $\frac{\% dX_t}{\% d(q_{L,t}B_t^{l,cb})} = \frac{dX_t}{d(q_{L,t}B_t^{l,cb})}\frac{q_L B^{l,cb}}{X}$ indicates the elasticity of $X_t = \{Y_t, C_t, I_t\}$ variable with respect to the value of long-term government bonds purchased by the central bank. To express the quantitative easing multipliers, we just need to adjust the stated ratio for the term $\frac{q_L B^{l,cb}}{X}$ and have the derivative part. The present value QE multiplier M_k for a given horizon k is given by

$$M_k = \frac{\sum_{k=0}^{\infty} \beta^k dX_k}{\sum_{k=0}^{\infty} \beta^k d(q_{L,k}B_k^{l,cb})}$$

For $k = 0$, the above expression refers to the impact QE multiplier.

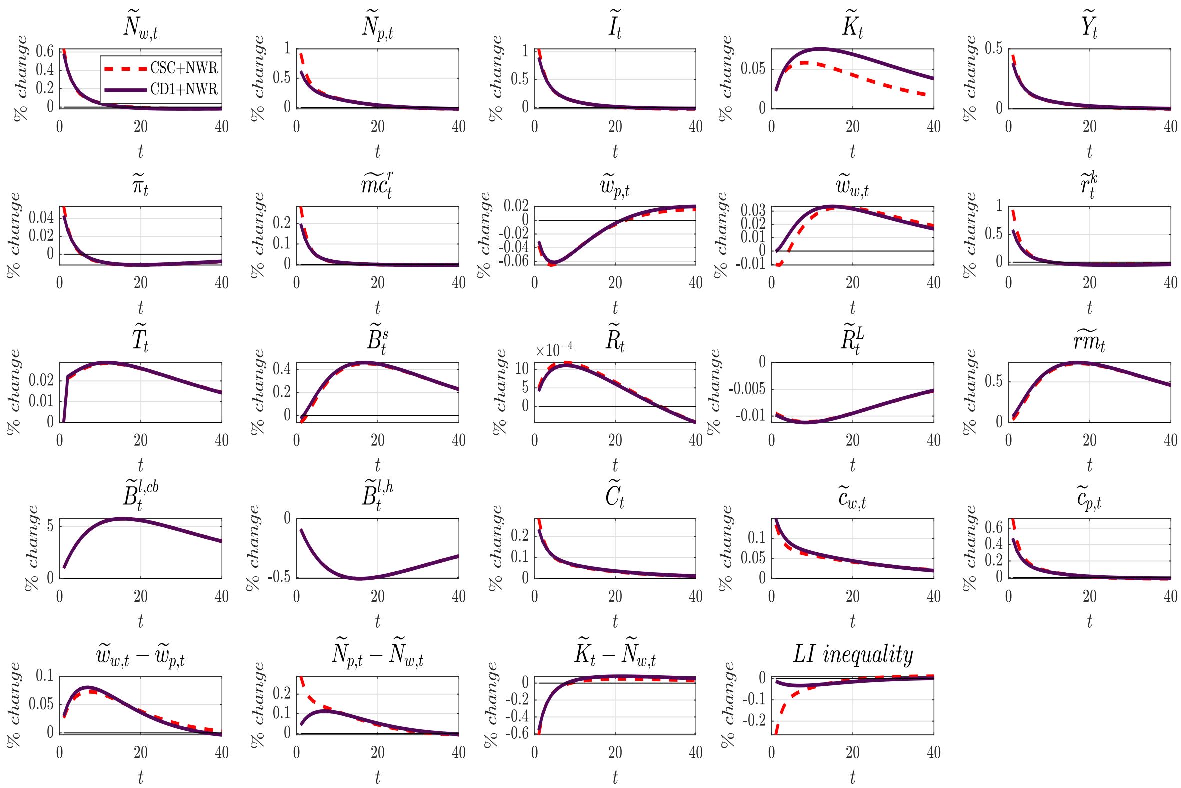

Figure 1.6: IRFs for the quantitative easing shock: CSC vs CD1 (the case of nominal wage rigidity)

Notes: For the CSC economy, the variables $\widetilde{I}_t$, $\widetilde{k}_t$, $\widetilde{K}_t$ - $\widetilde{N}_{w,t}$ and $\widetilde{r}_t^k$ stand for equipment investment, equipment capital, equipment to skilled labor ratio and equipment rental rate, respectively. This study takes the estimated parameters $\theta_{w,w} = 0.70$ and $\theta_{w,p} = 0.86$ from Bilbiie et al. (2022a), which measure the degree of nominal wage rigidity.

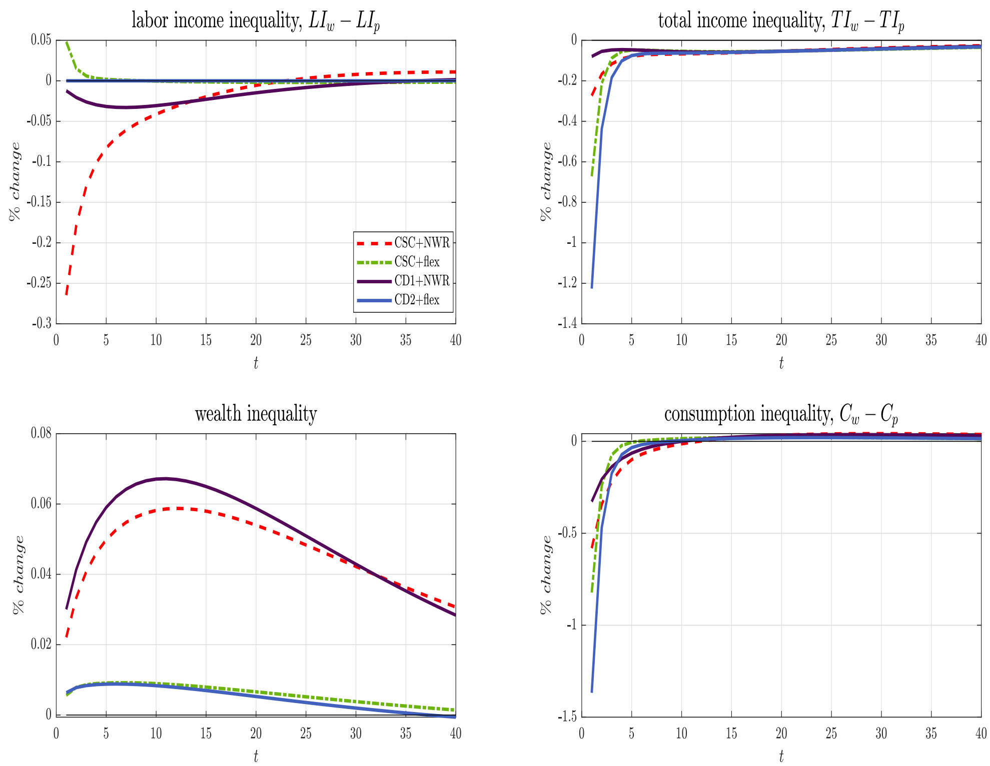

Figure 1.7: Inequality measures: The comparison of CSC and CD economies

Notes: Blue color indicates the portfolio rebalancing channel, while the other colors refer to the interaction of the earnings heterogeneity channel and the portfolio rebalancing channel.

Table 1.7: The impact and cumulated QE multipliers

	Horizon k					Horizon k			
Model	1Q	1Y	5Y	10Y	Model	1Q	1Y	5Y	10Y
Output					Output				
CSC+RW	2.030	0.464	0.038	0.017	CSC+NomWR	2.462	0.743	0.136	0.073
CSC+NRW	1.546	0.314	0.038	0.020	CSC+NRW	1.546	0.314	0.038	0.020
CD1+RW	1.738	0.427	0.038	0.017	CD1+NomWR	2.101	0.676	0.131	0.072
CD2+NRW	1.935	0.410	0.049	0.024	CD2+NRW	1.935	0.410	0.049	0.024
Consumption					Consumption				
CSC+RW	0.719	0.183	0.019	0.009	CSC+NomWR	0.828	0.257	0.055	0.033
CSC+NRW	0.750	0.154	0.020	0.011	CSC+NRW	0.750	0.154	0.020	0.011
CD1+RW	0.582	0.161	0.019	0.009	CD1+NomWR	0.687	0.230	0.053	0.033
CD2+NRW	1.164	0.248	0.031	0.016	CD2+NRW	1.164	0.248	0.031	0.016
Investment					Investment				
CSC+RW	1.312	0.281	0.020	0.008	CSC+NomWR	1.634	0.486	0.081	0.040
CSC+NRW	0.796	0.160	0.018	0.009	CSC+NRW	0.796	0.160	0.018	0.009
CD1+RW	1.151	0.265	0.020	0.008	CD1+NomWR	1.408	0.444	0.077	0.039
CD2+NRW	0.770	0.162	0.018	0.008	CD2+NRW	0.770	0.162	0.018	0.008

Notes: The left part of this table illustrates the economies with real wage rigidity (CSC+RW and CD1+RW), while the right part indicates the economies with nominal wage rigidity (CSC+NomWR and CD1+NomWR).

A.8 The elasticity of the investment to capital ratio with respect to Tobin's q

The Lagrangean function for the wealthy household's maximization problem in real terms:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Bigg\{ log c_{w,\tau} + \frac{\varphi_m}{1-\chi}(m_\tau)^{1-\chi} - \varphi_{n,w}\frac{(n_{w,\tau})^{1+\eta}}{1+\eta} - \lambda_{w,\tau}\Bigg(c_{w,t} + \frac{b_t^s}{R_t} +$$

$$+ \frac{b_t^{l,h}}{R_t^L}\left(1 + \frac{\phi_b}{2}\left(\kappa\frac{b_t^s}{b_t^{l,h}} - 1\right)^2\right) + t_{w,t} + i_{s,t} + i_{e,t} + m_t - w_{w,t}n_{w,t} -$$

$$- \frac{b_{t-1}^s}{\pi_t} - \frac{b_{t-1}^{l,h}}{R_t\pi_t} - r_{s,t}^k k_{s,t-1} - r_{e,t}^k k_{e,t-1} - \frac{m_{t-1}}{\pi_t} - tr_{w,t} - \Pi_t^{int} - \Pi_t^r \Bigg) -$$

$$- \sum_{\varsigma\in\{s,e\}} Q_{\varsigma,t}\left((k_{\varsigma,t} - (1-\delta_\varsigma)k_{\varsigma,t-1}) + \frac{\phi_k}{2}\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\right)^2 k_{\varsigma,t} - i_{\varsigma,t}\right)\Bigg\}$$

where $\lambda_{w,t}$ is the Lagrangean multiplier associated with the budget constraint of the wealthy household (i.e. the marginal utility of having extra consumption); $q_{\varsigma,t} = Q_{\varsigma,t}/\lambda_{w,t}$ is the Tobin's q marginal ratio with $Q_{\varsigma,t}$ being the Lagrangean multiplier associated with the law of motion of capital stock (i.e. the marginal utility from having additional installed capital). This ratio provides a measure of how much the wealthy household needs to sacrifice current consumption to have additional future capital.

The FOC for investment of the type $\varsigma \in \{s, e\}$ gives

$$-\lambda_{w,t} - Q_{\varsigma,t}\phi_k\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\right)\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} + Q_{\varsigma,t} = 0$$

$$\Leftrightarrow \frac{\lambda_{w,t}}{Q_{\varsigma,t}} = 1 - \phi_k\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\right)\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}}$$

$$\Leftrightarrow \frac{1}{q_{\varsigma,t}} = 1 - \phi_k\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\right)\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}}$$

$$\Leftrightarrow \frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} = \left(-\frac{1}{q_{\varsigma,t}} + 1\right)\frac{k_{\varsigma,t-1}}{\phi_k k_{\varsigma,t}} + \delta_\varsigma$$

$$\Leftrightarrow log\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}}\right) = log\left(\left(-e^{-log(q_{\varsigma,t})}+1\right)\frac{k_{\varsigma,t-1}}{\phi_k k_{\varsigma,t}}+\delta_\varsigma\right)$$

The elasticity of the investment to capital-ratio with respect to Tobin's q is

$$\frac{\partial log\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}}\right)}{\partial log(q_{\varsigma,t})} = \frac{1}{\left(-e^{-log(q_{\varsigma,t})}+1\right)\frac{k_{\varsigma,t-1}}{\phi_k k_{\varsigma,t}}+\delta_\varsigma}\left(-\frac{k_{\varsigma,t-1}}{\phi_k k_{\varsigma,t}}e^{-log(q_{\varsigma,t})}(-1)\right)$$

In the steady state, the above expression becomes

$$\frac{\partial log\left(\frac{i_\varsigma}{k_\varsigma}\right)}{\partial log(q_\varsigma)} = \frac{1}{\delta_\varsigma}\frac{1}{\phi_k}$$

If the elasticity of the investment to structure capital ratio with respect to Tobin's marginal q is $\varrho_{s,k} = 1/(\delta_s \cdot \phi_k) = 13.33$, then the elasticity of the investment-capital adjustment cost is:

$$\phi_k = \frac{1}{\delta_s \cdot \varrho_{s,k}} = \frac{1}{0.0142 \cdot 13.33} = 5.283$$

For simplicity, we assume that ϕ_k is the same for two types of capital, which implies $\varrho_{e,k} = 6.11$.

A.9 The log-linearised system of equations

This section specifies the log-linearised equations derived as first-order approximations around the model's nonstochastic steady state.

A.9.1 Wealthy households

1. FOC with respect to consumption:

$$\widetilde{\lambda}_{w,t} = \frac{-\sigma_c(c_w\widetilde{c}_{w,t} - hC_w\widetilde{C}_{w,t-1})}{(c_w - hC_w)}$$

2. FOC with respect to labor supply:

$$\eta\widetilde{n}_{w,t} = \widetilde{\lambda}_{w,t} + \widetilde{w}_{w,t}$$

3. FOC with respect to real money balances:

$$\widetilde{m}_t = \frac{1}{\chi}\left(-\frac{\pi}{\pi-\beta}\widetilde{\lambda}_{w,t} + \frac{\beta}{\pi-\beta}\mathbb{E}_t(\widetilde{\lambda}_{w,t+1} - \widetilde{\pi}_{t+1})\right)$$

4. FOC with respect to short-term bond holdings:

$$\mathbb{E}_t\frac{\beta}{\pi}(\widetilde{\lambda}_{w,t+1} - \widetilde{\pi}_{t+1}) = q(\widetilde{\lambda}_{w,t} + \widetilde{q}_t) + q_L\phi_b\kappa(\widetilde{b}^s_t - \widetilde{b}^{l,h}_t)$$

5. FOC with respect to long-term bond holdings:

$$\widetilde{q}_{L,t} = \widetilde{\lambda}_{w,t+1} - \widetilde{\lambda}_{w,t} - \widetilde{\pi}_{t+1} + \frac{\beta\varrho}{\pi}\widetilde{q}_{L,t+1} + \phi_b(\widetilde{b}^s_t - \widetilde{b}^{l,h}_t)$$

6. FOC with respect to physical capital:

$$\widetilde{\lambda}_{w,t} + \phi_k\widetilde{k}_{\varsigma,t} - \phi_k\widetilde{k}_{\varsigma,t-1} = \mathbb{E}_t\beta\Big((1-\delta_\varsigma)\widetilde{\lambda}_{w,t+1} + r^k_\varsigma(\widetilde{\lambda}_{w,t+1} + \widetilde{r}^k_{\varsigma,t+1}) + \phi_k\widetilde{k}_{\varsigma,t+1} - \phi_k\widetilde{k}_{\varsigma,t}\Big),\ \varsigma \in \{s,e\}$$

7. The price of long-term government bonds:

$$\widetilde{q}_{L,t} = -\frac{R^L}{R^L-\varrho}\widetilde{R}^L_t$$

8. The price of short-term government bonds:

$$\widetilde{q}_t = -\widetilde{R}_t$$

9. The budget constraint:

$$c_w\widetilde{c}_{w,t} + qb^s(\widetilde{q}_t + \widetilde{b}^s_t) + q_L b^{l,h}(\widetilde{q}_{L,t} + \widetilde{b}^{l,h}_t) + t_w\widetilde{t}_{w,t} + \sum_{\varsigma\in\{s,e\}} k_\varsigma(\widetilde{k}_{\varsigma,t} - (1-\delta_\varsigma)\widetilde{k}_{\varsigma,t-1}) + m\widetilde{m}_t$$

$$= w_w n_w(\widetilde{w}_{w,t} + \widetilde{n}_{w,t}) + \frac{b^s}{\pi}(\widetilde{b}^s_{t-1} - \widetilde{\pi}_t) + \frac{b^{l,h}}{\pi}(\widetilde{b}^{l,h}_{t-1} - \widetilde{\pi}_t) + \varrho q_L\frac{b^{l,h}}{\pi}(\widetilde{q}_{L,t} + \widetilde{b}^{l,h}_{t-1} - \widetilde{\pi}_t)$$

$$+ \sum_{\varsigma\in\{s,e\}} k_\varsigma r^k_\varsigma(\widetilde{k}_{\varsigma,t-1} + \widetilde{r}^k_{\varsigma,t}) + \frac{m}{\pi}(\widetilde{m}_{t-1} - \widetilde{\pi}_t) + tr_w\widetilde{tr}_{w,t} + \frac{\Pi^{int}}{s_w}\widetilde{\Pi}^{int}_t + \frac{\Pi^r}{s_w}\widetilde{\Pi}^r_t$$

10. Law of motion of capital:

$$\widetilde{i}^k_{\varsigma,t} = \frac{1}{\delta_\varsigma}(\widetilde{k}_{\varsigma,t} - (1-\delta_\varsigma)\widetilde{k}_{\varsigma,t-1}), \quad \text{for} \quad \varsigma \in \{s,e\}$$

A.9.2 Poor households

1. FOC with respect to consumption:

$$\widetilde{\lambda}_{p,t} = \frac{-\sigma_c(c_p\widetilde{c}_{p,t} - hC_p\widetilde{C}_{p,t-1})}{(c_p - hC_p)}$$

2. FOC with respect to labor supply:

$$\eta\widetilde{n}_{p,t} = \widetilde{\lambda}_{p,t} + \widetilde{w}_{p,t}$$

3. The budget constraint:

$$c_p\widetilde{c}_{p,t} + t_p\widetilde{t}_{p,t} = w_p n_p(\widetilde{w}_{p,t} + \widetilde{n}_{p,t}) + tr_p\widetilde{tr}_{p,t}$$

A.9.3 Intermediate goods firms

1. Production function:

$$\widetilde{Y}_{int,t} = \iota\widetilde{K}_{s,t-1} + (1-\iota)\left(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\right)^{-1} mN_p^\sigma\widetilde{N}_{p,t}+$$

$$+(1-\iota)\left(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\right)^{-1}\cdot$$

$$\cdot\,(1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}-1}\Big(\rho K_e^\nu\widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu\widetilde{N}_{w,t}\Big)$$

2. FOC with respect to structure capital:

$$\widetilde{r}_{s,t}^k = \widetilde{mc}_t^r + (\iota-1)\widetilde{K}_{s,t-1} + (1-\iota)\left(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\right)^{-1} mN_p^\sigma\widetilde{N}_{p,t}+$$

$$+(1-\iota)\left(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\right)^{-1}\cdot$$

$$\cdot\,(1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}-1}\Big(\rho K_e^\nu\widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu\widetilde{N}_{w,t}\Big)$$

3. FOC with respect to equipment capital:

$$\tilde{r}^k_{e,t} = \widetilde{mc}^r_t + \iota \widetilde{K}_{s,t-1} + \Big(\frac{1-\iota}{\sigma} - 1\Big) \Big(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\Big)^{-1} m\sigma N_p^\sigma \widetilde{N}_{p,t} +$$

$$+\Big(\frac{1-\iota}{\sigma} - 1\Big) \Big(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\Big)^{-1} \cdot$$

$$\cdot (1-m)\sigma\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}-1}\Big(\rho K_e^\nu \widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu \widetilde{N}_{w,t}\Big) +$$

$$+ (\frac{\sigma}{\nu} - 1)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{-1} \nu\Big(\rho K_e^\nu \widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu \widetilde{N}_{w,t}\Big) + (\nu - 1)\widetilde{K}_{e,t-1}$$

4. FOC with respect to demand for skilled labor:

$$\widetilde{w}_{w,t} = \widetilde{mc}^r_t + \iota \widetilde{K}_{s,t-1} + \Big(\frac{1-\iota}{\sigma} - 1\Big) \Big(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\Big)^{-1} m\sigma N_p^\sigma \widetilde{N}_{p,t} +$$

$$+\Big(\frac{1-\iota}{\sigma} - 1\Big) \Big(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\Big)^{-1} \cdot$$

$$\cdot (1-m)\sigma\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}-1}\Big(\rho K_e^\nu \widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu \widetilde{N}_{w,t}\Big) +$$

$$+ (\frac{\sigma}{\nu} - 1)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{-1} \nu\Big(\rho K_e^\nu \widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu \widetilde{N}_{w,t}\Big) + (\nu - 1)\widetilde{N}_{w,t}$$

5. FOC with respect to demand for unskilled labor:

$$\widetilde{w}_{p,t} = \widetilde{mc}^r_t + \iota \widetilde{K}_{s,t-1} + \Big(\frac{1-\iota}{\sigma} - 1\Big) \Big(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\Big)^{-1} m\sigma N_p^\sigma \widetilde{N}_{p,t} +$$

$$+\Big(\frac{1-\iota}{\sigma} - 1\Big) \Big(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\Big)^{-1} \cdot$$

$$\cdot (1-m)\sigma\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}-1}\Big(\rho K_e^\nu \widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu \widetilde{N}_{w,t}\Big) + (\sigma - 1)\widetilde{N}_{p,t}$$

6. Skill premium:

$$s\_premium = \widetilde{w}_{w,t} - \widetilde{w}_{p,t}$$

7. Unskilled to skilled labor ratio:

$$unskilled\_ls = \widetilde{n}_{p,t} - \widetilde{n}_{w,t}$$

8. Capital to skilled labor ratio:

$$k_e\_to\_l = \widetilde{k}_{e,t-1} - \widetilde{n}_{w,t}$$

9. Relative skilled labor income share or labor income inequality:

$$LI\_inequality = \widetilde{w}_{w,t} + \widetilde{n}_{w,t} - (\widetilde{w}_{p,t} + \widetilde{n}_{p,t})$$

10. Consumption inequality:

$$C\_inequality = \widetilde{c}_{w,t} - \widetilde{c}_{p,t}$$

11. Total income inequality:

$$TI\_inequality = \widetilde{TI}_{w,t} - \widetilde{TI}_{p,t}$$

$$\widetilde{TI}_{w,t} = \frac{NLI_w}{TI_w}\widetilde{NLI}_{w,t} + \frac{LI_w}{TI_w}\widetilde{LI}_{w,t}, \quad \widetilde{TI}_{p,t} = \frac{NLI_p}{TI_p}\widetilde{NLI}_{p,t} + \frac{LI_p}{TI_p}\widetilde{LI}_{p,t}$$

$$\widetilde{NLI}_{w,t} = \frac{1}{NLI_w}\Big(b^s\widetilde{b}^s_t - \frac{b^s}{R}(\widetilde{b}^s_t - \widetilde{R}_t) + \frac{b^{l,h}}{\pi}(\widetilde{b}^{l,h}_{t-1} - \widetilde{\pi}_t) + \sum_{\varsigma\in\{s,e\}} r^k_\varsigma k_\varsigma(\widetilde{r}^k_{\varsigma,t} + \widetilde{k}_{\varsigma,t-1}) + tr_w\widetilde{tr}_{w,t} - t_w\widetilde{t}_{w,t} + \frac{\Pi^r}{s_w}\widetilde{\Pi}^r_t\Big)$$

$$\widetilde{NLI}_{p,t} = \frac{1}{NLI_p}(tr_p\widetilde{tr}_{p,t} - t_p\widetilde{t}_{p,t})$$

$$\widetilde{LI}_{w,t} = \widetilde{w}_{w,t} + \widetilde{n}_{w,t}, \quad \widetilde{LI}_{p,t} = \widetilde{w}_{p,t} + \widetilde{n}_{p,t}$$

$$TI_p = NLI_p + LI_p, \quad LI_p = w_p n_p, \quad NLI_p = tr_p - t_p$$

$$TI_w = NLI_w + LI_w, \quad LI_w = w_w n_w, \quad NLI_w = b^s - \frac{b^s}{R} + \frac{b^{l,h}}{\pi} + \sum_{\varsigma\in\{s,e\}} r^k_\varsigma k_\varsigma + tr_w - t_w + \frac{\Pi^r}{s_w}$$

12. Wealth inequality:

$$W\_inequality = \left(\frac{b^s}{\pi} + \varrho q_L\frac{b^{l,h}}{\pi} + \frac{m}{\pi} + \sum_{\varsigma\in\{s,e\}}(1-\delta_\varsigma)k_\varsigma\right)^{-1}$$

$$\cdot\left(\frac{b^s}{\pi}(\widetilde{b}^s_{t-1} - \widetilde{\pi}_t) + \varrho q_L\frac{b^{l,h}}{\pi}(\widetilde{q}_{L,t} + \widetilde{b}^{l,h}_{t-1} - \widetilde{\pi}_t) + \frac{m}{\pi}(\widetilde{m}_{t-1} - \widetilde{\pi}_t) + \sum_{\varsigma\in\{s,e\}} k_\varsigma(1-\delta_\varsigma)\widetilde{k}_{\varsigma,t-1}\right)$$

A.9.4 Final goods firms

1. The New Keynesian Phillips Curve:

$$\widetilde{\pi}_t = \frac{(\epsilon - 1)}{\phi_p}\widetilde{mc}_t^r + \beta \mathbb{E}_t \widetilde{\pi}_{t+1}$$

2. Real profit of final goods firms:

$$\widetilde{\Pi}_t^r = \widetilde{Y}_t - \frac{mc^r}{1 - mc^r}\widetilde{mc}_t^r$$

A.9.5 The aggregate resource constraint

$$Y\widetilde{Y}_t = C\widetilde{C}_t + I\widetilde{I}_t + G\widetilde{G}_t$$

A.9.6 Fiscal policy

1. Fiscal Policy Rule:

$$\widetilde{T}_t = \rho_1 \left(\frac{q_L B^l}{q_L B^l + qB^s}(\widetilde{q}_{L,t-1} + \widetilde{B}_{t-1}^l) + \frac{qB^s}{q_L B^l + qB^s}(\widetilde{q}_{t-1} + \widetilde{B}_{t-1}^s) \right)$$

2. The real government budget constraint:

$$\begin{aligned} &T\widetilde{T}_t + qB^s(\widetilde{q}_t + \widetilde{B}_t^s) + q_L B^l(\widetilde{q}_{L,t} + \widetilde{B}_t^l) + M\widetilde{M}_t - \frac{M}{\pi}(\widetilde{M}_{t-1} - \widetilde{\pi}_t) - \\ &- \left(q_L B^{l,cb}(\widetilde{q}_{L,t} + \widetilde{B}_t^{l,cb}) - \frac{B^{l,cb}}{\pi}(\widetilde{B}_{t-1}^{l,cb} - \widetilde{\pi}_t) - \varrho q_L \frac{B^{l,cb}}{\pi}(\widetilde{q}_{L,t} + \widetilde{B}_{t-1}^{l,cb} - \widetilde{\pi}_t) \right) = \\ &= \frac{B^s}{\pi}(\widetilde{B}_{t-1}^s - \widetilde{\pi}_t) + \frac{B^l}{\pi}(\widetilde{B}_{t-1}^l - \widetilde{\pi}_t) + \varrho q_L \frac{B^l}{\pi}(\widetilde{q}_{L,t} + \widetilde{B}_{t-1}^l - \widetilde{\pi}_t) + G\widetilde{G}_t + TR\widetilde{TR}_t \end{aligned}$$

3. The distribution of lump-sum taxes:

$$\widetilde{T}_t = s_w \widetilde{t}_{w,t} + s_p \widetilde{t}_{p,t} \quad \text{and} \quad \widetilde{T}_t = \widetilde{t}_{p,t}$$

4. The distribution of lump-sum transfers:

$$\widetilde{TR}_t = \widetilde{tr}_{p,t} \quad \text{and} \quad \widetilde{tr}_{w,t} = 0$$

5. The decomposition of long-term government bonds:

$$B^l \widetilde{B}^l_t = B^{l,cb} \widetilde{B}^{l,cb}_t + B^{l,h} \widetilde{B}^{l,h}_t$$

A.9.7 The exogenous process

1. Central bank (nominal) money-market rate:

$$\widetilde{R}_t = \theta_r \widetilde{R}_{t-1} + (1-\theta_r)\left[\theta_\pi \widetilde{\pi}_t + \theta_y \widetilde{Y}_t\right] + \epsilon^r_t$$

2. The supply of long-term bonds:

$$\widetilde{B}^l_t = \phi_{b,l} \widetilde{B}^l_{t-1} + \epsilon^{b,l}_t$$

3. The central bank asset purchases:

$$\widetilde{B}^{l,cb}_t = (\phi_{cb1} + \phi_{cb2}) \widetilde{B}^{l,cb}_{t-1} - (\phi_{cb1}\phi_{cb2}) \widetilde{B}^{l,cb}_{t-2} + \epsilon^{l,cb}_t$$

4. Government expenditure:

$$\widetilde{G}_t = \phi_g \widetilde{G}_{t-1} + \epsilon^g_t$$

5. Transfers:

$$\widetilde{TR}_t = \phi_{tr} \widetilde{TR}_{t-1} + \epsilon^{tr}_t$$

A.9.8 Aggregate variables

1. Aggregate consumption:

$$C\widetilde{C}_t = C_w \widetilde{C}_{w,t} + C_p \widetilde{C}_{p,t} = s_w c_w \widetilde{c}_{w,t} + s_p c_p \widetilde{c}_{p,t}$$

2. Labor supply of the wealthy:

$$\widetilde{N}_{w,t} = \widetilde{n}_{w,t}$$

3. Labor supply of the poor:

$$\widetilde{N}_{p,t} = \widetilde{n}_{p,t}$$

4. Aggregate capital stock:

$$\widetilde{K}_{\varsigma,t} = \widetilde{k}_{\varsigma,t}, \quad \text{for } \varsigma \in \{s, e\}$$

5. Aggregate money holdings:

$$\widetilde{M}_t = \widetilde{m}_t$$

6. Aggregate long-term bond holdings:

$$\widetilde{B}_t^{l,h} = \widetilde{b}_t^{l,h}$$

7. Aggregate short-term bond holdings:

$$\widetilde{B}_t^{s} = \widetilde{b}_t^{s}$$

1.8.B IRFs for the CSC economy: investment and capital

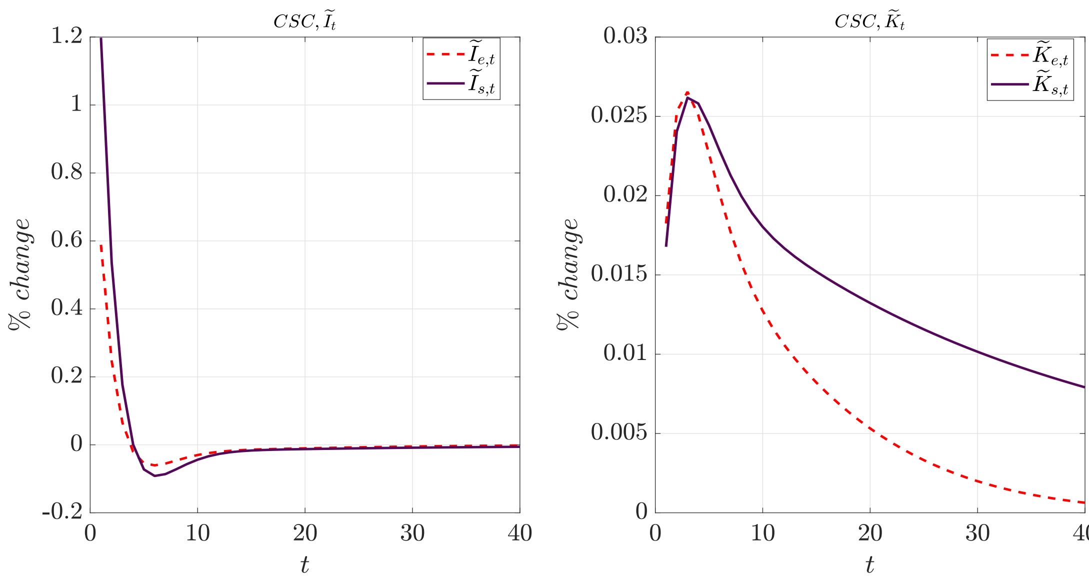

Figure 1.8.B: IRFs for the CSC economy: investment and capital

1.8.C Empirical evidence by Lenza and Slacalek (2018)

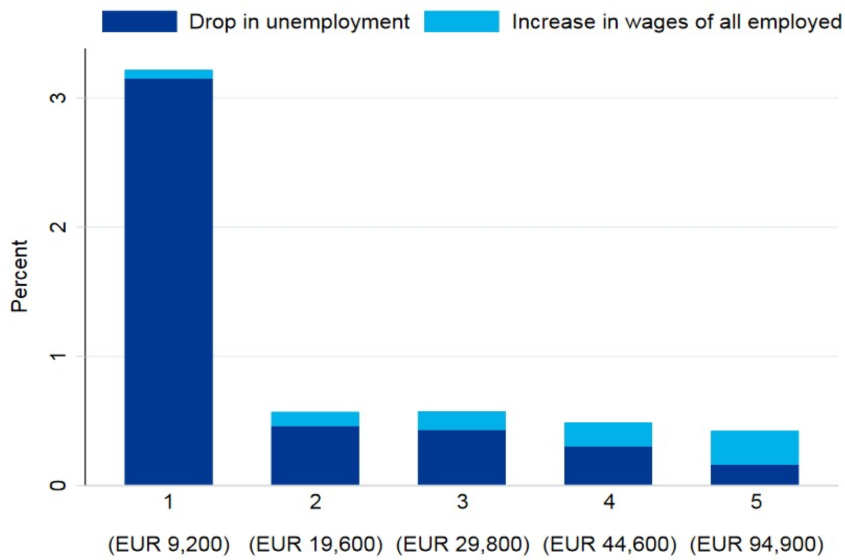

Figure 1.8.C: Decomposition of the Total Effect on Mean Income into the Extensive and the Intensive Margin

Figure 1.9 shows the percentage change in mean income across income quintiles in the EA four quarters after the impact of the QE shock in the EA. Although the whole EA population benefits from the rise in employment and wages, there is the drop in labor income inequality between wealthy and poor households. The poor experience a larger increase in employment after QE, while the wealthy benefit more from the rise in wages. Labor income inequality declines due to a stronger rise in employment than that in wages.

2 The Effects of Government Spending in Segmented Labor and Financial Markets

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2.1 Introduction

The U.S. economy experienced its largest contraction since the 1930s during the 2008 Great Recession. To spur aggregate demand and job creation, the U.S. fiscal authorities responded by implementing a large-scale fiscal stimulus in the form of government spending. According to Hagedorn et al. (2019), increased government spending follows almost every recession, but there is still plenty of room to improve our understanding of its effectiveness and propagation. Indeed, there is a lack of consensus about the estimated size of the fiscal multiplier (see e.g., Ramey, 2011 and Parker, 2011) and the sign of the fiscal multiplier (see e.g., Alesina et al., 2002 and Ilzetzki et al., 2013). Recent papers by Picco (2020) and Faccini and Yashiv (2022) indicate that large training costs of new hires in a representative agent framework are a crucial reason for the counterintuitive recessionary effects of expansionary policies. Our paper contributes to the literature by studying the role of training costs within the heterogeneous agent framework. Our main finding is that a rise in government spending induces an economic expansion despite large training costs.

When a hiring process includes training activities for new hires, production is disrupted. Specifically, a firm's ability to produce is lowered due to a temporal reallocation of some experienced workers from production to training activities. The output costs associated with production disruption can be large for a high value of output. This perfectly corresponds to the case of expansionary government spending, which under sticky prices generates excess aggregate demand pressure. In the representative agent New Keynesian (RANK) model à la Picco (2020) and Faccini and Yashiv (2022), a higher value of output, coupled with large training costs, leads to a larger rise in the marginal cost than the marginal benefit of hiring.

Consequently, firms decide to postpone hiring (or hire to a small extent). The presence of job separation and high savings induced by poor employment prospects translates into output contraction. This finding casts doubt on using countercyclical government spending as a general policymakers' tool to fight recessions.

By contrast, this paper considers a model economy populated with two types of workers, so that firms have a choice during the hiring process.[16] Differently skilled workers typically face asymmetric labor and financial market frictions, whose effects are reflected in the wage bargaining process and the job creation condition. When the value of output is high, the hiring of low-skilled workers is more attractive for firms due to their lower training (non-wage labor) costs. The output expansion occurs as the economy experiences an extensive hiring activity for low-skilled workers. When financial friction is added to the setting with training costs, low-skilled workers as liquidity-constrained households could become an even cheaper labor force. This is because financial friction makes low-skilled workers willing to accept lower wage payments as the hiring allows them to improve their lifetime utility. Hence, the stimulative effects of increased government spending are more pronounced when the interaction of labor and financial market frictions is considered.

To isolate the impact of asymmetric training costs on the transmission of increased government spending to the real economy, we build a two-agent New Keynesian model with a representative household (TANKrep). Differently skilled workers, who live together in one big family, face different levels of training costs. Additionally, we build a two-agent New Keynesian (TANK) model to examine the importance of the interaction between asymmetric training costs and financial friction, where the latter is characterized by no risk-sharing between high- and low-skilled workers. This model assumes that differently skilled workers live separately in two big families due to their different access to financial markets, as in Galí et al. (2007).[17] The presence of search and matching (SAM) frictions, which include

[16]The empirical studies that provide support for considering asymmetric training costs for differenlty skilled workers are, among others, Blatter et al. (2012) and Belo et al. (2017).

[17]High-skilled workers have access to financial markets and provide high-skilled labor services, while

matching efficiency, separation rates, and bargaining power, as in Dolado et al. (2021), are common to both the TANKrep and TANK models.

This paper contributes to the analytical heterogeneous agent New Keynesian (HANK) literature by developing a TANK framework with segmented labor and financial markets. Although a recently growing quantitative HANK literature is characterized by richer households' heterogeneity on the basis of idiosyncratic risk and incomplete markets, Debortoli and Galí (2018) find that a TANK model captures the implications of aggregate shocks in a full-scale HANK model reasonably well. In addition, the analytical and quantitative HANK literature abstracts from firm-specific hiring frictions, which are essentially the hallmark of the literature with a representative agent framework.

Our paper provides a bridge between the two strands of literature. In the first strand, hiring frictions are traditionally modeled as pecuniary costs (vacancy posting costs) within the heterogeneous agent setting (see, e.g., Gornemann et al., 2021 and Ravn and Sterk, 2021). The second strand emphasizes the non-pecuniary nature of hiring (training) costs in the representative agent framework (see, e.g., Picco, 2020, Faccini and Yashiv, 2022 and Faccini and Melosi, 2022). With respect to the first strand, hiring costs are expressed as asymmetric non-pecuniary costs and SAM frictions are asymmetric across skills. With respect to the second strand, segmented labor and financial markets are introduced to study the heterogeneous responses of households to higher government spending. Note that this paper follows Dolado et al. (2021) in modelling a segmented labor market, and additionally considers non-pecuniary training costs and a segmented financial market.

In our quantitative results, both government consumption and government investment gen-

low-skilled workers do not have access to financial markets and supply low-skilled labor services. Two well-established premises provide the justification for considering these two groups of workers. First, the existence of employment and earnings polarization by skill level (Goos and Manning, 2007 and Autor and Dorn, 2013). Second, the difference in financial literacy (Lusardi and Mitchell, 2007) and participation costs (Vissing-Jorgensen, 2002), which underlies the unequal access of workers to financial markets.

erate an economic expansion, with the latter having a much larger fiscal multiplier. The impulse response analysis of government consumption is divided into three parts.[18] The first part focuses on the output responses to an expansionary government consumption shock in models with flexible wages. In the RANK model, the output expansion is recorded for vacancy posting costs in non-pecuniary terms, but the fiscal multiplier is small on impact (0.039) and stays positive for another twenty quarters. By contrast, the RANK model with internal training costs characterizes persistent recessionary effects, with a multiplier of -0.101 after forty quarters. If vacancy posting costs are expressed in pecuniary terms, a rise in output is more pronounced, leading to a multiplier of 0.095 on impact. The rationale for the opposing output responses is that non-pecuniary hiring costs are related to production disruption, while pecuniary hiring costs imply payments for hiring services to an external labor agency. In contrast, TANKrep and TANK models with flexible wages report the expansionary output effects despite modelling hiring costs as internal training costs. The TANKrep model[19] generates a multiplier of 0.055 on impact, which gradually declines. With the exception of a small, initially negative multiplier of -0.015, the TANK model[20] also shows expansionary output effects with a peak multiplier of 0.236 after forty quarters.

The second part of the analysis examines the output responses under rigid wages. In the TANKrep model, rigid wages initially amplify the multiplier to 0.092. However, a large increase in demand for labor in the first two quarters, with a training costs specification, implies a more expensive hiring of new workers in the next five quarters. Consequently, output drops and the multiplier becomes lower. Later, the low value of output induced by lower aggregate demand pressure stimulates firms to hire more (productive) high-skilled workers so that output starts to rise. To determine the influence of financial friction in the TANK model,

[18] In these three parts of the analysis, government investment is not an integral part of government spending, and thus government consumption corresponds to government spending.

[19] The asymmetric training costs in this TANKrep model are specified with asymmetric hiring cost scaling parameters, $e_w = 5.07$ and $e_p = e_w/5.25$.

[20] This TANK model includes workers who are differently skilled due to their different skill intensity in production, have different access to financial markets but face symmetric SAM frictions and symmetric training costs.

we assume that workers face symmetric labor market frictions. In this case, firms would still have lower wage labor costs by hiring low-skilled workers. As a result of increased hiring and associated investment activity, the initial drop in output is significantly limited in the TANK model relative to the RANK model. Moreover, the TANK model shows that output returns to its pre-crisis average level after nine quarters, while output in the RANK model does not complete its recovery even after forty quarters. In addition, adding asymmetric SAM frictions to the TANKrep and TANK models only slightly amplifies the effects of asymmetric training costs and financial friction through an improved labor market position of high-skilled workers. After forty quarters, the size of the cumulative multiplier is 0.158.

The third part of the analysis investigates the responses of several real economic variables in addition to output. In the RANK model, higher government spending leads to a rise in aggregate demand pressures, which under large training costs increase the marginal cost of hiring more than the marginal benefit and accordingly discourage firms from hiring new workers. As this fiscal stimulus is followed by increasing taxes, there are standard negative wealth effects that induce wealthy households to decrease consumption and to increase labor supply. However, they face a problem of finding a job due to reduced hiring incentives for firms. In addition to poor employment prospects for wealthy households, the real interest rate rises as a government reacts to increased aggregate demand pressures, which in turn has crowding-out effects on capital investment. The combination of increasing taxes, low employment and decreasing capital investment puts downward pressure on output. By contrast, in the TANKrep and TANK models there is greater hiring activity, particularly of low-skilled workers, which has stimulative effects on investment and production activities.

In addition to our analysis of government consumption, we investigate output responses to expansionary government investment. There are two important observations regarding the effects of government investment when real wages are rigid. First, government investment generates stronger expansionary effects than government consumption because of a higher marginal productivity of labor inputs, which stimulates firms' labor demand. Thus, from the perspective of policy makers, government investment is a more efficient tool in dealing with recessions than government consumption. Second, the expansionary effects of government investment in the TANK model are larger and more persistent than in the RANK model. The size of the fiscal multiplier is 0.128 and 0.055 on impact in the TANK and the RANK models, respectively. After forty quarters, the cumulative fiscal multiplier is 0.755 in the TANK model, while it is 0.317 in the RANK model.

The rest of the chapter is organized as follows. Section 2.2 presents the model economy. Section 2.3 accounts for the transmission mechanism. Section 2.4 is dedicated to the calibration, while Section 2.5 shows the impulse response analysis regarding expansionary government spending. Section 2.6 concludes.

2.2 Model Economy

The model economy denoted as TANK has household, production, and government sectors.[21] The household sector includes a continuum of wealthy w and poor households p on the unit interval. These households are different in terms of the frictions they face in the financial and labor markets. In this regard, households have differential access to financial markets, in the spirit of Galí et al. (2007).[22] In addition, households may have different productivity levels, reflected in skill intensity in production, and face asymmetric SAM frictions (matching efficiency, separation rates, and bargaining power), as in Dolado et al. (2021), as well as asymmetric training costs internal to intermediate goods firms. Taking financial and labor market segmentation together, a constant share $s_w \in [0, 1]$ of the household population[23]

[21] The description of the TANKrep model is provided in Appendix A.5.

[22] In this model economy, household members can be perfectly insured against unemployment risk (induced by SAM frictions and hiring costs) within a particular skill group, but not between them. As in Merz (1995), the head of each household provides perfect risk sharing within a given household type by pooling the income of all its members and then allocating it to consumption, so that all members consume the same amount of consumption goods regardless of their employment state.

[23] When $s_w = 1$, our two-agent model collapses to a standard representative agent model with only wealthy households.

participates in financial/capital markets and provide high-skilled labor services, while the remaining fraction $s_p = 1 - s_w$ are non-participants in financial/capital markets and provides low-skilled labor services. In the production sector, there exists a distinction between intermediate and final goods firms to avoid difficulties arising from having the hiring and pricing decisions within the same firm. Perfectly competitive intermediate producers hire labour and rent capital from households to produce a homogeneous intermediate good, which is then differentiated by final goods firms that face price-setting rigidities. The final output is used for private consumption, investment, and public consumption. In the government sector, the monetary authority sets the short-term nominal interest rate following a standard Taylor rule, while the fiscal authority conducts government spending that is financed with lump-sum taxes and issuing short-term bonds.

2.2.1 Labor Market

There is a large number of households, which are classified into two groups by the skill level of their members: high- and low-skilled workers. Workers can only participate in the labor market they belong to the basis of their skill level; high- or low-skill labor markets. In addition, we assume that workers cannot change their skill level over time, which makes their respective population share constant.

Following Galí (2010), in each period household members can be in one of three different employment states: employed, unemployed but actively looking for a job, and unemployed but inactive. The sum of those members who are employed $N_{k,t}$ and those who are unemployed but actively looking for a job $U_{k,t}$ constitutes a pool of people who participate in the labor market or the total workforce

$$L_{k,t} = N_{k,t} + U_{k,t}, \quad k \in \{w, p\} \tag{2.1}$$

The labor market as a place of interaction between (intermediate goods) firms and workers is characterized by labor market frictions in the form of SAM frictions and training costs. To find new workers, firms post job vacancies $v_{k,t}$ for which $U^k_{0,t}$ apply. The variable $U^k_{0,t}$ is the notation for the pool of unemployed people at the beginning of period t who are actively searching for a job. Only the beginning-of-period job seekers from the unemployment pool can be hired, while employed workers cannot search for jobs. The matching technology for new gross hires $H_{k,t}$ takes the standard Cobb-Douglas form:

$$H_{k,t}(\upsilon_{k,t}, U^k_{0,t}) = \psi_k \left(\upsilon_{k,t}\right)^{\varsigma} \left(U^k_{0,t}\right)^{1-\varsigma}, \quad k \in \{w, p\} \tag{2.2}$$

where $\psi_k > 0$ captures the matching efficiency and $\varsigma \in (0,1)$ is the elasticity of the new hires to the beginning-of-period job seekers.

Labor market tightness $\theta_{k,t}$, vacancy filling probabilities $\nu_{k,t}$ and hiring probabilities $\mu_{k,t}$ differ by the skill type of workers $k \in \{w, p\}$:

$$\theta_{k,t} = \frac{\upsilon_{k,t}}{U^k_{0,t}} \tag{2.3}$$

$$\nu_{k,t} = \frac{H_{k,t}}{\upsilon_{k,t}} \tag{2.4}$$

$$\mu_{k,t} = \frac{H_{k,t}}{U^k_{0,t}} \tag{2.5}$$

Aggregate employment in the wholesale sector evolves according to the following law of motion:

$$N_{k,t} = (1-\sigma_k)N_{k,t-1} + H_{k,t}, \quad k \in \{w, p\} \tag{2.6}$$

where $\sigma_k \in (0,1)$ is a constant exogenous separation rate, which indicates a share of employed workers who leave the firm and consequently become unemployed until the next period. Note that equation (2.6) indicates that newly hired workers become productive (or start working) immediately in the same period in which they are hired. This is in line with Blanchard and Galí (2010) timing specification, where employment is a choice variable that can contemporaneously respond to shocks in the economy.

In addition to SAM frictions, intermediate goods firms face hiring costs. Faccini and Yashiv (2022) provide micro-evidence that around 80% of hiring costs are post-match and expressed in intermediate goods or as foregone output. Accordingly, in our benchmark model specification, hiring costs are treated as internal training costs. These costs occur after the establishment of a job relationship, and assume the discrepancy between newly hired workers and experienced workers regarding the level of productivity. To close the gap between them, the new hires pass through the training process. If the training activity is not delegated to some third-party labor agency, firms resort to internal training. With this internal training activity, production disruption takes place as some experienced workers are diverted from production to training the new hires.

2.2.2 Households

2.2.2.1 Ricardian High-Skilled Households (the Wealthy)

Wealthy households maximize their expected lifetime utility, which is a separably additive function of consumption $c_{w,t}$ and labor supply $\ell_{w,t}$:

$$\max_{c_{w,t},i_t,k_t,b_t,\ell_{w,t},n_{w,t},u_{w,t}} \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \frac{1}{1-\sigma_c} (c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} - \varphi_{n,w} \frac{(\ell_{w,\tau})^{1+\eta}}{1+\eta} \right\}$$

where $\mathbb{E}_t$ is the conditional expectations operator in period t, $\beta \in (0,1)$ is the subjective discount factor, $c_{w,t}(C_{w,t})$ is the time-t individual (aggregate) level of consumption of the final good, $\sigma_c \geq 0$ is the inverse of the intertemporal elasticity of substitution, $h < 1$ measures the degree of external consumption habits, $\eta > 0$ is the inverse Frisch elasticity of labour supply, and $\varphi_{n,w} > 0$ specifies the weight on the disutility of labor market activities $\ell_{w,t}$.[24]

The real budget constraint of a wealthy household in every period t is:

$$c_{w,t} + t_{w,t} + i_t + b_t \leq w_{w,t} n_{w,t} + r_t^k k_{t-1} + \frac{R_{t-1} b_{t-1}}{\pi_t} + \frac{\Pi_t^{int}}{s_w} + \frac{\Pi_t^r}{s_w}$$

[24] Similarly to Galí (2010), we focus on the extensive margin (the changes in the number of workers), and abstract from the intensive margin (the changes in the working time). Moreover, Dossche et al. (2019) indicate that firms in the US adjust their labor input mainly along the extensive margin as only 6% of variation in aggregate hours is attributed to the variation in hours per worker, which is much less than the 48% in the Euro Area.

and the employment law of motion:

$$n_{w,t} = (1-\sigma_w)n_{w,t-1} + \frac{\mu_{w,t}}{1-\mu_{w,t}}u_{w,t}(= h_{w,t})$$

and the law of motion of physical capital:

$$i_t = k_t - (1-\delta_k)k_{t-1} + \frac{\phi_k}{2}\Big(\frac{k_t}{k_{t-1}} - 1\Big)^2 k_{t-1}$$

Note that the nominal variables are transformed in real terms by being divided with the price of the final composite good P_t, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

Wealthy households receive real labor income $w_{w,t}n_{w,t}$ when employed, income from renting capital stock $r_t^k k_{t-1}$, real return on government bonds $\frac{R_{t-1}b_{t-1}}{\pi_t}$ (where R_t is the nominal interest rate set by the central bank), and real profits in the form of dividends $\Pi_t^{int} + \Pi_t^r$ from ownership of intermediate and final goods firms. The household chooses to save these total resources in the form of risk-free government bonds b_t and physical capital i_t, and to spend them by purchasing consumption goods $c_{w,t}$ and paying real lump-sum taxes $t_{w,t}$ to the government.

The Lagrangian function associated with the maximization problem of wealthy household is:

$$\begin{aligned}\mathcal{L} \;=\; \mathbb{E}_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\Bigg\{&\frac{1}{1-\sigma_c}(c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} - \varphi_{n,w}\frac{(\ell_{w,\tau})^{1+\eta}}{1+\eta} - \lambda_{w,\tau}^c\Bigg(c_{w,t} + t_{w,t} + i_t + b_t - \\ &- w_{w,t}n_{w,t} - r_t^k k_{t-1} - \frac{R_{t-1}b_{t-1}}{\pi_t} - \frac{\Pi_t^{int}}{s_w} - \frac{\Pi_t^r}{s_w} + \lambda_{w,\tau}^n\Big(n_{w,t} - (1-\sigma_w)n_{w,t-1} - \frac{\mu_{w,t}}{1-\mu_{w,t}}u_{w,t}\Big)\Bigg) + \\ &+ \lambda_{w,\tau}^l(n_{w,t} + u_{w,t} - \ell_{w,t})\Bigg\}\end{aligned}$$

Let $\lambda_{w,t}^c$, $\lambda_{w,t}^n$, $\lambda_{w,t}^l$ be the Lagrangian multipliers corresponding to the budget constraint, the employment law of motion and the labor force participation, respectively. The first-order conditions for the intertemporal problem of wealthy households are

$$[c_{w,t}]: \quad \lambda_{w,t}^c = \frac{1}{(c_{w,t} - hC_{w,t-1})^{\sigma_c}} \tag{2.7}$$

$$[n_{w,t}]: \quad \lambda^n_{w,t} = \frac{\lambda^l_{w,t}}{\lambda^c_{w,t}} + w_{w,t} + \mathbb{E}_t \beta \frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}} \lambda^n_{w,t+1}(1-\sigma_w) \tag{2.8}$$

$$[\ell_{w,t}]: \quad \lambda^l_{w,t} = -\varphi_{n,w} \cdot \ell^{\eta}_{w,t} \tag{2.9}$$

$$[u_{w,t}]: \quad \lambda^l_{w,t} = -\lambda^c_{w,t} \cdot \lambda^n_{w,t} \cdot \frac{\mu_{w,t}}{1-\mu_{w,t}} \tag{2.10}$$

$$[k_t]: \quad \lambda^c_{w,t}\left(1 + \phi_k\left(\frac{k_t}{k_{t-1}} - 1\right)\right) = \mathbb{E}_t \beta \lambda^c_{w,t+1}\left((1-\delta_k) + r^k_{t+1} + \frac{\phi_k}{2}\left(\left(\frac{k_{t+1}}{k_t}\right)^2 - 1\right)\right) \tag{2.11}$$

$$[b_t]: \quad \lambda^c_{w,t} = \mathbb{E}_t \beta \lambda^c_{w,t+1} \frac{R_t}{\pi_{t+1}} \tag{2.12}$$

The first optimality condition states that the Lagrange multiplier $\lambda^c_{w,t}$ must equal the marginal utility of private consumption. The next three conditions determine the real marginal values of being employed and participating in the labor market. The last two conditions are arbitrage conditions related to the returns on capital and bonds.

The real marginal value of a job for a skilled worker $\lambda^n_{w,t}$ is a function of the disutility of labor market participation (forgone leisure), the real wage and the continuation value of a job (or the expected discounted value of staying employed in the next period). Note that the disutility from labor supply is divided by the marginal utility of consumption to transform utils into consumption goods. In the absence of labor market frictions, there is no surplus for the household of having one more employed member $\lambda^n_{w,t} = 0$. In this case, equation (2.8) reduces to the standard labor supply condition, where the marginal rate of substitution between consumption and leisure $\frac{\lambda^l_{w,t}}{\lambda^c_{w,t}}$ equals the real wage.

2.2.2.2 Non-Ricardian Low-Skilled Households (the Poor)

A continuum of infinitely-lived poor households maximizes their expected lifetime utility:

$$\max_{c_{p,t},l_{p,t},n_{p,t},u_{p,t}} \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c} (c_{p,\tau} - hC_{p,\tau-1})^{1-\sigma_c} - \varphi_{n,p} \frac{(\ell_{p,\tau})^{1+\eta}}{1+\eta} \Big\}$$

subject to the real budget constraint in every period t:

$$c_{p,t} + t_{p,t} \leq w_{p,t} n_{p,t}$$

and subject to the constraint on employment flows:

$$n_{p,t} = (1-\sigma_p) n_{p,t-1} + \frac{\mu_{p,t}}{1-\mu_{p,t}} u_{p,t}$$

The Lagrangian function associated with the maximization problem of a poor household is:

$$\begin{aligned} \mathcal{L} =& \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Bigg\{ \frac{1}{1-\sigma_c} (c_{p,\tau} - hC_{p,\tau-1})^{1-\sigma_c} - \varphi_{n,p} \frac{(\ell_{p,\tau})^{1+\eta}}{1+\eta} - \lambda^c_{p,\tau} \Bigg(c_{p,t} + t_{p,t} - w_{p,t} n_{p,t} + \\ & + \lambda^n_{p,\tau} (n_{p,t} - (1-\sigma_p) n_{p,t-1} - \frac{\mu_{p,t}}{1-\mu_{p,t}} u_{p,t}) \Bigg) + \lambda^l_{p,\tau} (n_{p,t} + u_{p,t} - \ell_{p,t}) \Bigg\} \end{aligned}$$

The optimization with respect to the choice variables of the poor gives the following optimality conditions:

$$[c_{p,t}]: \quad \lambda^c_{p,t} = \frac{1}{(c_{p,t} - hC_{p,t-1})^{\sigma_c}} \tag{2.13}$$

$$[n_{p,t}]: \quad \lambda^n_{p,t} = \frac{\lambda^l_{p,t}}{\lambda^c_{p,t}} + w_{p,t} + \mathbb{E}_t \beta \frac{\lambda^c_{p,t+1}}{\lambda^c_{p,t}} \lambda^n_{p,t+1} (1-\sigma_p) \tag{2.14}$$

$$[\ell_{p,t}]: \quad \lambda^l_{p,t} = -\varphi_{n,p} \cdot \ell^{\eta}_{p,t} \tag{2.15}$$

$$[u_{p,t}]: \quad \lambda^l_{p,t} = -\lambda^c_{p,t} \cdot \lambda^n_{p,t} \cdot \frac{\mu_{p,t}}{1-\mu_{p,t}} \tag{2.16}$$

The Lagrangian multipliers associated with the budget constraint, the employment law of motion and the labor force participation have the same interpretation as in the optimality problem of wealthy households.

The poor can only participate in the labor market as they are excluded from financial/capital markets. For supplying low-skilled labor services to intermediate goods firms, employed poor households receive real labor income $w_{p,t}n_{p,t}$. This total disposable income is used for purchases of consumption goods $c_{p,t}$ and the payment of real lump-sum taxes $t_{p,t}$ to the government. Given that poor households spend all their net disposable income each period in a hand to-mouth manner as in Galí et al. (2007), they are expected to have a larger marginal propensity to consume than wealthy households, and thus be more sensitive to transitory labor income changes. Differently to Galí et al. (2007), hand-to-mouth workers do not have pure myopic behavior due to the dynamic nature of the employment law of motion. They consider the benefits of being employed today. If they get a job today, they are likely to stay employed in the future due to a relatively low separation rate. They will enjoy labor income from employment, which will be used for consumption tomorrow and according improvement of their lifetime utility.

2.2.3 Producers

2.2.3.1 Intermediate (Wholesale) Goods Producers

There is a unit continuum of perfectly competitive firms that produce a homogeneous good $f_{int,t}$ and sell it to retail firms at price $P_{int,t}$ in a competitive market. In the production process, wholesale firms rent the aggregate stock of capital K_t, and hire aggregate skilled labor $N_{w,t}$, and aggregate unskilled labor $N_{p,t}$. The production function takes a standard Cobb-Douglas form with a nested CES composite of two labor inputs:

$$f_{int,t} = F(K_t, N_{w,t}, N_{p,t}) = AK_t^{\iota}\Big[m(N_{w,t})^{\sigma} + (1-m)(N_{p,t})^{\sigma}\Big]^{\frac{1-\iota}{\sigma}} \qquad (2.17)$$

where $A > 0$ stands for the level of aggregate productivity, the parameter ι indicates the income share of physical capital, the parameter m determines the skill intensity (or the productivity level) of labor input, and the parameter σ governs the elasticity of substitution between skilled and unskilled labor in the production process.

When making hiring decisions, intermediate goods firms face labor adjustment costs, which are modelled as training costs and expressed in non-pecuniary terms. Differently to Faccini and Yashiv (2022), the hiring cost function is specified to be asymmetric for differently skilled workers:

$$\tilde{g}^k_{int,t} = \frac{e_k}{2}\left(\frac{H_{k,t}}{N_{k,t}}\right)^2, \quad k \in \{w, p\}$$

where e_k measures the degree of curvature of hiring cost, and $H_{k,t}/N_{k,t}$ is the hiring rate.

The net output of an intermediate goods firm is:

$$Y_{int,t} = f_{int,t}\Bigg(1 - \sum_{k\in\{w,p\}} \tilde{g}^k_{int,t}\Bigg) = f_{int,t} - g_{int,t} \tag{2.18}$$

Intermediate goods producers seek to maximize their nominal profits subject to the employment law of motion (2.6) and the production function net of hiring costs (2.18):

$$P_t \Pi^{int}_t = P_{int,t} Y_{int,t} - W_{w,t} N_{w,t} - W_{p,t} N_{p,t} - R^k_t K_t$$

The real profit of the intermediate goods firms is expressed as follows:

$$\Pi^{int}_t = \frac{Y_{int,t}}{x_t} - w_{w,t} N_{w,t} - w_{p,t} N_{p,t} - r^k_t K_t,$$

where $x_t = \frac{P_t}{P_{int,t}}$ is the retail-price markup defined as a ratio of the price of the final good P_t and the price of the intermediate good $P_{int,t}$. The inverse of retail-price markup $\frac{1}{x_t}$ is the real marginal cost for retail firms.

The present discounted value of real profits of intermediate goods firms is:

$$\max_{K_t,N_{w,t},N_{p,t},H_{w,t},H_{p,t}} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda^c_{t,\tau}\Pi^{int}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\frac{\lambda^c_{w,\tau}}{\lambda^c_{w,t}}\Bigg\{\frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r^k_t K_t -$$

$$- \sum_{k\in\{w,p\}} Q^N_{k,t}\Big(N_{k,t} - (1-\sigma_k)N_{k,t-1} - H_{k,t}\Big)\Bigg\}$$

where $\Lambda^c_{t,t+1} = \beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}$ is the real stochastic discount factor of wealthy households who only own the intermediate goods firms, and $Q^N_{k,t}$ is the Lagrange multiplier on the employment constraint (2.6).

The first order conditions of the real profit function with respect to the firm's choice variables are

$$[K_t] : r^k_t = \frac{1}{x_t}(f_{K,t} - g_{K,t}) \tag{2.19}$$

$$[N_{w,t}] : Q^N_{w,t} = \frac{1}{x_t}(f_{N_{w,t}} - g_{N_{w,t}}) - w_{w,t} + (1-\sigma_w)\mathbb{E}_t\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}Q^N_{w,t+1} \tag{2.20}$$

$$[N_{p,t}] : Q^N_{p,t} = \frac{1}{x_t}(f_{N_{p,t}} - g_{N_{p,t}}) - w_{p,t} + (1-\sigma_p)\mathbb{E}_t\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}Q^N_{p,t+1} \tag{2.21}$$

$$[H_{w,t}] : Q^N_{w,t} = \frac{1}{x_t}g_{H_{w,t}} \tag{2.22}$$

$$[H_{p,t}] : Q^N_{p,t} = \frac{1}{x_t}g_{H_{p,t}} \tag{2.23}$$

The derivatives of the production function and the hiring cost function are given by

$$f_{K,t} = A\iota K_t^{\iota-1}\Big[m(N_{w,t})^\sigma + (1-m)(N_{p,t})^\sigma\Big]^{\frac{1-\iota}{\sigma}}$$

$$f_{N_{w,t}} = AK_t^{\iota}(1-\iota)\Big[m(N_{w,t})^\sigma + (1-m)(N_{p,t})^\sigma\Big]^{\frac{1-\iota}{\sigma}-1} mN_{w,t}^{\sigma-1}$$

$$f_{N_{p,t}} = AK_t^{\iota}(1-\iota)\Big[m(N_{w,t})^{\sigma} + (1-m)(N_{p,t})^{\sigma}\Big]^{\frac{1-\iota}{\sigma}-1}(1-m)N_{p,t}^{\sigma-1}$$

$$g_{N_{w,t}} = -e_w\left(\frac{H_{w,t}}{N_{w,t}}\right)^2\frac{1}{N_{w,t}}f_{int,t} + f_{N_{w,t}}\left(\frac{e_w}{2}\left(\frac{H_{w,t}}{N_{w,t}}\right)^2 + \frac{e_p}{2}\left(\frac{H_{p,t}}{N_{p,t}}\right)^2\right)$$

$$g_{N_{p,t}} = -e_p\left(\frac{H_{p,t}}{N_{p,t}}\right)^2\frac{1}{N_{p,t}}f_{int,t} + f_{N_{p,t}}\left(\frac{e_w}{2}\left(\frac{H_{w,t}}{N_{w,t}}\right)^2 + \frac{e_p}{2}\left(\frac{H_{p,t}}{N_{p,t}}\right)^2\right)$$

$$g_{H_{w,t}} = e_w\left(\frac{H_{w,t}}{N_{w,t}}\right)\frac{1}{N_{w,t}}f_{int,t}$$

$$g_{H_{p,t}} = e_p\left(\frac{H_{p,t}}{N_{p,t}}\right)\frac{1}{N_{p,t}}f_{int,t}$$

$$g_{K,t} = \frac{e_w}{2}\left(\frac{H_{w,t}}{N_{w,t}}\right)^2 f_{K,t} + \frac{e_p}{2}\left(\frac{H_{p,t}}{N_{p,t}}\right)^2 f_{K,t}$$

The first optimality condition is related to the demand for capital, which equates the rental rate of capital with the marginal revenue from using an additional unit of capital. The latter term is the net marginal product of capital multiplied by the real marginal costs. The next two conditions specify the labor demand for two types of workers. In equations (2.20) and (2.21), the real value of a marginal job for a firm is the sum of current real profits from an additional worker and the expected continuation value. Note that the current profits consist of the marginal revenue from employing an additional worker less the real wage payment, while the continuation value presents the expected discounted future profits provided that the worker remains employed. The last two optimality conditions define the firm's hiring decision, which relates the real marginal value of employment for a firm to the real marginal cost of hiring. Accordingly, a wholesale firm tends to hire a new worker until the benefit of hiring equals the cost of hiring that worker.

2.2.3.2 Final (Retail) Goods Producers

A continuum of retail firms indexed by $j \in [0,1]$ operate in a monopolistically competitive market. Each firm purchases the quantity $Y_t(j)$ of the homogeneous intermediate good $Y_t = Y_{int,t}$, which is then used as an input in the production of the final differentiated good $Y_t^f(j)$. The transformation technology is linear, $Y_t^f(j) = Y_t(j)$, so that aggregate final output is given by:

$$Y_t^f = \left[\int_0^1 Y_t^f(j)^{\frac{\epsilon}{\epsilon-1}} dj \right]^{\frac{\epsilon}{\epsilon-1}} = \left[\int_0^1 Y_t(j)^{\frac{\epsilon}{\epsilon-1}} dj \right]^{\frac{\epsilon}{\epsilon-1}} = Y_t$$

where $\epsilon > 1$ is the elasticity of substitution across varieties. It can be shown that the final consumption bundle Y_t^f gives the aggregate price index P_t by solving the standard cost-minimization problem of the firm.

Final goods firms buy intermediate goods at wholesale price $P_{int,t}$, costlessly differentiate them, and then sell a variety of final goods at price $P_t(j)$. When changing their prices, retailers have to pay quadratic price adjustment costs in terms of the final good as in the Rotemberg (1982) model specification. Specifically, the cost is present whenever the ratio between the current price and the price set in the previous period, $P_t(j)/P_{t-1}(j)$, deviates from the steady state inflation rate π:

$$\frac{\phi_p}{2} \left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 Y_t \tag{2.24}$$

where $\phi_p \geq 0$ is a parameter that measures the extent of price adjustment costs.

Each final goods firm chooses its own price $P_t(j)$ to maximize real profits

$$\max_{P_t(j)} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau}^c \Pi_t^r(j) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{\lambda_{w,\tau}^c}{\lambda_{w,t}^c} \left(\left(\frac{P_\tau(j)}{P_\tau} - \frac{P_{int,\tau}}{P_\tau} \right) Y_t(j) - \frac{\phi_p}{2} \left(\frac{P_\tau(j)}{\pi P_{\tau-1}(j)} - 1 \right)^2 Y_\tau \right)$$

subject to (2.24) and the demand of households for final goods variety

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t$$

Taking the derivative with respect to the price $P_t(j)$ gives

$$[P_t(j)]: \ (1-\epsilon)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t} - (-\epsilon)\left(\frac{P_{int,t}}{P_t}\right)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon-1}\frac{Y_t}{P_t} - \phi_p\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)\frac{Y_t}{\pi P_{t-1}(j)}$$

$$+ \mathbb{E}_t\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}\phi_p\left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1\right)\frac{P_{t+1}(j)Y_{t+1}}{\pi P_t(j)^2} = 0$$

Since all retailers produce the same quantity of output in equilibrium, they all set the same price. Given this statement and the aggregate price level in the economy $P_t = (\int_0^1 P_t(j)^{1-\epsilon}dj)^{\frac{1}{1-\epsilon}}$, it follows that $P^*_t(j) = P^*_t$. Accordingly, the optimal pricing condition for retailers is given by

$$(1-\epsilon) + \epsilon\frac{P_{int,t}}{P_t} - \phi_p\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)\frac{P_t}{\pi P_{t-1}(j)} + \mathbb{E}_t\Lambda^c_{t,t+1}\phi_p\left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1\right)\frac{P_{t+1}(j)}{\pi P_t(j)}\frac{Y_{t+1}}{Y_t} = 0$$

$$\Leftrightarrow \quad (1-\epsilon) + \epsilon\ mc^r_t - \phi_p\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} + \mathbb{E}_t\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}\phi_p\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{\pi_{t+1}}{\pi}\frac{Y_{t+1}}{Y_t} = 0$$

When log-linearized around a zero-inflation steady state, the above pricing equation becomes the log-linearized New Keynesian Phillips Curve

$$\widetilde{\pi}_t = \frac{(\epsilon-1)}{\phi_p}\widetilde{mc}^r_t + \beta\mathbb{E}_t\widetilde{\pi}_{t+1}$$

In symmetric equilibrium where all retailers are identical, the value of aggregate real profits distributed to all wealthy households is defined as follows

$$\Pi^r_t = \int_0^1 \Pi^r_t(j)dj = \int_0^1\left(\frac{P_t(j)}{P_t}Y_t(j) - mc^r_t\cdot Y_t(j) - \frac{\phi_p}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)^2 Y_t\right)dj$$

$$\Leftrightarrow \quad \Pi^r_t = \left(1 - mc^r_t - \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2\right)Y_t$$

2.2.4 Monetary and Fiscal Policies

The monetary authority implements monetary policy through a standard Taylor rule that takes the following form:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\theta_r}\left[\left(\frac{\pi_t}{\pi}\right)^{\theta_\pi}\left(\frac{Y_t}{Y}\right)^{\theta_y}\right]^{1-\theta_r} \tag{2.25}$$

where R is the steady-state short term (gross) nominal interest rate, $0 \leq \theta_r \leq 1$ is a parameter associated with interest rate smoothing, $\theta_\pi > 0$ and $\theta_y > 0$ capture the interest rate response to deviations of inflation and output from their respective steady states.

The fiscal authority collects lump-sum taxes from households and issues one-period bonds to finance (unproductive) government purchases and interest payments on its outstanding debt.

The intertemporal government budget constraint expressed in aggregate real terms is:

$$T_t + B_t = \frac{R_{t-1}B_{t-1}}{\pi_t} + G_t \tag{2.26}$$

The real government spending G_t evolves exogenously over time and follows an AR(1) process

$$G_t = G^{1-\phi_g}(G_{t-1})^{\phi_g} exp(\epsilon_t^g), \quad \epsilon_t^g \sim \mathcal{N}(0, \sigma_g)$$

where G is the steady state fraction of government spending, $\phi_g \in (0, 1)$ is the persistence parameter, and ϵ_t^g is the government spending shock.

Lump-sum taxes $T_t = \sum_k s_k t_{k,t}$ follow the passive fiscal policy rule specified as:

$$\frac{T_t}{Y} - \frac{T}{Y} = \phi_B\Big(\frac{B_{t-1}}{Y} - \frac{B}{Y}\Big) + \phi_{BG}\Big(\frac{G_t}{Y} - \frac{G}{Y}\Big)$$

where $\phi_B > 0$ and $\phi_{BG} > 0$ stand for the tax-feedback parameters related to the government debt and spending, respectively. Lump-sum taxes are assumed to be the same for both types of workers.

2.2.5 Wage Bargaining

Following a successful job match between a wholesale firm and a worker, real wages are determined by a standard Nash bargaining process. The negotiation of real wages takes place separately for the two distinct labor markets $k \in \{w, p\}$. The Nash wages maximise the joint match surplus of a worker and a firm weighted by the parameter $\vartheta^k \in [0, 1]$, which refers to the bargaining power of a worker:

$$\max_{w_{k,t}} \left(\frac{\lambda^l_{k,t}}{\lambda^c_{k,t}}+w_{k,t}+\mathbb{E}_t\beta\frac{\lambda^c_{k,t+1}}{\lambda^c_{k,t}}\lambda^n_{k,t+1}(1-\sigma_k)\right)^{\vartheta^k}\left(\frac{1}{x_t}(f_{N_{k,t}}-g_{N_{k,t}})-w_{k,t}+(1-\sigma_k)\mathbb{E}_t\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}Q^N_{k,t+1}\right)^{1-\vartheta^k}$$

The optimality condition to this problem characterizes the surplus sharing rule:

$$\vartheta^k Q^{*,N}_{k,t} = (1-\vartheta^k)\lambda^{*,n}_{k,t}$$

The real value of a marginal job for a firm $Q^{*,N}_{k,t}$ and for a household $\lambda^{*,n}_{k,t}$ are specified as follows:

$$Q^{*,N}_{k,t} = \frac{1}{x_t}(f_{N_{k,t}} - g_{N_{k,t}}) - w^*_{k,t} + (1-\sigma_k)\mathbb{E}_t\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}Q^{*,N}_{k,t+1}$$

$$\lambda^{*,n}_{k,t} = \frac{\lambda^l_{k,t}}{\lambda^c_{k,t}} + w^*_{k,t} + (1-\sigma_k)\mathbb{E}_t\beta\frac{\lambda^c_{k,t+1}}{\lambda^c_{k,t}}\lambda^{*,n}_{k,t+1}$$

The substitution of $Q^{*,N}_{k,t}$ and $\lambda^{*,n}_{k,t}$ in the bargaining solution leads to the real wage for $k \in \{w, p\}$:

$$\begin{aligned} w^*_{k,t} \equiv w^{NASH}_{k,t} &= \vartheta^k\frac{1}{x_t}(f_{N_{k,t}} - g_{N_{k,t}}) - (1-\vartheta^k)\frac{\lambda^l_{k,t}}{\lambda^c_{k,t}} + \\ &+ \vartheta^k(1-\sigma_k)\mathbb{E}_t\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}Q^{*,N}_{k,t+1} - (1-\vartheta^k)(1-\sigma_k)\mathbb{E}_t\beta\frac{\lambda^c_{k,t+1}}{\lambda^c_{k,t}}\lambda^{*,n}_{k,t+1} \end{aligned}$$

The Nash bargained wage includes two terms that are common to both types of workers. The first term is a fraction ϑ^k of the marginal revenue product of a worker. The second term is a fraction $1-\vartheta^k$ of the worker's reservation wage (or the outside option), which is the MRS between consumption and leisure. There is also another term that constitutes a part of the reservation wage distinctive only to the low-skilled. Boscá et al. (2011) call this third term an inequality term in utility. Different access of the two types of workers to financial markets induces this third term. As risk-sharing exists within the household type, but not between them, a difference in the intertemporal MRS is present, $\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}} - \frac{\lambda^c_{p,t+1}}{\lambda^c_{p,t}} \gtrless 0$. Note that the inequality term in utility disappears in the steady state.

Although Boscá et al. (2011) specify different reservation wages for Ricardian and non-Ricardian workers, both types of workers receive the same wage and have the same employment level. The reason is the assumption of the same skill level for those two types of workers. Accordingly, the union structure can pool together both types of workers in the labor market and then bargain with firms about the wage and employment. Our model, however, assumes a segmented labor market, so differently skilled workers have different Nash bargained wages in addition to different reservation wages.

We also introduce real wage rigidity, as in Hall (2005), such that the actual real wage is a weighted average between the actual real wage from the previous period and the Nash wage:

$$w_{k,t} = \rho_w^k w_{k,t-1} + (1 - \rho_w^k) w_{k,t}^*$$

where ρ_w^k controls the degree of real wage rigidity and refers to the fraction of wages not adjusted each period. The importance of sticky wages is to make a search and matching model better in terms of matching empirically observed high volatility of unemployment and low volatility of wages (Shimer, 2005).

2.2.6 Aggregate Variables and Market Clearing

In equilibrium, the market clearing conditions for skilled and unskilled labour, physical capital, bonds, and goods markets are respectively

$$N_{w,t} = s_w n_{w,t}$$

$$N_{p,t} = s_p n_{p,t}$$

$$K_t = s_w k_{t-1}$$

$$B_t = s_w b_t$$

$$Y_t = C_t + I_t + G_t + \frac{\phi_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 Y_t$$

In the aggregate resource constraint, the aggregate consumption is defined as $C_t = s_w c_{w,t} + s_p c_{p,t}$, and aggregate investment as $I_t = s_w i_t$. The goods market clearing condition[25] stems

[25]Derivation of the goods market clearing is provided in Appendix A.1.

from the combination of the budget constraints of the two types of households, the government budget constraint, and the definition of firms' profits. It requires the net aggregate output to be equal to aggregate demand plus the resources allocated to the cost of price adjustment.

2.3 Transmission Mechanism

This section highlights that hiring costs are essential for transmitting the effects of increased government spending on the real economy. How hiring costs are modelled and the composition of hiring costs largely determine whether the government spending induces expansionary or recessionary effects. Accordingly, the following question arises: What are the effects of government spending in the presence of large hiring costs?

In general, there are two strands of literature that take different approaches to modelling the hiring costs. One strand of literature (see e.g. Gertler et al., 2008 and Mayer et al., 2010) generates the expansionary effects of government spending using the standard NK model with DMP framework. The reason behind this result lies in treating hiring costs as *vacancy posting costs*. To see this, let us analyse the (log-linearized) labor demand equation that relates the marginal revenue product of labor to the real wage:

$$(\widetilde{mc}_t^r + \widetilde{f}_{N_t}) - \widetilde{w}_t = 0$$

When the fiscal authorities increase government consumption, the demand for intermediate goods increases. Under the sticky price setting, the difference between intermediate and final good prices increases. The higher aggregate demand pressures, which are reflected in higher real marginal costs for retailers, stimulate intermediate goods firms to hire more workers. As a result of higher employment in the economy, the amount of goods produced becomes larger. In addition to this direct influence of employment on production, there is an indirect stimulus. Specifically, higher labor demand is followed by larger vacancy posting costs, which form a part of aggregate demand as they are pecuniary third-party payments for the provision of hiring services.

The other strand of literature generates the recessionary effects of government spending. The reason lies in modelling hiring costs as *training costs*. To provide the rationale for this result, we can use the job creation condition that equates the marginal benefit of hiring to the marginal cost of hiring:

$$mc^r f_N(\widetilde{mc}_t^r + \widetilde{f}_{N_t}) - mc^r g_N(\widetilde{mc}_t^r + \widetilde{g}_{N_t}) - w\widetilde{w}_t + \mathbb{E}_t\beta(1-\sigma)Q^N(\widetilde{\lambda}_{t+1}^c - \widetilde{\lambda}_t^c + \widetilde{Q}_{t+1}^N) =$$

$$mc^r g_H(\widetilde{mc}_t^r + \widetilde{g}_{H_t})$$

In the above expression, the derivative of the hiring cost function with respect to the new hires in the steady state is

$$g_H = e \cdot \frac{H}{N} \cdot \frac{f_{int}}{N}$$

The higher level of government spending leads to a rise in real marginal costs, which appear on both sides of the job creation condition. Thus, the net output effect is ambiguous. Two studies stand out by assessing the response of the economy to aggregate demand shocks when hiring costs are larger. First, Faccini and Yashiv (2022) perform a sensitivity analysis on the scaling parameter e in the hiring cost function. They find that a higher value of parameter e makes the marginal benefit of hiring lower that the marginal cost of hiring. This reduces the incentives of firms for hiring, which under job separation may translate into a lower level of employment and then output. Second, Picco (2020) examines what happens if the economy starts with a higher steady state level of hiring rate H/N. She also finds recessionary effects of government spending.

Despite the presence of large training costs, the model of this paper may generate expansionary effects. The reason lies in the coexistence of two different types of workers in the production process. The population division is a result of asymmetric training costs in the TANKrep model or the interaction between financial and labor market frictions in the TANK model. Firms do not need to postpone their hiring decision for the times when $\widetilde{mc}_t^r$ are lower, which is the case with the second strand of literature. Instead, at the times of higher $\widetilde{mc}_t^r$ firms may choose to hire low-skilled workers whose hiring (training) costs are lower. We can identify the intuition by analysing the optimal hiring condition for the skill level $k \in \{w, p\}$:

$$mc^r f_{N_k}(\widetilde{mc}_t^r + \widetilde{f}_{N_{k,t}}) - mc^r g_{N_k}(\widetilde{mc}_t^r + \widetilde{g}_{N_{k,t}}) - w_k \widetilde{w}_{k,t} +$$

$$+ \mathbb{E}_t \beta(1-\sigma_k) Q_k^N (\widetilde{\lambda}_{k,t+1}^c - \widetilde{\lambda}_{k,t}^c + \widetilde{Q}_{k,t+1}^N) = mc^r g_{H_k}(\widetilde{mc}_t^r + \widetilde{g}_{H_{k,t}})$$

This job creation condition stems from the combination of firm's optimality conditions for employment and hiring.

What is noticeable in the second strand of literature is that it only considers the effects coming from differences in the steady state values of marginal hiring costs $g_H = e \cdot \frac{H}{N} \cdot \frac{f_{int}}{N}$ by changing the values of parameter e and the hiring rate H/N. Similarly, our paper considers how different steady state values of marginal hiring costs for high-skilled workers g_{H_w} and low-skilled workers g_{H_p} affect the job creation condition of respective workers.

Our paper also highlights the importance of dynamic responses of variables. Introducing financial friction in the TANK model causes population decomposition. Poor households who have restricted access to financial/capital markets exhibit high MPC, which increases the sensitivity of real marginal costs depending on their net disposable income. Moreover, the members of poor households are low-skilled workers with no pure myopic behaviour. They perceive the chance of improving their lifetime utility by being hired, which lowers their reservation wage and then their market wage. With higher real marginal costs and lower wage payments, the hiring of low-skilled workers becomes more attractive even in the presence of training costs. The marginal benefit is higher than the marginal cost of hiring low-skilled workers, incentivizing intermediate goods firms to hire them more. Increased hiring activity is also closely followed by higher investment in capital due to the complementarity between inputs in the CD production function. As a result, an economic expansion may arise.

A lower reservation wage of low-skilled workers is associated with a decreasing inequality term in utility. This claim is supported by two important reasons. First, the discrepancy between today's and tomorrow's level of consumption is much more pronounced for low-skilled workers,

$\widetilde{\lambda}^c_{w,t+1} - \widetilde{\lambda}^c_{w,t} < \widetilde{\lambda}^c_{p,t+1} - \widetilde{\lambda}^c_{p,t}$. The rationale is that low-skilled workers cannot use wealth to smooth their consumption over time. Second, low-skilled workers enjoy higher consumption today than tomorrow due to a larger net disposable income today, $\widetilde{\lambda}^c_{p,t+1} - \widetilde{\lambda}^c_{p,t} > 0$. As increased government expenditure is financed by an increasing level of taxes, net disposable income becomes relatively higher today than tomorrow. Hence, poor household tends to have as many (low-skilled) workers employed as possible in order to collect more labor income sources tomorrow, which will then be used for consumption. Taking both reasons together, the gap in the intertemporal MRS can be interpreted as a readiness of low-skilled workers to accept a lower wage payment.

2.4 Calibration

Table 2.1 shows the calibrated values of structural parameters for the US economy at a quarterly frequency. The values of these parameters are determined internally by solving a non-stochastic steady state corresponding to the long run pre-crisis average values of targets, and externally in accordance with the estimates from the existing literature.

There are two baseline model economies, TANKrep and TANK, which are populated with high- and low-skilled workers. These workers either live together in one big family (TANKrep) or separately in two big families, due to their different access to financial markets (TANK). Population shares in both models are set to $s_w = 0.5$ and $s_p = 0.5$, which implies that 50 percent of the total population provides high-skilled labor services to intermediate goods firms. This is in line with Wolcott (2021) who indicates that 56 percent of the US population in 2007 can be regarded as high-skilled as they have at least one year of college education and accordingly search for high-skilled jobs. In addition, the TANK model is characterized by high-skilled workers who only have access to financial markets, and thus allows us to examine the role of financial friction in driving the output responses to government spending. The same population share for high-skilled workers who are treated as 'Ricardian' and low-skilled workers who are 'hand-to-mouth' can be found in Bhattarai et al. (2022).

In calibrating the parameters related to SAM frictions in the labor market, this paper closely follows Dolado et al. (2021). Accordingly, the parameters $\varphi_{n,k}$ and ϑ^k are jointly determined by matching the pre-crisis average values of participation and unemployment rates for the two types of workers $k \in \{w, p\}$

$$partic\_k = \frac{N_k + U_k}{s_k} \quad \text{and} \quad unemp\_k = \frac{U_k}{N_k + U_k}$$

The parameters $\varphi_{n,k}$ and ϑ^k stand for the weight on the disutility of labor market activities and the bargaining power of workers, respectively.

The baseline calibration in Table 2.1 specifies symmetry in participation and unemployment rates for the two types of workers. As in Dolado et al. (2021), this implies a participation rate of 0.675 and an unemployment rate of 0.053. There is also a symmetry in the training costs scaling parameters $e_w = e_p$ and the two parameters associated with SAM frictions: the job separation rate $\sigma_w = \sigma_p$ and the matching efficiency $\psi_w = \psi_p$. In the matching function, the matching elasticity $\varsigma = 0.5$ is assumed to be the same for both types of workers.

We also consider two additional models, Model 3 and Model 4 in Table 2.2, which examine asymmetric training costs and the interaction of financial friction with asymmetric training costs and SAM frictions, respectively. Model 3 relies on the calibrated values of parameters from the TANKrep and includes asymmetric training costs. Model 4 uses the parameter values from the TANK and incorporates both asymmetric training costs and asymmetric SAM frictions. In these two additional models, keeping the same values of parameters from the TANKrep and the TANK implies that the skill premium, participation rate and unemployment rate become non-targeted. Table 2.2 reports three results that suggest the good performance of the models. First, the model-induced values of the stated variables are close to their real-data counterparts. Second, the model non-targeted steady state ratios θ_w/θ_p, μ_w/μ_p, and ν_w/ν_p match well the estimates of Wolcott (2021). Third, training costs are close to one percent of aggregate output, which is comparable to the aggregate hiring costs in Blanchard and Galí (2010) due to the small vacancy costs in the data.

In addition to the internally calibrated parameters $\varphi_{n,w}$ and $\varphi_{n,p}$, the other parameters in the utility function specification include β, σ_c, h, η. The subjective discount factor, $\beta = 0.9945$, is calibrated to match a quarterly gross interest rate of around 1 percent ($R = 1 + \frac{2.21}{4 \cdot 100} = 1.0055$). The inverse of the intertemporal elasticity of substitution σ_c is set to 1, giving the log form of the utility function in consumption. The degree of external habit formation h takes the conventional value of 0.75. The inverse Frisch elasticity of labor supply η on the extensive margin is set to 1. Chang et al. (2019) indicate that the value of 1 for η is quite a reasonable value.

In the production process of intermediate goods firms, the steady-state value of the technological process A is normalized to 1. To match an investment rate of 2.5%, the quarterly depreciation rate of physical capital δ_k is set to 0.025, which corresponds to 10% in annual terms. As is standard in the literature, the income share of capital is $\iota = 0.35$. This choice implies the elasticity of substitution between capital and high-skilled/low-skilled labor of $1/(1 - \iota) = 1.538$. The parameter governing the income share of high-skilled labor input $m = 0.6241$ is calibrated to match a skill (wage) premium of 1.55, the value provided by Bhattarai et al. (2022). Following Katz and Murphy (1992), the parameter σ is set to 0.2908, which implies the elasticity of substitution between high- and low-skilled labor of $1/(1 - \sigma) = 1.41$. The degree of real wage rigidity for both types of workers is set to 0.8 to be consistent with Dolado et al. (2021).

Silva and Toledo (2009) report that average training costs are 55% of quarterly wages in the US, while only around 5% of quarterly wages goes to average vacancy costs. According to Faccini and Melosi (2022), the corresponding value of the scaling parameter for training costs is $e = 5.0417$. Given that Faccini and Yashiv (2022) take this value as an approximation of high training costs, we assume that $e_w = e$. For the case of symmetric SAM frictions and symmetric training costs, the ratio of the scaling parameters is $e_w/e_p = 1$. If the case of asymmetric training costs is considered, this ratio is determined from the ratio of average hiring (training) costs in terms of wages.

There are two observations that the ratio of average hiring costs in terms of wages equals one. First, Blatter et al. (2012) compare the construction sector with the industrial (and service) sectors in terms of hiring costs in weeks of wage payments. They find that hiring costs in the construction sector are around 1/1.55 of those in the industrial (and service) sectors. Second, the construction sector is known to be characterized by lower skill requirements. Accordingly, the ratio of average hiring costs in terms of wages is given by:

$$\frac{(g^{w}_{int} \cdot mc^{r}/H_{w})/w_{w}}{(g^{p}_{int} \cdot mc^{r}/H_{p})/w_{p}} = 1, \quad \text{where} \quad g^{k}_{int} = \frac{e_{k}}{2}\left(\frac{H_{k}}{N_{k}}\right)^{2} f_{int}, \quad \text{for} \quad k \in \{w, p\}$$

From the above equation, we can express a ratio of scaling parameters:

$$\frac{e_{w}}{e_{p}} = \frac{w_{w}}{w_{p}}\frac{\sigma_{p}}{\sigma_{w}}\frac{N_{w}}{N_{p}}$$

For the case of symmetric SAM frictions and asymmetric training costs, this ratio is $e_{w}/e_{p} = 1.55$. We will also consider the alternative values for the ratio e_{w}/e_{p} as Belo et al. (2017) indicate that the ratio of the labor adjustment costs parameters in the high- and low-skill industries is 10.5. A ratio of similar value can be found in Blatter et al. (2012) when comparing average hiring costs of occupations with the highest labor skills (an automation technician) and the lowest labor skills (a medical assistant).

The steady state gross inflation rate is normalized to 1. The elasticity of substitution across varieties ϵ is set to 11, which refers to a final good price mark-up of 10% over the intermediate good ($\mu^{p} = \frac{\epsilon}{\epsilon-1} = 1.1$). The Rotemberg quadratic adjustment cost parameter is set to 118.0521 to be consistent with the Calvo (1983) price stickiness model, where prices change on average once every fourth quarter. If the share of retailers that can adjust their prices is given by $1-\theta$, then the value for parameter ϕ_{p} is

$$\phi_{p} = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\beta\theta)} = \frac{11 \cdot 0.75}{(1-0.75)(1-0.75 \cdot 0.9945)} = 118.0521$$

The capital adjustment costs parameter ϕ_{k} is set to 4 as in Dolado et al. (2021), which together with $\delta_k = 0.025$ implies that the elasticity of the investment-to-capital ratio with respect to Tobin's q is 10. The detailed derivation of this elasticity is given in Appendix A.2.

The steady state share of government expenditure in output is set to 20%, while a ratio of government debt to output is set to 2.8 or to 70% in annual terms. As for the fiscal and monetary policy parameters, they take common values in the literature. Specifically, the tax-feedback parameters related to government debt and spending are set to 0.33 and 0.1. In addition, the interest rate responsiveness to the inflation and output gaps are set to 1.5 and 0.5/4, while the interest rate smoothing parameter is 0.75.

Table 2.1: Parameter values for Model 1 (TANKrep: SAM+TC) and Model 2 (TANK: SAM+TC+FF)

Notation	Description	Model1 (Model2)	Source
Households			
β	Subjective discount factor	0.9945	quarterly R of 1%
η	Elasticity of labor supply	1	Convention
σ_c	Coefficient of relative risk aversion	1	Convention
$\varphi_{n,w}$	Relative weight on ℓ_w	23.1420 (2.9992)	Target is $partic\_w = 0.675$
$\varphi_{n,p}$	Relative weight on ℓ_p	14.9304 (6.5372)	Target is $partic\_p = 0.675$
s_w	Population share of the wealthy	0.5	Bhattarai et al. (2022)
Inter goods firms			
A	Production scale parameter	1	Convention
δ_k	Capital depreciation rate	0.025	quarterly I/K of 2.5%
ι	Income share of capital	0.35	Convention
m	Income share of high-skilled labor	0.6075	Target is $w_w/w_p = 1.55$
σ	Measure of elas of subs b/w N_w and N_p	0.2908	Katz and Murphy (1992)
ϕ_k	Capital adjustment cost	4	Dolado et al. (2021)
Final goods firms			
ϕ_p	Price adjustment cost	118.05	reset prices every 4 quarters
ϵ	Elas of subs between retail goods	11	mark-up of 10%
Labor market			
σ_w	Separation rate-wealthy	0.0404	Dolado et al. (2021)
σ_p	Separation rate-poor	0.0404	Dolado et al. (2021)
ρ_w^k	Wage stickeness	0.8	Dolado et al. (2021)
e_w	Hiring friction parameter-wealthy	5.079	Average hiring cost/wage
e_p	Hiring friction parameter-poor	5.079	Average hiring cost/wage
ϑ^w	Bargaining power-wealthy	0.7131	Target is $unemp\_w = 0.053$
ϑ^p	Bargaining power-poor	0.6159	Target is $unemp\_p = 0.053$
ς	Matching elasticity	0.5	Dolado et al. (2021)
ψ_w	Matching efficiency-wealthy	0.5875	Dolado et al. (2021)
ψ_p	Matching efficiency-poor	0.5875	Dolado et al. (2021)
Fis and mon policy			
ϕ_B	Tax response to debt	0.33	Convention
ϕ_{BG}	Tax response to gov spending	0.1	Convention
θ_π	Monetary policy response to inflation	1.5	Convention
θ_y	Monetary policy response to output	0.125	Convention
θ_r	Monetary policy inertia	0.75	Convention
ϕ_g	Gov spending persistence	0.9	Faccini and Melosi (2022)
σ_g	Volatility of gov spending shock	0.01	Faccini and Melosi (2022)

Notes: TANKrep - two types of workers live together in one representative household, TANK - two types of workers live separately in their own representative household, SAM - symmetric search and matching frictions, TC - symmetric training costs, and FF - financial friction.

Table 2.2: Selected Steady-state values of variables

Notation	Description	Model 1	Model 2	Model 3	Model 4	Data
Targeted						
G/Y	Government consumption to GDP ratio	0.2	0.2	0.2	0.2	0.2
$B/4Y$	Debt to GDP ratio (annualised)	0.7	0.7	0.7	0.7	0.7
ci_share	Capital income share	0.35	0.35	0.35	0.35	0.35
w_w/w_p	Skill premium	1.55	1.55			1.55
$partic\_w$	Participation rate - wealthy	0.675	0.675			0.675
$partic\_p$	Participation rate - poor	0.675	0.675			0.675
$unemp\_w$	Unemployment rate - wealthy	0.053	0.053			0.053
$unemp\_p$	Unemployment rate - poor	0.053	0.053			0.053
Non-targeted						
μ_w/μ_p	Ratio of job finding rates			0.78	1.15	1.15
θ_w/θ_p	Ratio of labor market tightness			0.61	0.53	0.65
ν_w/ν_p	Vacancy filling probabilities			1.28	2.17	1.74
w_w/w_p	Skill premium			1.58	1.50	1.55
$partic\_w$	Participation rate - wealthy			0.674	0.677	0.69
$partic\_p$	Participation rate - poor			0.678	0.677	0.66
$unemp\_w$	Unemployment rate - wealthy			0.053	0.019	0.028
$unemp\_p$	Unemployment rate - poor			0.034	0.067	0.078
$\frac{(g_{int}^{w} \cdot mc^r/H_w)/w_w}{(g_{int}^{p} \cdot mc^r/H_p)/w_p}$	Ratio of average hiring cost to wage	0.645	0.645	1	0.429	1
g_{int}/Y	Hiring cost to GDP ratio	0.008	0.008	0.007	0.007	0.01

Notes: Model 1 is TANKrep: SAM+TC. Model 2 is TANK: SAM+TC+FF. Model 3 is TANKrep: SAM+ATC, which has the same parameter values from Model 1 and $e_w = 5.079 > e_p = 3.277$. Model 4 is TANK: ASAM+ATC+FF, which has the same parameter values from Model 2 and $\psi_w = 0.720 > \psi_p = 0.455$, $\sigma_w = 0.025 < \sigma_p = 0.056$ and $e_w = 5.079 > e_p = 3.277$.

2.5 Results

This section is divided into three parts. The first part reports the responses of the real economy in the TANKrep and TANK models and how the change in the form of hiring costs[26] (vacancy costs or training costs) affects the propagation of a government spending shock to the real economy. The second part examines whether the symmetric or asymmetric forms of search and matching frictions interacted with training costs play a dominant role in driving the responses of the real output. In this part of the analysis, the focus is on training costs rather than vacancy costs as they may generate counterintuitive recessionary effects of expansionary policies (see, for instance, Picco, 2020 and Faccini and Yashiv, 2022). The third part provides a more general picture of the dynamic responses of many real economic variables of interest. The analysis in all three parts is conducted in both the representative and heterogeneous agent frameworks. The consideration of the latter setting is the novel contribution of this paper to the literature.

Figure 2.1 shows impulse responses of output, hiring, employment and the value of output in the TANKrep model to an expansionary fiscal policy shock, which corresponds to a rise in government spending of one percent of steady-state output. As opposed to the RANK model, which shows recessionary effects, the TANKrep model reports economic expansion. The rise in output in the TANKrep model is dependent on the extent of training costs of the new hires. With smaller training costs for low-skilled workers, the output records a stronger expansion. When the value of output is high, firms decide to hire low-skilled workers to a large extent, while the hiring of high-skilled workers is negative or small. When the value of output is low, firms choose to hire more high-skilled workers whose marginal productivity is higher.

In Figure 2.2, the expansionary effects of government spending can be observed for the RANK models with hiring costs modelled as vacancy posting costs, and the TANK model with training costs. By contrast, the persistent recessionary effects are distinctive to the model

[26] A formal presentation of different forms of hiring costs is provided in Appendix A.3.

with one type of workers where hiring frictions are expressed as training costs or in terms of foregone output. These results are in line with the section dedicated to the transmission mechanism. Additionally, the RANK model with vacancy costs in pecuniary terms or in units of final good generates larger expansionary effects than the RANK model with vacancy costs in non-pecuniary terms or in units of intermediate good. The reason is that non-pecuniary hiring costs cause disruption in production, while pecuniary hiring costs are characterized by third-party payments for the provision of hiring services.

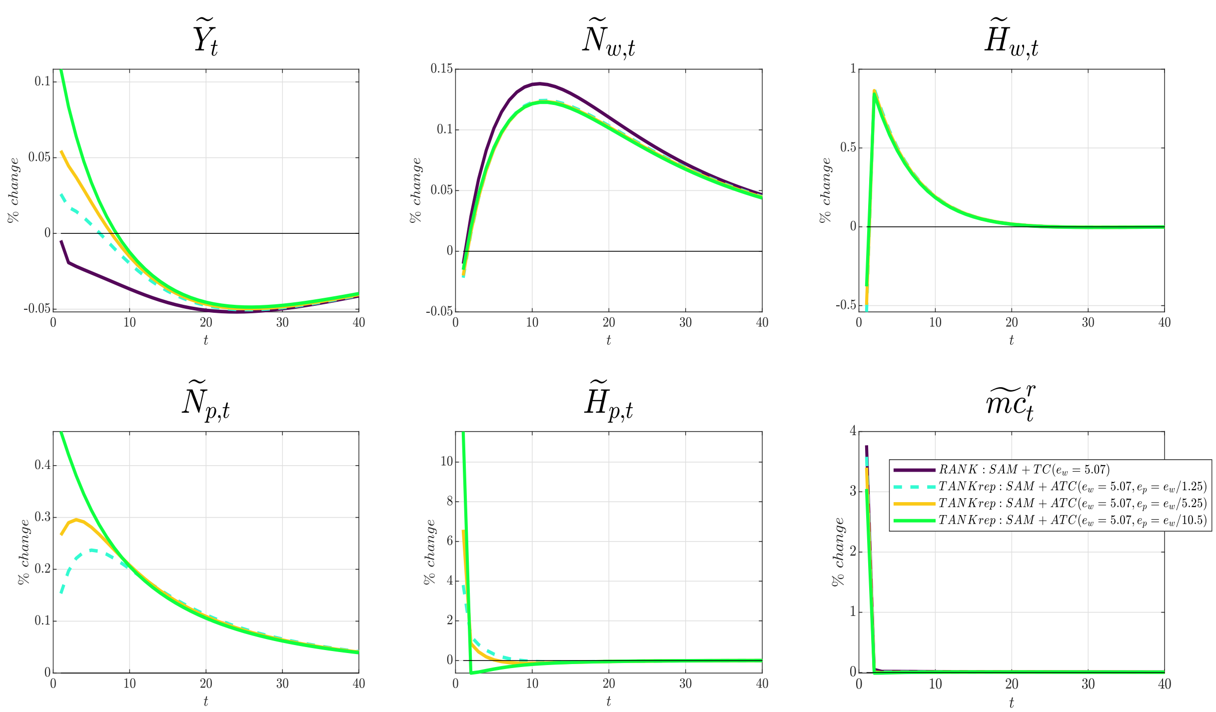


Figure 2.1: Impulse responses to a fiscal expansion when real wages are flexible

Notes: RANK: SAM + TC refers to the representative agent New Keynesian model with search and matching frictions SAM and training cost TC. TANKrep: SAM + ATC is the two agent New Keynesian model with a representative household. SAM frictions include matching efficiency ψ_k, separation rate σ_k, and bargaining power ϑ^k. The results of these models are generated for the case of absent real wage rigidity, $\rho_w^k = 0$.

The impulse response analysis in Figure 2.1 and Figure 2.2 emphasizes that expansionary effects are still present despite modelling hiring costs in terms of foregone output, and even surpass the effects of vacancy costs. Note that these results are generated under the assumption of flexible wages. In the next part of the analysis, we examine how the responses of the real economy change when real wage rigidity is introduced.

Figure 2.3 displays the output responses to an increase in government spending assuming real wage stickiness and the interaction of SAM frictions with training costs. The left panel differs from the right panel in that it considers only the economies characterized by symmetry in SAM frictions and training costs. If we focus on the left panel, despite the symmetry in labor market frictions, the response of output is markedly different. Initially, the economies with two types of workers (blue and red solid lines) experience a drop in output, which is then followed by an expansion. However, this output reduction is much less persistent and pronounced relative to the economy with only one type of workers (purple solid line). In addition, models with two types of workers document output recovery to its pre-crisis average level after around two years, while output in a model with one type of workers does not complete its recovery even after 10 years.

The firm's hiring decision lies at the core of the output responses. The models with one type and two types of workers both report a rise in output in the first period, which is an indication of a greater marginal benefit than the marginal cost of hiring. However, this output expansion is small, as hiring is associated with training costs that swallow up output. In the next period, the assumption of sticky wages and a training costs specification imply a more expensive hiring of new workers. Specifically, a higher aggregate demand pressure in the first period leads to a rise in labor demand and wages, which are largely transmitted to wages in the next period due to wage rigidity. In addition, training activity causes production disruption so that firms would rather choose to postpone hiring and focus on sales that are more profitable at the time of a high value of output. With relatively high hiring costs in the RANK model as in Faccini and Yashiv (2022) and Picco (2020), weak employment and output occur. However, in the economy populated with two types of workers, firms have a choice in a hiring process. When aggregate demand pressure is large, a cheaper labor force

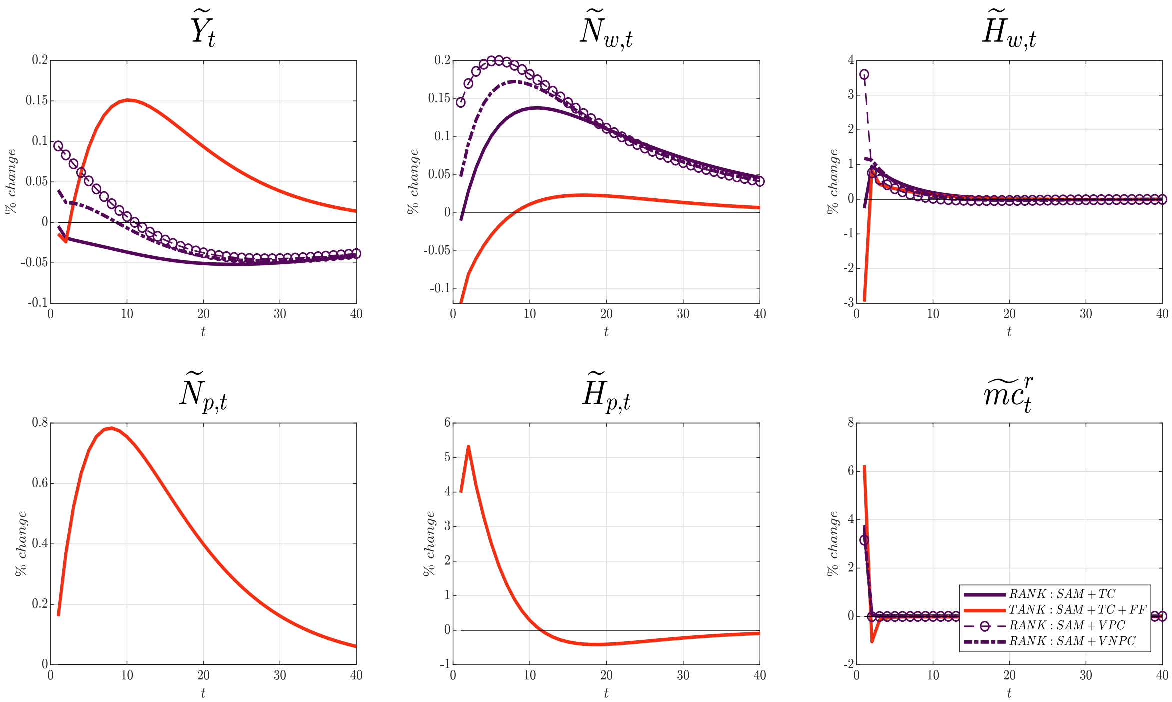


Figure 2.2: Impulse responses to a fiscal expansion when real wages are flexible

Notes: RANK: SAM + TC refers to the representative agent New Keynesian model with search and matching frictions SAM and training cost TC. TANK: SAM + TC + FF is the two agent New Keynesian model where households face asymmetric financial friction. SAM frictions include matching efficiency ψ_k, separation rate σ_k, and bargaining power ϑ^k. VPC is vacancy posting costs in pecuniary terms while VNPC is vacancy posting costs in non-pecuniary terms. The results of the stated models are generated for the case of absent real wage rigidity, $\rho_w^k = 0$.

such as low-skilled workers can be used to sustain production until the period of relatively low value of output.

In the left panel of Figure 2.3, a blue solid line represents the contribution of adding financial friction (FF) to the RANK model. To measure the influence of FF, workers need to be equally productive, alongside the symmetry in SAM frictions and training costs. This is achieved by adding symmetry in their skill intensity $m = 0.5$ and perfect substitutability in the production function $\sigma = 1$. With no skill mismatch in production, the fall in output and subsequent expansion are mitigated compared to the red solid line. Note that higher skill intensity and higher complementarity of labor inputs are incorporated in the model in line with observed labor market dynamics in the US economy.[27] When $m > 0.5$, high-skilled workers are more present in production. However, initial periods feature a higher value of output, which under higher marginal costs of hiring of high-skilled workers leads to a larger output contraction.[28] In later periods, when the value of output is lower, firms are incentivized to hire more for two reasons: the lower value of foregone output and larger production capacity, as higher skill intensity is associated with higher marginal productivity of high-skilled workers. Firms also hire more low-skilled workers due to higher complementarity between the two types of workers, $\sigma < 1$.

The right panel of Figure 2.3 focuses on the interaction of the symmetric and asymmetric forms of SAM frictions and training costs. In comparison with the RANK model, all models presented with two groups of workers characterize the expansionary effects of government spending. Asymmetric SAM frictions go in favour of high-skilled workers, but generate a slightly stronger economic expansion (yellow solid line) than symmetric SAM frictions (red solid line). A stronger economic expansion is recorded for asymmetric training costs that favor low-skilled workers (compare the green and red solid lines).

The third part of this section provides the impulse response analysis of several real economic variables of interest besides the real output. A government spending shock, which can be interpreted as an aggregate demand shock, gives rise to elevated aggregate demand pressures in the economy (see the dynamics of $\widetilde{mc}_t^r$ in Figure 2.4). To keep the budget balanced over time, the government finances an increased demand for goods by raising lump-sum taxes and issuing debt. The negative wealth effect of government spending comes into play as agents in

[27]As stated in calibration, skill premium and imperfect substitutability between high-skilled and low-skilled workers underlie the labor market in the US.

[28]A larger skill intensity in production leaves less space for low-skilled workers, who are a cheaper labor force, due to financial friction. The role of this type of workers is especially important for sustaining production in the period of a high value of output.

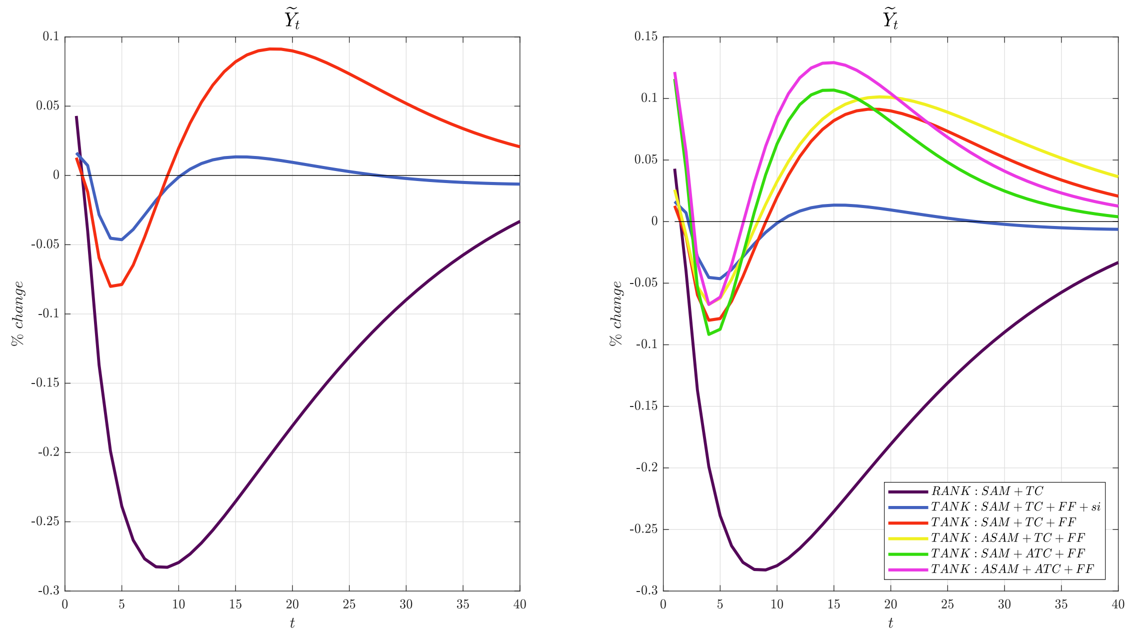


Figure 2.3: Impulse responses of the economy to a fiscal expansion when real wages are rigid

Notes: SAM - search and matching frictions, TC - training costs, ASAM - asymmetric search and matching frictions in all three parameters ($\psi_w \neq \psi_p$, $\vartheta^w \neq \vartheta^p$, $\sigma_w \neq \sigma_p$), ATC - asymmetric training costs ($e_p = e_w/5.25$), si - symmetric skill intensity, and FF - financial friction. The results of the stated models are generated for the case of real wage rigidity, $\rho_w^k = 0.8$.

the economy perceive that the fiscal stimulus goes hand-in-hand with higher tax payments. In response to a lower disposable income, workers participate more actively in the labor market. In addition to consuming less leisure, high-skilled workers decide to consume less consumption goods and save more. More precisely, they save more in the form of government bonds at the expense of capital investment due to a greater demand of the government for bonds and a greater real interest rate.

In Figure 2.4, the four model specifications display qualitatively similar results regarding the crowding-out of private consumption and capital investment. However, the quantitative effects are different, especially those related to the response of investment in capital. A faster recovery of investment is observed for the TANKrep and TANK models. These models

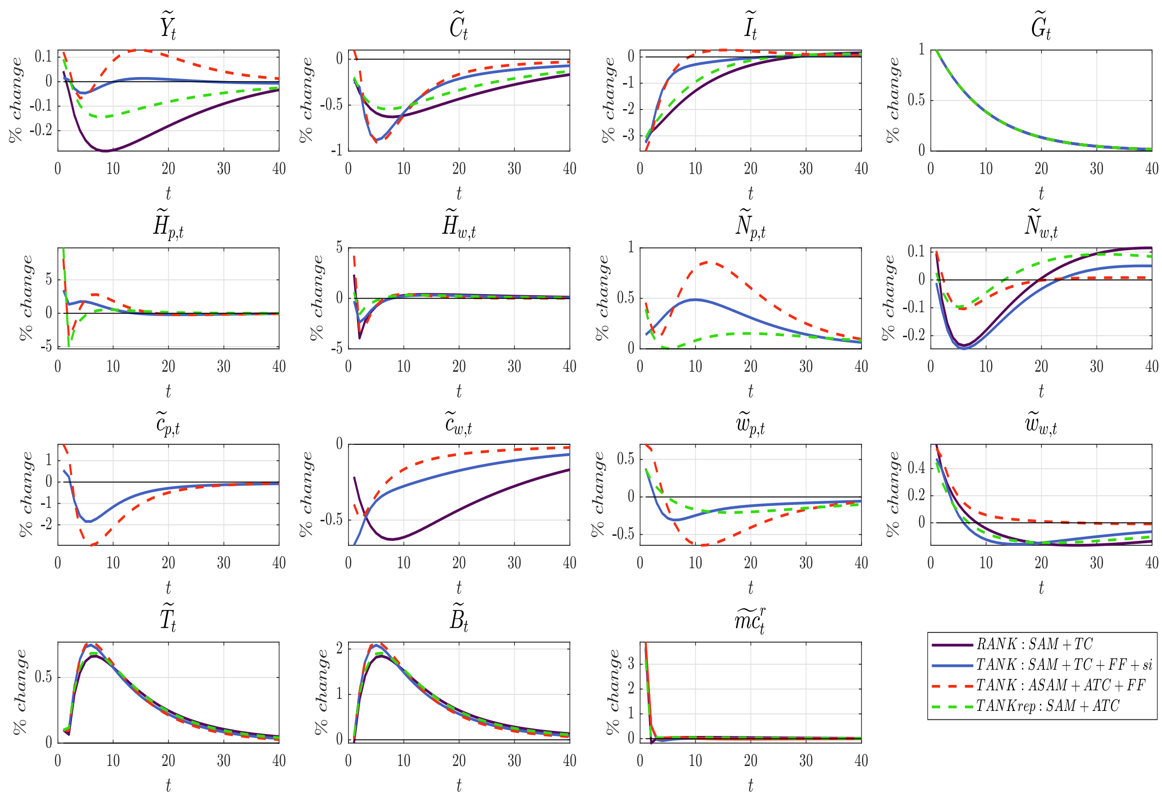


Figure 2.4: Impulse responses of the economy to a fiscal expansion when real wages are rigid

Notes: SAM - search and matching frictions, TC - training costs, ASAM - asymmetric search and matching frictions in all three parameters ($\psi_w \neq \psi_p$, $\vartheta^w \neq \vartheta^p$, $\sigma_w \neq \sigma_p$), ATC - asymmetric training costs ($e_p = e_w/5.25$), si - symmetric skill intensity, and FF - financial friction. The results of the stated models are generated for the case of real wage rigidity, $\rho_w^k = 0.8$.

underlie a higher level of hiring and associated employment, which limits a drop in investment. Firms accumulate more capital to ensure that an increasing number of workers in production is equipped with a sufficient level of capital. Better investment opportunities exert a stimulative impact on production activities.

2.5.1 Productivity-Enhancing Government Spending and the Fiscal Multiplier

Given a close relationship between capital investment and job creation in the economy, it is useful to perform a counterfactual analysis on whether that relationship can be improved. Specifically, this analysis compares the effectiveness unproductive government spending considered so far with productive spending. Productivity-enhancing government spending assumes that government capital enters the aggregate production function of intermediate goods firms

$$f_{int,t} = AK_t^{\iota}\Big[m(N_{w,t})^{\sigma} + (1-m)(N_{p,t})^{\sigma}\Big]^{\frac{1-\iota}{\sigma}} K_{g,t}^{\zeta}$$

where $K_{g,t}$ is productive government capital and ζ is a parameter that determines the productivity of government capital. Moreover, we specify the law of motion of government investment

$$G_{I,t} = K_{g,t} - (1-\delta_{k_g})K_{g,t-1}$$

and an AR(1) process of government investment

$$G_{I,t} = G_I^{1-\phi_{g_I}}(G_{I,t-1})^{\phi_{g_I}} exp(\epsilon_t^{g_I}), \quad \epsilon_t^{g_I} \sim \mathcal{N}(0, \sigma_{g_I})$$

We follow Sims and Wolff (2018) in setting a value of parameter $\zeta = 0.05$, the depreciation rate on government capital $\delta_{k_g} = 0.025$, a value of parameter $\phi_{g_I} = 0.9338$, and the value of the steady-state capital ratio $\frac{K_{g,t}}{K_t} = 0.165$. Note that a value of parameter $\zeta = 0$ returns the benchmark specification with only unproductive government spending. There are two useful observations in Figure 2.5 regarding the effects of government investment. First, productivity-enhancing government spending generates larger expansionary effects in both models (compare dashed lines with solid lines). This is because government investment leads to a higher marginal productivity of labor inputs, which is then transmitted to increased labor demand of firms. Hence, from a policy maker's perspective, it is better to use government investment than government consumption to deal with recessions. Second, the expansionary effects of government investment in the TANK model are stronger and more persistent than in the RANK model.

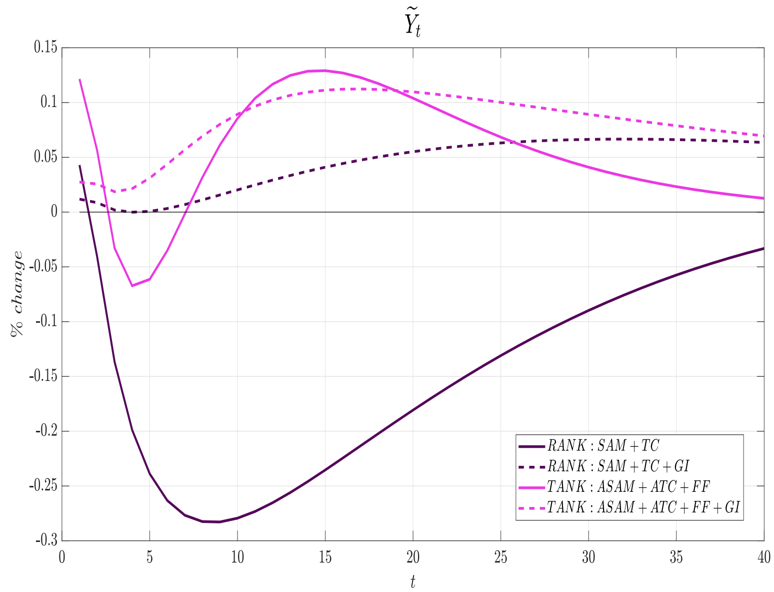


Figure 2.5: Impulse responses of the economy to a fiscal expansion when real wages are rigid

Notes: SAM - search and matching frictions, TC - training costs, ASAM - asymmetric search and matching frictions in all three parameters ($\psi_w \neq \psi_p$, $\vartheta^w \neq \vartheta^p$, $\sigma_w \neq \sigma_p$), ATC - asymmetric training costs ($e_p = e_w/5.25$), and FF - financial friction. GI is government investment. The results of the stated models are generated for the case of real wage rigidity, $\rho_w^k = 0.8$. Solid lines indicate impulse responses of output to government consumption shocks, while dashed lines indicate impulse responses of output to government investment shocks.

The next part of the analysis focuses on the cumulative fiscal multiplier, which is defined as a ratio of the cumulative sum of the discounted percentage output changes and that of government spending changes for a given horizon k

$$fm = \frac{\sum_{k=0}^{\infty} \beta^k dY_k}{\sum_{k=0}^{\infty} \beta^k dG_k}$$

For $k = 0$, the above expression refers to the impact fiscal multiplier.

Table 2.3 shows that the RANK model has negative fiscal multipliers of government consumption for both types of wage specifications and over all horizons (the exception is the first period in the model with rigid wages). The reason lies in strong crowding-out of aggregate demand components such as private consumption and capital investment. When flexible wages are considered, the TANK model shows positive (cumulative) fiscal multipliers of government consumption. As for the rigid wages specification, positive fiscal multipliers are documented for the TANK model with asymmetric search and matching frictions and asymmetric training costs. In addition, the TANK model with government investment has positive fiscal multipliers over all horizons. Compared to the RANK model with government investment, the fiscal multiplier in the TANK model is more than twice as large.

Table 2.3: Fiscal multipliers across different models and different horizons

	Horizon k				
Model	1Q	1Y	5Y	10Y	250Y
Flexible wages					
$RANK : SAM + TC$	-0.005	-0.017	-0.055	-0.101	-0.245
$TANKrep : SAM + ATC$	0.055	0.050	0.009	-0.042	-0.186
$TANK : SAM + TC + FF$	-0.015	0.001	0.167	0.236	0.288
$RANK : SAM + VPC$	0.095	0.092	0.058	0.010	-0.130
$RANK : SAM + VNPC$	0.039	0.033	0.008	-0.038	-0.179
Rigid wages					
$RANK : SAM + TC$	0.042	-0.061	-0.363	-0.493	-0.561
$TANKrep : SAM + ATC$	0.092	0.015	-0.161	-0.225	-0.302
$TANK : SAM + TC + FF + si$	0.016	-0.005	-0.019	-0.012	-0.028
$TANK : SAM + TC + FF$	0.012	-0.026	-0.009	0.069	0.145
$TANK : ASAM + TC + FF$	0.026	-0.017	0.012	0.102	0.216
$TANK : SAM + ATC + FF$	0.116	0.032	0.042	0.103	0.136
$TANK : ASAM + ATC + FF$	0.122	0.048	0.080	0.158	0.216
$RANK : SAM + TC + GI$	0.055	0.035	0.113	0.317	1.055
$TANK : ASAM + ATC + FF + GI$	0.128	0.121	0.437	0.755	1.550

Notes: SAM - search and matching frictions, TC - training costs, ASAM - asymmetric search and matching frictions in all three parameters ($\psi_w \neq \psi_p$, $\vartheta^w \neq \vartheta^p$, $\sigma_w \neq \sigma_p$), ATC - asymmetric training costs ($e_w = 5.07, e_p = e_w/5.25$) and FF - financial friction, VPC and VNPC are vacancy posting costs in pecuniary and non-pecuniary terms, si - symmetric skill intensity. GI is government investment. Real wage rigidity is introduced with $\rho_w^k = 0.8$.

2.6 Conclusion

This study examines the effects of increased government spending on the real economy in the presence of large training costs of newly hired workers. For that purpose, we build a TANKrep model with asymmetric training costs and a TANK model that includes the interaction between asymmetric training costs and financial friction. The heterogeneous market structure of these models shows the expansionary effects of the fiscal stimulus, in contrast to the recessionary effects indicated by the literature that relies on the representative agent setting. A different hiring decision of firms plays an essential role in shaping a different response of the real economy to the fiscal stimulus. The firms' investment in training activity for new hires causes production disruption, as some experienced workers are diverted from production to training the new hires. Training costs are a common feature of both the representative and heterogeneous agent frameworks. In the period of high aggregate demand pressure, the value of forgone output is large, which under large training costs reduces the incentives of firms to hire. What makes two frameworks different is the firms' chance to choose the cheaper type of workers at a time of high marginal costs of hiring driven by high aggregate demand pressure. This is the case with the heterogeneous agent framework, where firms choose low-skilled workers and postpone hiring of high-skilled workers. Lower training costs for low-skilled workers stimulates their hiring, and the addition of financial friction further amplifies this hiring. Financial friction constrains the access of low-skilled workers to financial markets, which leads to their lower reservation wage and then market wage. There are two broad types of government spending that fiscal authorities can implement: government consumption and government investment. Given that government investment generates more expansionary effects in terms of the output multiplier, fiscal authorities may use it as a more efficient tool to deal with recessions.

2.7 Appendix

2.7.A Model Derivation

A.1 Derivation of the aggregate resource constraint

If $n-1$ market clearing conditions are satisfied in equilibrium, then by Walras's Law, the n^{th} (goods) market clears in equilibrium too.

To derive the aggregate resource constraint, we combine the following equations:

1. The real budget constraint of wealthy households:

$$s_w\left(c_{w,t}+t_{w,t}+i_t+b_t=w_{w,t}n_{w,t}+r_t^k k_{t-1}+\frac{R_{t-1}b_{t-1}}{\pi_t}+\frac{\Pi_t^{int}}{s_w}+\frac{\Pi_t^r}{s_w}\right)$$

2. The real budget constraint of poor households:

$$s_p\Big(c_{p,t}+t_{p,t}=w_{p,t}n_{p,t}\Big)$$

3. The definition of real profits and output:

$$\Pi_t^{int}=\frac{P_{int,t}}{P_t}Y_{int,t}-w_{w,t}N_{w,t}-w_{p,t}N_{p,t}-r_t^k K_t,$$

$$\Pi_t^r=\left(1-\frac{1}{x_t}-\frac{\phi_p}{2}(\frac{\pi_t}{\pi}-1)^2\right)Y_t,$$

$$Y_t=Y_{int,t}$$

4. The real government budget constraint:

$$T_t+B_t=\frac{R_{t-1}B_{t-1}}{\pi_t}+G_t$$

The distribution of lump-sum taxes can be expressed from the government budget constraint:

$$T_t \equiv s_w t_{w,t} + s_p t_{p,t} = \frac{R_{t-1}B_{t-1}}{\pi_t} + G_t - B_t$$

Substitution of lump-sum taxes paid by households into the government budget constraint yields:

$$s_w w_{w,t} n_{w,t} + s_w r_t^k k_{t-1} + s_w \frac{R_{t-1}b_{t-1}}{\pi_t} + \Pi_t^{int} + \Pi_t^r - s_w(c_{w,t} + i_t + b_t) +$$

$$+ s_p w_{p,t} n_{p,t} - s_p c_{p,t} = \frac{R_{t-1}B_{t-1}}{\pi_t} + G_t - B_t$$

Aggregating terms in the previous expression

$$w_{w,t}N_{w,t} + r_t^k K_t + \frac{R_{t-1}B_{t-1}}{\pi_t} - C_{w,t} - I_t - B_t$$

$$+ \left[\frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r_t^k K_t\right] + \left[\left(1 - \frac{1}{x_t} - \frac{\phi_p}{2}(\frac{\pi_t}{\pi} - 1)^2\right) Y_t\right] +$$

$$+ w_{p,t}N_{p,t} - C_{p,t} = \frac{R_{t-1}B_{t-1}}{\pi_t} + G_t - B_t$$

and given that market clearing conditions hold for labor, capital and bond markets, the aggregate resource constraint (or the goods market clearing condition) becomes:

$$Y_t = C_t + I_t + G_t + \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t$$

A.2 Derivation of the elasticity of the investment to capital ratio with respect to Tobin's q

The Lagrangean function for the optimization problem of wealthy households:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Bigg\{ \frac{1}{1-\sigma_c}(c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} - \varphi_{n,w}\frac{(\ell_{w,\tau})^{1+\eta}}{1+\eta} - \lambda_{w,\tau}^c \Bigg(c_{w,t} + t_{w,t} + i_t + b_t -$$

$$- w_{w,t}n_{w,t} - r_t^k k_{t-1} - \frac{R_{t-1}b_{t-1}}{\pi_t} - \frac{\Pi_t^{int}}{s_w} - \frac{\Pi_t^r}{s_w} + \lambda_{w,\tau}^n \left(n_{w,t} - (1-\sigma_w)n_{w,t-1} - \frac{\mu_{w,t}}{1-\mu_{w,t}} u_{w,t} \right) \Bigg) +$$

$$+ \lambda_{w,\tau}^l (n_{w,t} + u_{w,t} - \ell_{w,t}) - Q_t \Bigg((k_t - (1-\delta_k)k_{t-1}) + \frac{\phi_k}{2}\Big(\frac{i_t}{k_{t-1}} - \delta_k\Big)^2 k_t - i_t \Bigg) \Bigg\}$$

where $q_t = Q_t/\lambda^c_{w,t}$ is the Tobin's q marginal ratio.

The derivative of the optimization problem of wealthy households with respect to investment is:

$$-\lambda^c_{w,t} - Q_t\phi_k\Big(\frac{i_t}{k_{t-1}} - \delta_k\Big)\frac{k_t}{k_{t-1}} + Q_t = 0$$

$$\frac{\lambda_{w,t}}{Q_t} = 1 - \phi_k\Big(\frac{i_t}{k_{t-1}} - \delta_k\Big)\frac{k_t}{k_{t-1}}$$

$$\frac{1}{q_t} = 1 - \phi_k\Big(\frac{i_t}{k_{t-1}} - \delta_k\Big)\frac{k_t}{k_{t-1}}$$

$$\frac{i_t}{k_{t-1}} = \Big(-\frac{1}{q_t} + 1\Big)\frac{k_{t-1}}{\phi_k k_t} + \delta_k$$

$$log\Big(\frac{i_t}{k_{t-1}}\Big) = log\Big(\Big(-e^{-log(q_t)} + 1\Big)\frac{k_{t-1}}{\phi_k k_t} + \delta_k\Big)$$

The elasticity of the investment to capital ratio with respect to Tobin's q is

$$\frac{\partial log\Big(\frac{i_t}{k_{t-1}}\Big)}{\partial log(q_t)} = \frac{1}{\Big(-e^{-log(q_t)} + 1\Big)\frac{k_{t-1}}{\phi_k k_t} + \delta_k}\Big(-\frac{k_{t-1}}{\phi_k k_t}e^{-log(q_t)}(-1)\Big)$$

In steady state, the previous expression is evaluated as

$$\frac{\partial log\left(\frac{i}{k}\right)}{\partial log(q)} = \frac{1}{\delta_k}\frac{1}{\phi_k}$$

For $\delta_k = 0.025$ and $\phi_k = 4$, we have:

$$\varrho_k = \frac{1}{\delta_k \cdot \phi_k} = \frac{1}{0.025 \cdot 4} = 10$$

A.3 Different forms of hiring costs

The real profit of intermediate goods firms given different forms of hiring costs:

1. training costs

$$\Pi_t^{int} = \frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r_t^k K_t,$$

$$Y_{int,t} = f_{int,t}\Big(1 - \sum_{k\in\{w,p\}} \tilde{g}_{int,t}^k\Big) \quad \text{for} \quad \tilde{g}_{int,t}^k = \frac{e_k}{2}\left(\frac{H_{k,t}}{N_{k,t}}\right)^2$$

2. vacancy costs in pecuniary terms

$$\Pi_t^{int} = \frac{f_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r_t^k K_t - \sum_{k\in\{w,p\}} \frac{e_k}{2}\left(\frac{\upsilon_{k,t}}{N_{k,t}}\right)^2 f_{int,t}$$

3. vacancy costs in non-pecuniary terms

$$\Pi_t^{int} = \frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r_t^k K_t,$$

$$Y_{int,t} = f_{int,t}\Big(1 - \sum_{k\in\{w,p\}} \tilde{g}_{int,t}^k\Big) \quad \text{for} \quad \tilde{g}_{int,t}^k = \frac{e_k}{2}\left(\frac{\upsilon_{k,t}}{N_{k,t}}\right)^2$$

A.4 The log-linearized system of equations

To study the dynamics of the model, this section specifies the log-linearized version of the model, where $\widetilde{x}_t$ indicates the log deviation of any variable x_t from its non-stochastic steady state x, i.e. $\widetilde{x}_t = log(x_t/x) \simeq (x_t - x)/x$. The exception holds for the fiscal variables (government spending, taxes and bonds) and profits, which are measured in percentage deviation relative to the non-stochastic steady-state level of output.

A.4.1 Labor market

1. Aggregate labor force participation:

$$L_k\widetilde{L}_{k,t} = N_k\widetilde{N}_{k,t} + U_k\widetilde{U}_{k,t}$$

2. Aggregate number of newly hired workers:

$$\widetilde{H}_{k,t} = \varsigma \widetilde{\upsilon}_{k,t} + (1-\varsigma)\widetilde{U}^k_{0,t}$$

3. Labor market tightness:

$$\widetilde{\theta}_{k,t} = \widetilde{\upsilon}_{k,t} - \widetilde{U}^k_{0,t}$$

4. Vacancy filling probabilities:

$$\widetilde{\nu}_{k,t} = \widetilde{H}_{k,t} - \widetilde{\upsilon}_{k,t}$$

5. Hiring probabilities:

$$\widetilde{\mu}_{k,t} = \widetilde{H}_{k,t} - \widetilde{U}^k_{0,t}$$

6. The aggregate job seekers at the beginning of period t:

$$U^k_0 \widetilde{U}^k_{0,t} - \mu_k U^k_0 (\widetilde{\mu}_{k,t} + \widetilde{U}^k_{0,t}) = U\widetilde{U}_t$$

7. Law of motion for employment:

$$\widetilde{N}_{k,t} = (1-\sigma_k)\widetilde{N}_{k,t-1} + \sigma_k \widetilde{H}_{k,t}$$

8. The Nash bargained wage for households $k \in \{w, p\}$:

$$w^*_k \widetilde{w}^*_{k,t} = \vartheta^k \left(\frac{f_{N_k}}{x}(\widetilde{f}_{N_{k,t}} - \widetilde{x}_t) - \frac{g_{N_k}}{x}(\widetilde{g}_{N_{k,t}} - \widetilde{x}_t) \right) - (1-\vartheta^k)\frac{\lambda^l_k}{\lambda^c_k}\left(\widetilde{\lambda}^l_{k,t} - \widetilde{\lambda}^c_{k,t}\right) +$$

$$+ \vartheta^k(1-\sigma_k)\beta Q^{*,N}_k(\widetilde{\lambda}^c_{w,t+1} - \widetilde{\lambda}^c_{w,t} + \widetilde{Q}^{*,N}_{k,t+1}) - (1-\vartheta^k)(1-\sigma_k)\beta\lambda^{*,n}_k(\widetilde{\lambda}^c_{k,t+1} - \widetilde{\lambda}^c_{k,t} + \widetilde{\lambda}^{*,n}_{k,t+1})$$

In Nash bargaining process, the surplus sharing rule is $\widetilde{Q}^{*,N}_{k,t} = \widetilde{\lambda}^{*,n}_{k,t}$.

9. Inertial real wage for households with skill level $k \in \{w, p\}$:

$$\widetilde{w}_{k,t} = \rho^k_w \widetilde{w}_{k,t-1} + (1-\rho^k_w)\widetilde{w}^*_{k,t}$$

A.4.2 Wealthy households

1. FOC with respect to consumption:

$$\widetilde{\lambda}^c_{w,t} = \frac{-\sigma_c(c_w\widetilde{c}_{w,t} - hC_w\widetilde{C}_{w,t-1})}{(c_w - hC_w)}$$

2. FOC with respect to employment:

$$\lambda^n_w\widetilde{\lambda}^n_{w,t} = \frac{\lambda^l_w}{\lambda^c_w}(\widetilde{\lambda}^l_{w,t} - \widetilde{\lambda}^c_{w,t}) + w_w\widetilde{w}_{w,t} + \mathbb{E}_t\beta(1-\sigma_w)\lambda^n_w(\widetilde{\lambda}^c_{w,t+1} - \widetilde{\lambda}^c_{w,t} + \widetilde{\lambda}^n_{w,t+1})$$

3. FOC with respect to the participation in the labor market:

$$\widetilde{\lambda}^l_{w,t} = \eta\widetilde{\ell}_{w,t}$$

4. FOC with respect to unemployment:

$$\widetilde{\lambda}^l_{w,t} = \widetilde{\lambda}^c_{w,t} + \widetilde{\lambda}^n_{w,t} + \frac{1}{1-\mu_w}\widetilde{\mu}_{w,t}$$

5. FOC with respect to bonds:

$$\widetilde{\lambda}^c_{w,t} = \mathbb{E}_t\frac{\beta R}{\pi}(\widetilde{\lambda}^c_{w,t+1} + \widetilde{R}_t - \widetilde{\pi}_{t+1})$$

6. FOC with respect to physical capital:

$$\widetilde{\lambda}^c_{w,t} + \phi_k\widetilde{k}_t - \phi_k\widetilde{k}_{t-1} = \mathbb{E}_t\beta\Big((1-\delta_k)\widetilde{\lambda}^c_{w,t+1} + r^k(\widetilde{\lambda}^c_{w,t+1} + \widetilde{r}^k_{t+1}) + \phi_k\widetilde{k}_{t+1} - \phi_k\widetilde{k}_t\Big)$$

7. The budget constraint of wealthy households:

$$\begin{aligned} & c_w\widetilde{c}_{w,t} + Y\widetilde{t}_{w,t} + k(\widetilde{k}_t - (1-\delta_k)\widetilde{k}_{t-1}) + Y\widetilde{b}_t \\ = \;& w_w n_w(\widetilde{w}_{w,t} + \widetilde{n}_{w,t}) + kr^k(\widetilde{k}_{t-1} + \widetilde{r}^k_t) + \frac{Rb}{\pi}(\widetilde{R}_{t-1} + \frac{Y}{b}\widetilde{b}_{t-1} - \widetilde{\pi}_t) + \frac{Y}{s_w}\widetilde{\Pi}^{int}_t + \frac{Y}{s_w}\widetilde{\Pi}^r_t \end{aligned}$$

8. The law of motion of capital:

$$\widetilde{i}^k_t = \frac{1}{\delta_k}(\widetilde{k}_t - (1-\delta_k)\widetilde{k}_{t-1})$$

A.4.3 Poor households

1. FOC with respect to consumption:

$$\widetilde{\lambda}^c_{p,t} = \frac{-\sigma_c(c_p\widetilde{c}_{p,t} - hC_p\widetilde{C}_{p,t-1})}{(c_p - hC_p)}$$

2. FOC with respect to employment:

$$\lambda^n_p\widetilde{\lambda}^n_{p,t} = \frac{\lambda^l_p}{\lambda^c_p}(\widetilde{\lambda}^l_{p,t} - \widetilde{\lambda}^c_{p,t}) + w_p\widetilde{w}_{p,t} + \mathbb{E}_t\beta(1-\sigma_p)\lambda^n_p(\widetilde{\lambda}^c_{p,t+1} - \widetilde{\lambda}^c_{p,t} + \widetilde{\lambda}^n_{p,t+1})$$

3. FOC with respect to the participation in the labor market:

$$\widetilde{\lambda}^l_{p,t} = \eta\widetilde{\ell}_{p,t}$$

4. FOC with respect to unemployment:

$$\widetilde{\lambda}^l_{p,t} = \widetilde{\lambda}^c_{p,t} + \widetilde{\lambda}^n_{p,t} + \frac{1}{1-\mu_p}\widetilde{\mu}_{p,t}$$

5. The budget constraint of poor households:

$$c_p\widetilde{c}_{p,t} + Y\widetilde{t}_{p,t} = w_pn_p(\widetilde{w}_{p,t} + \widetilde{n}_{p,t})$$

A.4.4 Intermediate goods firms

1. Production function:

$$\widetilde{f}_{int,t} = \iota\widetilde{K}_t + (1-\iota)\left(mN^\sigma_w + (1-m)N^\sigma_p\right)^{-1}(mN^\sigma_w\widetilde{N}_{w,t} + (1-m)N^\sigma_p\widetilde{N}_{p,t})$$

2. The net output of intermediate goods firm:

$$Y\widetilde{Y}_t = f_{int}\widetilde{f}_{int,t} - g_{int}\widetilde{g}_{int,t}$$

$$g_{int}\widetilde{g}_{int,t} = f_{int}\frac{e_w}{2}\left(\frac{H_w}{N_w}\right)^2(\widetilde{f}_{int,t} + 2\widetilde{H}_{w,t} - 2\widetilde{N}_{w,t}) + f_{int}\frac{e_p}{2}\left(\frac{H_p}{N_p}\right)^2(\widetilde{f}_{int,t} + 2\widetilde{H}_{p,t} - 2\widetilde{N}_{p,t})$$

3. FOC with respect to capital:

$$r^kx(\widetilde{r}^k_t + \widetilde{x}_t) = f_K\widetilde{f}_{K,t} - g_K\widetilde{g}_{K,t}$$

4. FOC with respect to skilled labor:

$$Q_w^N \widetilde{Q}_{w,t}^N = \frac{f_{N_w}}{x}(\widetilde{f}_{N_{w,t}} - \widetilde{x}_t) - \frac{g_{N_w}}{x}(\widetilde{g}_{N_{w,t}} - \widetilde{x}_t) - w_w \widetilde{w}_{w,t} + \mathbb{E}_t \beta (1-\sigma_w) Q_w^N (\widetilde{\lambda}_{w,t+1}^c - \widetilde{\lambda}_{w,t}^c + \widetilde{Q}_{w,t+1}^N)$$

5. FOC with respect to unskilled labor:

$$Q_p^N \widetilde{Q}_{p,t}^N = \frac{f_{N_p}}{x}(\widetilde{f}_{N_{p,t}} - \widetilde{x}_t) - \frac{g_{N_p}}{x}(\widetilde{g}_{N_{p,t}} - \widetilde{x}_t) - w_p \widetilde{w}_{p,t} + \mathbb{E}_t \beta (1-\sigma_p) Q_p^N (\widetilde{\lambda}_{w,t+1}^c - \widetilde{\lambda}_{w,t}^c + \widetilde{Q}_{p,t+1}^N)$$

6. FOC with respect to hiring of skilled labor:

$$\widetilde{Q}_{w,t}^N = \widetilde{g}_{H_{w,t}} - \widetilde{x}_t$$

7. FOC with respect to hiring of unskilled labor:

$$\widetilde{Q}_{p,t}^N = \widetilde{g}_{H_{p,t}} - \widetilde{x}_t$$

8. The derivatives of the production function and hiring cost function:

$$\widetilde{f}_{K_t} = (\iota - 1)\widetilde{K}_t + (1-\iota)(mN_w^\sigma + (1-m)N_p^\sigma)^{-1}\Big(mN_w^\sigma \widetilde{N}_{w,t} + (1-m)N_p^\sigma \widetilde{N}_{p,t}\Big)$$

$$\widetilde{f}_{N_{w,t}} = \iota \widetilde{K}_t + \Big(\frac{1-\iota}{\sigma} - 1\Big)(mN_w^\sigma + (1-m)N_p^\sigma)^{-1}\Big(m\sigma N_w^\sigma \widetilde{N}_{w,t} + (1-m)\sigma N_p^\sigma \widetilde{N}_{p,t}\Big) + (\sigma - 1)\widetilde{N}_{w,t}$$

$$\widetilde{f}_{N_{p,t}} = \iota \widetilde{K}_t + \Big(\frac{1-\iota}{\sigma} - 1\Big)(mN_w^\sigma + (1-m)N_p^\sigma)^{-1}\Big(m\sigma N_w^\sigma \widetilde{N}_{w,t} + (1-m)\sigma N_p^\sigma \widetilde{N}_{p,t}\Big) + (\sigma - 1)\widetilde{N}_{p,t}$$

$$\begin{aligned} g_{N_w}\widetilde{g}_{N_{w,t}} =& - e_w\Big(\frac{H_w}{N_w}\Big)^2 \frac{1}{N_w} f_{int}\Big(2\widetilde{H}_{w,t} - 2\widetilde{N}_{w,t} - \widetilde{N}_{w,t} + \widetilde{f}_{int,t}\Big) + \\ & f_{N_w}\frac{e_w}{2}\Big(\frac{H_w}{N_w}\Big)^2(\widetilde{f}_{N_{w,t}} + 2\widetilde{H}_{w,t} - 2\widetilde{N}_{w,t}) + f_{N_w}\frac{e_p}{2}\Big(\frac{H_p}{N_p}\Big)^2(\widetilde{f}_{N_{w,t}} + 2\widetilde{H}_{p,t} - 2\widetilde{N}_{p,t}) \end{aligned}$$

$$\begin{aligned} g_{N_p}\widetilde{g}_{N_{p,t}} =& - e_p\Big(\frac{H_p}{N_p}\Big)^2 \frac{1}{N_p} f_{int}\Big(2\widetilde{H}_{p,t} - 2\widetilde{N}_{p,t} - \widetilde{N}_{p,t} + \widetilde{f}_{int,t}\Big) + \\ & f_{N_p}\frac{e_w}{2}\Big(\frac{H_w}{N_w}\Big)^2(\widetilde{f}_{N_{p,t}} + 2\widetilde{H}_{w,t} - 2\widetilde{N}_{w,t}) + f_{N_p}\frac{e_p}{2}\Big(\frac{H_p}{N_p}\Big)^2(\widetilde{f}_{N_{p,t}} + 2\widetilde{H}_{p,t} - 2\widetilde{N}_{p,t}) \end{aligned}$$

$$\widetilde{g}_{H_{w,t}} = \widetilde{H}_{w,t} - 2\widetilde{N}_{w,t} + \widetilde{f}_{int,t}$$

$$\widetilde{g}_{H_{p,t}} = \widetilde{H}_{p,t} - 2\widetilde{N}_{p,t} + \widetilde{f}_{int,t}$$

$$g_K \widetilde{g}_{K,t} = e_w \left(\frac{H_w}{N_w}\right)^2 f_K \left(\widetilde{H}_{w,t} - \widetilde{N}_{w,t} + \frac{1}{2}\widetilde{f}_{K,t}\right) + e_p \left(\frac{H_p}{N_p}\right)^2 f_K \left(\widetilde{H}_{p,t} - \widetilde{N}_{p,t} + \frac{1}{2}\widetilde{f}_{K,t}\right)$$

A.4.5 Final goods firms

1. The New Keynesian Phillips Curve:

$$\widetilde{\pi}_t = \frac{(\epsilon - 1)}{\phi_p}\widetilde{mc}^r_t + \beta \mathbb{E}_t \widetilde{\pi}_{t+1}$$

2. Real profit of the final good firms:

$$\widetilde{\Pi}^r_t = \widetilde{Y}_t - mc^r(\widetilde{mc}^r_t + \widetilde{Y}_t)$$

A.4.6 Monetary and fiscal policies

1. Monetary Policy Rule:

$$\widetilde{R}_t = \theta_r \widetilde{R}_{t-1} + (1 - \theta_r)\left[\theta_\pi \widetilde{\pi}_t + \theta_y \widetilde{Y}_t\right]$$

2. Fiscal Policy Rule:

$$\widetilde{T}_t = \phi_B \widetilde{B}_{t-1} + \phi_{BG} \widetilde{G}_t$$

3. The real government budget constraint:

$$\widetilde{T}_t + \widetilde{B}_t = \frac{RB}{\pi Y}(\widetilde{R}_{t-1} + \frac{Y}{B}\widetilde{B}_{t-1} - \widetilde{\pi}_t) + \widetilde{G}_t$$

4. The distribution of lump-sum taxes:

$$\widetilde{T}_t = s_w \widetilde{t}_{w,t} + s_p \widetilde{t}_{p,t}$$

A.4.7 Aggregate resource constraint

$$Y\widetilde{Y}_t = C\widetilde{C}_t + I\widetilde{I}_t + Y\widetilde{G}_t$$

A.4.8 The exogenous process

1. Government spending:

$$\widetilde{G}_t = \phi_g \widetilde{G}_{t-1} + \epsilon_t^g$$

A.4.9 Aggregate variables

1. Aggregate consumption:

$$C\widetilde{C}_t = C_w\widetilde{C}_{w,t} + C_p\widetilde{C}_{p,t} = s_w c_w \widetilde{c}_{w,t} + s_p c_p \widetilde{c}_{p,t}$$

2. Labor supply of the wealthy:

$$\widetilde{N}_{w,t} = \widetilde{n}_{w,t}$$

3. Labor supply of the poor:

$$\widetilde{N}_{p,t} = \widetilde{n}_{p,t}$$

4. Aggregate capital stock:

$$\widetilde{K}_t = \widetilde{k}_{t-1}$$

5. Aggregate bonds:

$$\widetilde{B}_t = s_w \widetilde{b}_t$$

A.5 TANKrep model

The head of the household maximises discounted lifetime household utility choosing $\{c_t,\ i_t,\ k_t,\ b_t,\ \ell_{w,t},\ \ell_{p,t},\ n_{w,t},\ n_{p,t},\ u_{w,t},\ u_{p,t}\}$

$$\max\ \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c}(c_\tau - hC_{\tau-1})^{1-\sigma_c} - \varphi_{n,w}\frac{(s_w \ell_{w,\tau})^{1+\eta}}{1+\eta} - \varphi_{n,p}\frac{(s_p \ell_{p,\tau})^{1+\eta}}{1+\eta} \Big\}$$

The real budget constraint of a wealthy household in every period t is:

$$c_t + t_t + i_t + b_t \leq s_w w_{w,t} n_{w,t} + s_p w_{p,t} n_{p,t} + r_t^k k_{t-1} + \frac{R_{t-1} b_{t-1}}{\pi_t} + \Pi_t^{int} + \Pi_t^r$$

and the employment law of motion:

$$n_{w,t} = (1 - \sigma_w) n_{w,t-1} + \frac{\mu_{w,t}}{1 - \mu_{w,t}} u_{w,t} (= h_{w,t})$$

$$n_{p,t} = (1 - \sigma_p) n_{p,t-1} + \frac{\mu_{p,t}}{1 - \mu_{p,t}} u_{p,t} (= h_{p,t})$$

and the law of motion of physical capital:

$$i_t = k_t - (1 - \delta_k) k_{t-1} + \frac{\phi_k}{2} \left(\frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1}$$

3 Corporate Income Tax Changes and Aggregate Productivity

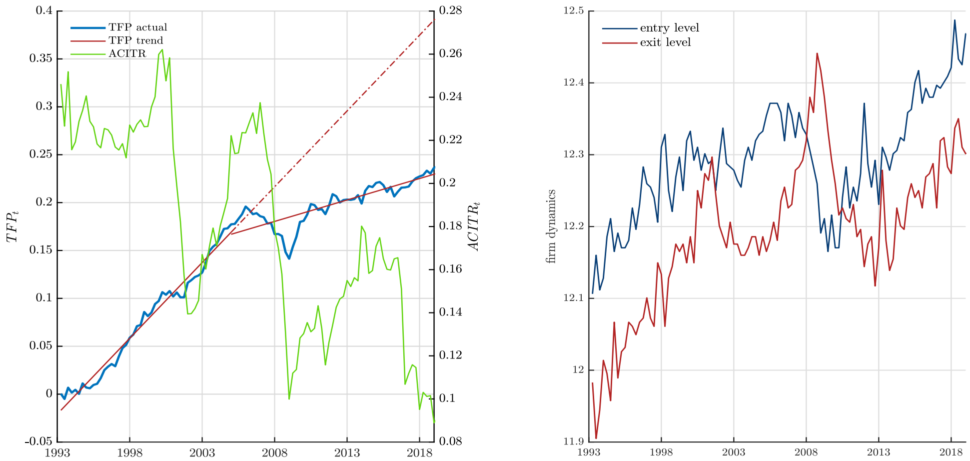
Co-authored with Danilo Stojanović (CERGE)

3.1 Introduction

Over the past three decades, the average corporate income tax rates (ACITRs) in the U.S. economy have steadily reduced from 25% to 10%. Concurrently, aggregate total factor productivity (TFP) and the net entry of firms recorded an increase (see Figure 3.1). The standard macro theory predicts that tax cuts boost productive capital investment by reducing the user cost of capital, which then increases aggregate TFP. However, the intended benefits of tax cuts may be offset by several other factors. For instance, tax cuts may enable low-productive firms to remain profitable and continue operating. In addition, many firms may remain financially constrained to finance their growth potential at an early stage of their life. This particularly applies to high-productive firms that are discouraged from entering the economy due to restricted borrowing capacity. The existing literature is salient about whether the productivity-enhancing effects of the tax cuts can occur and persist in the presence of firms' entry and exit dynamics and corporate borrowing.

To evaluate the dynamic effects of corporate income tax shocks on TFP and other U.S aggregates, we apply two methodologies commonly used in the literature. First, we use a proxy structural vector autoregression model developed by Mertens and Ravn (2013) to identify tax shocks. Second, we implement the approach of Wong (2015) to construct a counterfactual economy in which firms are restricted from borrowing. Specifically, we generate a sequence of borrowing shocks of sufficient magnitude to fully offset the response of borrowing to a 1% tax shock for a period of 40 quarters. This counterfactual framework allows us to evaluate the role of borrowing in the transmission of tax shocks to changes in firms' composition, aggregate productivity and output.

Figure 3.1: Aggregate productivity, average taxes and firm dynamics, 1993q2-2019q4



Note: ACITRs refer to average corporate income tax rates. ACITRs come from NIPA, while aggregate TFP is from table of John G. Fernald, entry and exit levels are from BLS, BED.

Our paper provides two novel empirical findings for the U.S. economy. First, we document that when there is increased net entry of firms and borrowing in the capital market, the average corporate income tax cuts lead to a temporary rise in aggregate TFP and real GDP. Second, these expansionary effects persist only if firms have the ability to borrow additional external funds. The intuition is that increased capital accumulation, stimulated by tax cuts, relaxes collateral constraints, providing existing firms with additional funds to sustain previously increased aggregate TFP and output growth. The availability of external funds allows new entrants to finance their high growth potential, amplifying the positive effects of tax cuts.

The remainder of this chapter is organized as follows. Section 3.2 reviews the related literature. In Section 3.3, we present the empirical results. Section 3.4 concludes. In the Appendix, we conduct a set of robustness checks of our empirical results.

3.2 Related Literature

The objective of this paper is to provide a novel empirical analysis of the dynamic relationship between average corporate income tax rates (ACITRs) and aggregate productivity gains and other macroeconomic aggregates in the presence of firms' entry and exit dynamics and corporate borrowing. To achieve this goal, we connect two strands of literature.

The first strand of literature evaluates tax effects using external instruments in VAR models. While Mertens and Ravn (2013) and Romer and Romer (2010) document short-run stimulative tax effects on output, Cloyne et al. (2022) find long-run positive tax effects on productivity and output through R&D expenditure. The recent work by Colciago et al. (2023) studies the tax effects on labor market outcomes in the context of entry and exit of firms, and show an increase in productivity and output in the short run. We contribute to this literature by highlighting the role of the interaction of firms' entry and exit with corporate borrowing in the transmission of stimulating tax effects on aggregates over the short and long term.

We claim that there is no guarantee that the tax effects are productivity enhancing because lower ACITRs increase the after-tax income of existing low-productive firms, enabling them to continue operating. In addition, new firms are discouraged from entering the economy as their access to external funds remaines restricted. Consequently, the change in the number of firms in the economy (extensive margin) and the reallocation of resources to firms with low productivity (intensive margin) may slow down the rise in aggregate productivity and output growth.[29] Further, we argue that it matters how firms finance their capital investment. Corporate borrowing could magnify tax effects through the close interaction between capital and collateral constraint. Our results indicate that the interplay between firms' entry and exit dynamics and borrowing makes tax effects productivity enhancing over a forty-quarter time horizon.

[29] The inclusion of firms' entry and exit in our analysis is justified by Foster et al. (2018), who highlights their important role in explaining innovation in the capital market.

A second strand of literature deals with approaches to evaluate the empirical relevance of the mechanisms in a SVAR framework. While the literature mostly relies on estimated impulse response functions or historical decomposition, Wong (2015) and Sims and Zha (2006) propose generating counterfactual impulse response functions to shocks. Specifically, to explore inflation expectations as a channel for transmitting real oil price shocks on actual inflation, Wong (2015) conducts a counterfactual experiment where inflation expectations are set insensitive to oil price shocks. We follow the idea about forming a counterfactual experiment but focus on the channels between tax shocks and macroeconomic aggregates.

3.3 Empirical Evidence

In Figure 3.1, we observe a slowdown in aggregate productivity growth over the past three decades. This could be attributed to many factors, including depleted innovations, global recession, etc.[30] Our study investigates whether ACITRs, in the presence of firms' entry and exit, is behind this slow down in productivity. Corporate income tax changes are one of the most polarizing topics in fiscal policy due to different channels at work with potentially opposing effects. We contribute to the debate on tax policy changes by addressing the following two questions. How effective are ACITRs in stimulating aggregate productivity and output growth across different time horizons? What role do firms' entry and exit dynamics and borrowing play in transmitting the effects of ACITRs?

Empirical Model. To isolate exogenous variation in taxes, we use a proxy Structural Vector Autoregressive (SVAR) developed by Mertens and Ravn (2013). It combines a SVAR with the narrative approach. We use narrative measures of tax changes by Romer and Romer (2010) as our proxy, which imposes the restrictions that they are correlated with the structural tax shock but are not correlated with other structural shocks. The benchmark proxy SVAR model includes the following variables: ACTIRs, nonresidential fixed investment, real GDP, aggregate TFP, entry and exit levels, and corporate debt.

[30] For more information on the reasons behind week aggregate productivity growth, look at Akcigit and Ates (2021).

Data. We analyze quarterly observations from 1993q2 to 2019q4. All variables are expressed in real per capital terms. We use U.S. data on firms' entry and exit from the Bureau of Labor Statistics (BLS). Availability of these data from 1993q2 determines the start of our sample. The average corporate income tax rates are computed as:

$$ACITRs = \frac{\text{federal taxes on corporate income}}{\text{corporate profits} - \text{Federal Reserve Bank profits}}$$

Table 3.1: Aggregate US data, 1993q2-2019q4

Variable	Description	Source
ACITR	average corporate income tax rate	NIPA
ACITB	average corporate income tax base	NIPA
NRI	nonresidential fixed investment	NIPA
GDP	gross domestic product	NIPA
TFP	total factor productivity	John G. Fernald
entry	entry level	BLS BED
exit	exit level	BLS BED
corp_debt	corporate debt	FoF
m_CI	narratively-identified shock	MR(2013) and HHP(2021)

Notes: NIPA is National Income and Product Accounts; BLS is Bureau of Labor Statistics; BED is Business Employment Dynamics; FoF is Flow of Funds; MR refers to Mertens and Ravn (2013); HHP is Hanson et al. (2021).

Results. Figure 3.2 illustrates the impulse responses of selected variables to a one-percentage-point decrease in ACITR. The blue solid line represents the point estimates, while the blue and red dashed lines represent 90% and 68% bootstrap confidence intervals, respectively. It is evident that the unexpected shock significantly reduces ACITR for the first three quarters before going back to zero, its historical average. This tax change can be interpreted as a temporary reduction in taxes.

We observe several aggregate responses to the tax shock within our benchmark model. First, investment as a GDP component reacts significantly, with an impact increase of 1%.[31] Second, despite the transitory nature of the corporate tax cut, short-term increases in real GDP and aggregate TFP persist over the ten-year period. Third, firms' entry and exit levels initially respond in opposite directions. Specifically, entry increases, while exit decreases.[32] Given that the fiscal stimulus and the associated boom fade away over time, low-productive incumbents become unprofitable and tend to exit the economy. In addition, lower profits per firm discourage the creation of new firms, reducing competition by entering firms. Fourth, corporate borrowing exhibits a hump-shaped response to the tax shock. Responses of all aggregates to the tax shock are statistically significant.

Table 3.2 suggests how our main results contribute to the understanding of the dynamic effects of tax shocks in the empirical literature. In contrast to Cloyne et al. (2022), who focus on tax effects along the intensive margin of investment, we highlight the importance of both intensive and extensive effects of the tax shocks. Relative to Colciago et al. (2023), who explore the labor market, our paper focuses on the capital market, emphasizing the role of corporate borrowing in the long run. We focus on the capital market because capital investment is the most volatile component of aggregate output, and most firms in the US rely on borrowing, with capital serving as collateral.

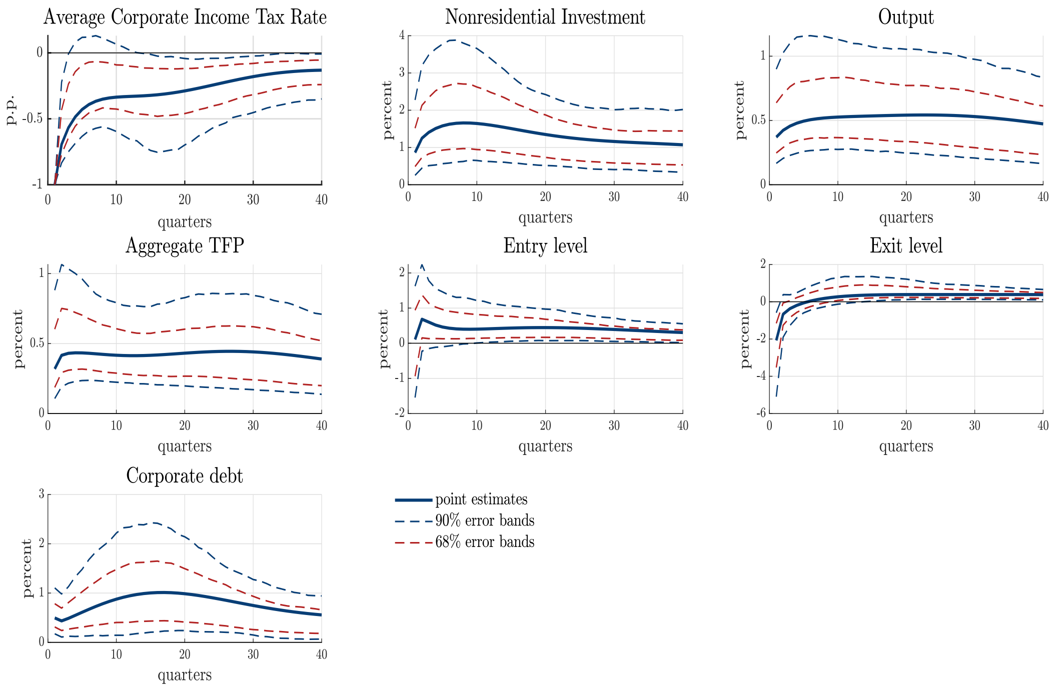
We find that, on impact, a temporary reduction in corporate taxes increases aggregate investment and aggregate output in the presence of an increase in after-tax internal funds.[33] As the corporate taxes gradually increase, their stimulative effects on investment become smaller, but their positive effects on output persist. In the long run, investment remains at

[31] According to standard macroeconomic theory, lower corporate tax rates reduce the rental rate of capital, stimulating firms to increase capital investment.

[32] The rise in net entry of firms primarily drives the increase in aggregate TFP on impact. This claim is clearly justified in Figure 3.3 when another important financial channel (borrowing) is excluded from the analysis.

[33] In the left upper panel of Figure 3.4 in Appendix, we show that the drop in ACITR increases corporate profits by 1.16% on impact and remains significantly above the pre-shock level for one year, and then gradually reduces as the tax cuts are reduced.

Figure 3.2: Impulse Responses to ACITRs, 1993q2-2019q4



its initial response level because the initial capital accumulation increases future cash-flows and allows firms to relax borrowing constraints. Real GDP remains at high levels because of a strong rise in consumption, as depicted in Figure 3.4 in the Appendix.

Robustness Checks. To confirm the robustness of our empirical findings, we perform a set of additional checks. The results are presented in the figures in Appendix. Figure 3.4 shows a statistically significant rise in consumption, indicating potentially strong demand effects on the economy that push up production and profits. As regards the labor market outcomes, we observe that higher capital investment increases wages in spite of the reduced employment rate.[34]

Given the importance of changes in a firm's composition for the transmission of tax shocks, we reestimate the tax effects in the model where firms' entry and exit levels are replaced

[34]As for the employment rate, the transmission of the tax shocks is fully absorbed.

Table 3.2: Responses to a one percentage point cut in the ACITR

	Mertens and Ravn (2013)	Cloyne et al. (2022)	Colciago et al. (2023)	Our paper
Statistics	non-resid inv	R&D inv	firms' entry & exit and labor market	firms' entry & exit and capital market
Yagg on impact	0.4	0.4	0.7	0.4
Yagg in q20	0.5	0.7	1.2	0.5
TFPagg on impact	-	0.2	-	0.3
TFPagg in q20	-	0.4	-	0.4
Iagg on impact	0.5	0.8	-	0.9
Iagg in q20	0.2	2.0	-	1.3

Notes: The sample period in Mertens and Ravn (2013) and Cloyne et al. (2022) is from 1950q1 to 2006q4. Colciago et al. (2023) consider the period from 1979q1 to 2006q1. Our paper covers the period from 1993q2 to 2019q4.

with firms' entry and exit rates. In Figure 3.5, we observe that the initial drop in exit rates is stronger than the drop in entry rates, leading to a rise in net entry rates upon impact. However, as ACITRs gradually increase, exit rates increase, which for relatively constant entry rates diminish the total number of active firms.

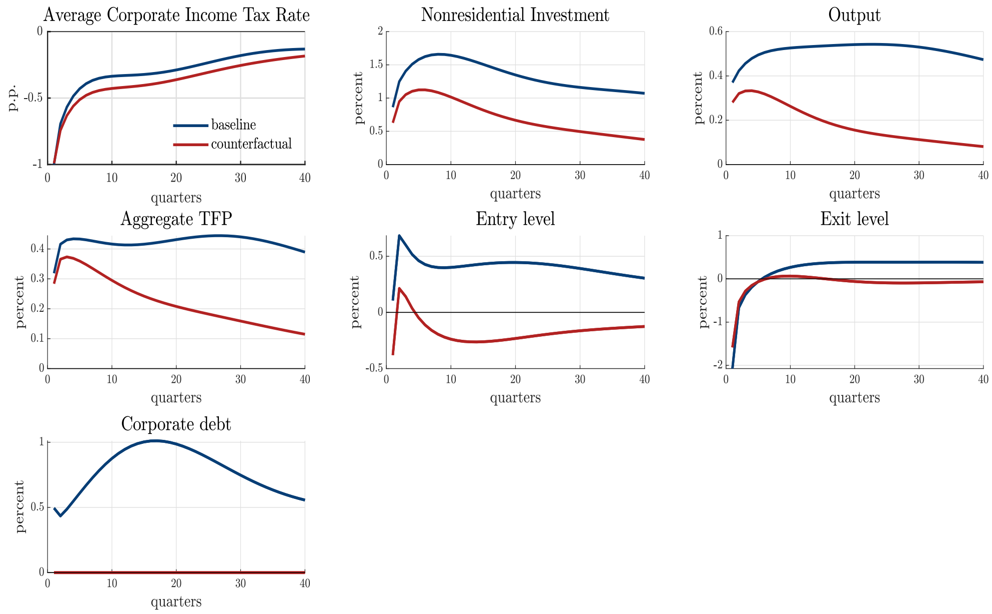
We also test the responses of alternative measures of productivity, including the adjusted TFP and labor productivity. The utilization-adjusted TFP measure from Fernald (2014) aims to isolate changes in productivity that are not influenced by endogenous changes in factor utilization. The estimation results, depicted in Figure 3.6 in the Appendix, are similar to those observed with our baseline measure of productivity, aggregate TFP.

The role of corporate borrowing. We construct a counterfactual scenario to simulate the effects of a tax shock in the absence of corporate borrowing. Following the approach by Wong (2015), we generate a sequence of corporate borrowing shocks just large enough to fully offset the response of corporate borrowing to a 1% tax shock for all 40 quarters. If corporate borrowing serves as an important mechanism in transmitting tax shocks, the counterfactual impulse response functions of macro aggregates tend to deviate significantly from their baseline estimates. This exercise answers the question about the role of firm dynamics and borrowing in propagating the tax shock.

Figure 3.3 compares the estimated IRFs of the baseline proxy SVAR model (blue line) with the counterfactual IRFs (red line). Without borrowing, the red line shows that the responses of aggregate TFP and output are small on impact. However, their responses are significantly mitigated in the long run, accompanied by a strong decline in the net entry of firms. This highlights the significant contribution of borrowing to the transmission of a tax shock to the economy through changes in entry and exit of firms. We also generate a counterfactual impulse response function to a shock by setting all coefficients in the borrowing equation to zero. The estimation results of this alternative approach are shown in Figure 3.7.

In Figure 3.3, we also observe that the distance between the blue and red lines is larger for entry levels than exit levels. Five quarters after the shock, incumbent firms have sufficient time to accumulate internal funds to be away from the exit decision, stabilizing exit around its historical average. Conversely, entering firms face lower internal funds due to a relatively higher tax rate and a fully restricted access to external funds. Reduced competition, which is mainly driven by a reduced entry level, leaves a larger space for active firms to continue their operation in the economy, pushing down aggregate TFP. Our findings complement the study by Hamano and Zanetti (2022), which shows that a contractionary *monetary* policy decreases the entry of new firms. This shields incumbent firms from the competition of new entrants and reduces aggregate productivity.

Figure 3.3: Counterfactual analysis with zero borrowing, 1993q2-2019q4



Note: Counterfactual analysis is in the spirit of Wong (2015) and Sims and Zha (2006).

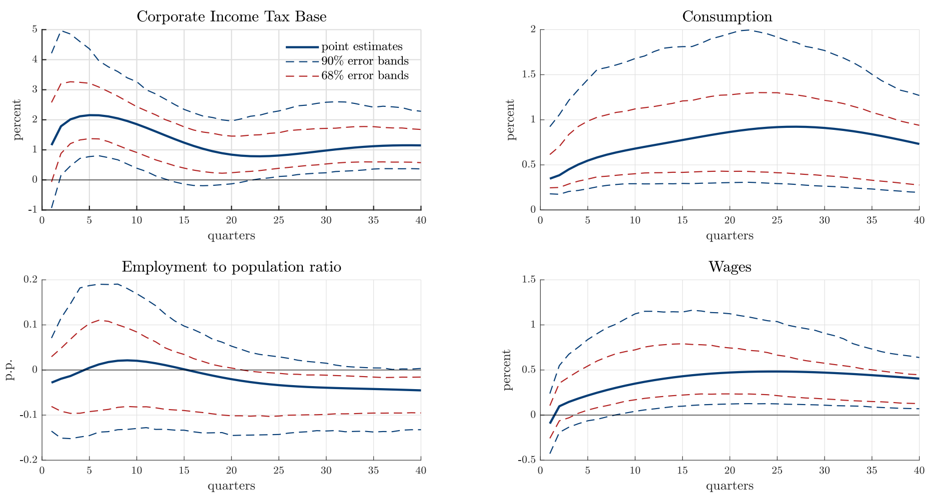
3.4 Conclusion

This paper provides new insights into the aggregate effects of tax cuts and their transmission through firms' entry and exit dynamics and borrowing over time. Our empirical results reject a theoretical consideration that the corporate income tax cuts may reduce aggregate TFP and output growth in the short-run, showing instead that these positive responses persist in the long run. Specifically, we find that the corporate income tax cuts generate the cleansing of low-productive firms from the market, enhancing aggregate output growth. The ability of firms to borrow amplifies the influence of the increased net entry of firms. For future research, it would be interesting to explore the interplay between firms' entry and exit dynamics and borrowing on the basis of micro-level data. This additional exercise is important for understanding the effects of tax cuts on reallocating resources from low to high productivity firms, which may drive a large portion of productivity growth.

3.5 Appendix

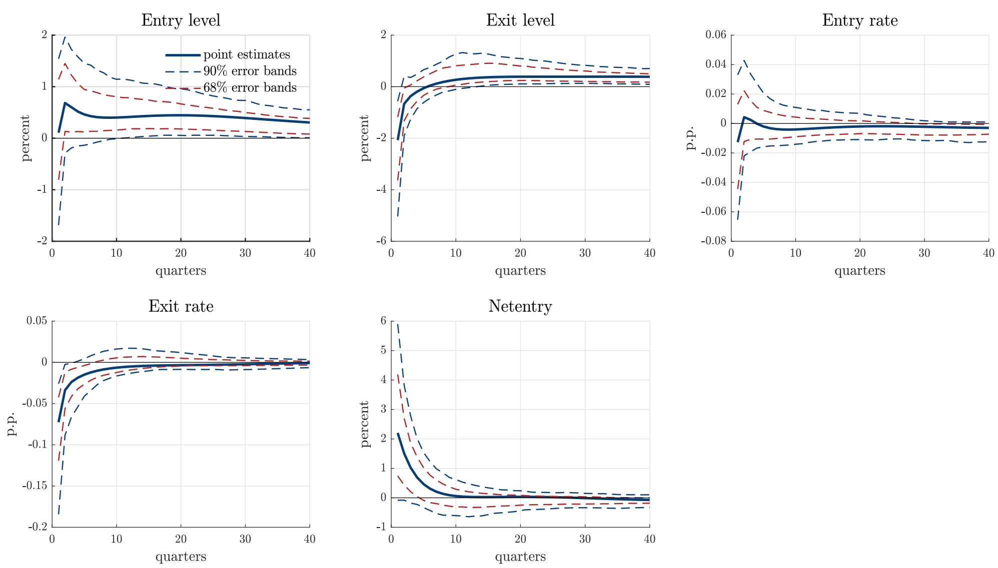
3.5.A Robustness Checks

Figure 3.4: Impulse Responses to ACITR, 1993q2-2019q4



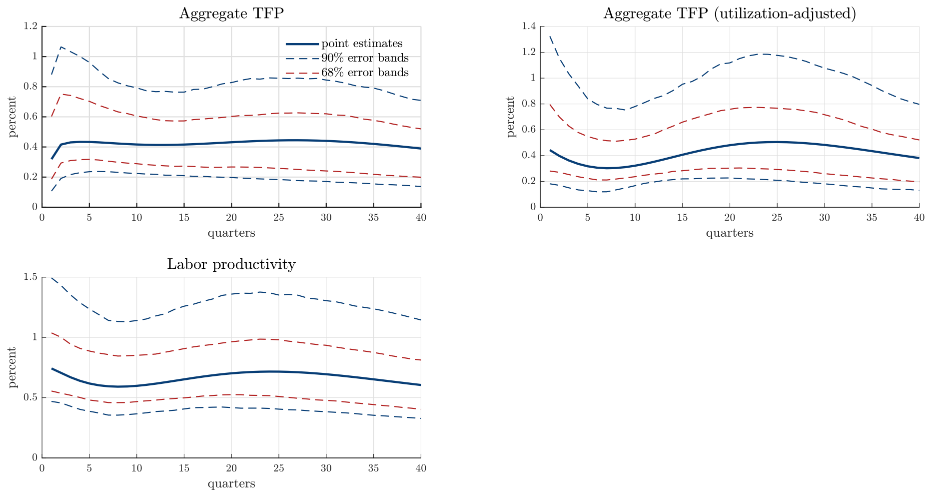
Note: Each additional variable is added to the baseline data vector one at the time to avoid a sharp increase in the number of parameters to be estimated.

Figure 3.5: Firms' entry and exit, 1993q2-2019q4



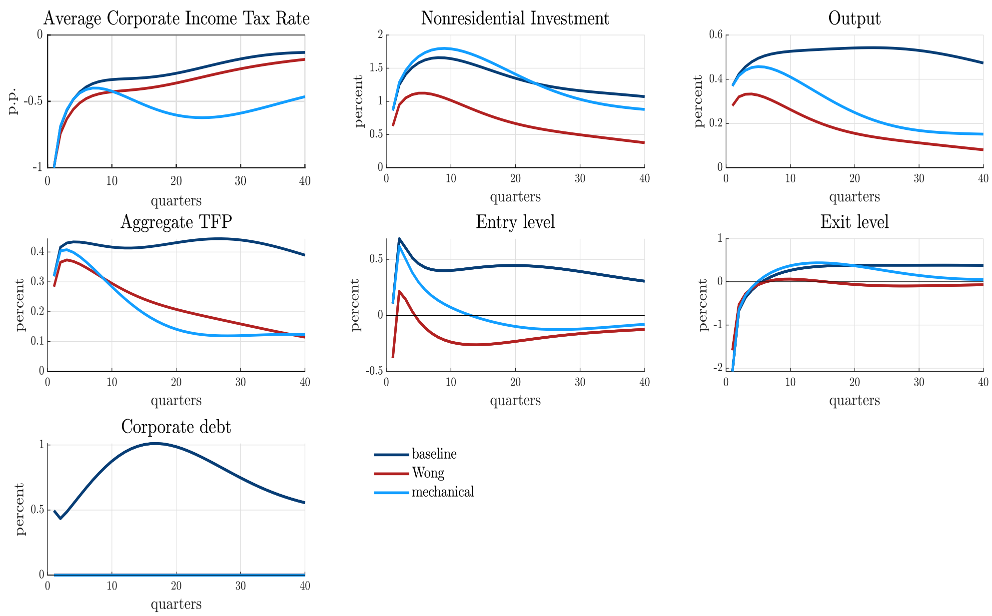
Note: Each additional variable is added to the baseline data vector one at the time to avoid a sharp increase in the number of parameters to be estimated.

Figure 3.6: Aggregate productivity, 1993q2-2019q4



Note: Each additional variable is added to the baseline data vector one at the time to avoid a sharp increase in the number of parameters to be estimated.

Figure 3.7: Different approaches to constructing counterfactual IRFs, 1993q2-2019q4



Note: Counterfactual analysis is in the spirit of Wong (2015).

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