**Charles University in Prague**
**Faculty of Social Sciences**

**Institute of Economic Studies**

# DIPLOMA THESIS

**2010** **Gorkem YAZICIOGLU**

**CHARLES UNIVERSITY IN PRAGUE**
**FACULTY OF SOCIAL SCIENCES**

**INSTITUTE OF ECONOMIC STUDIES**

**MASTER'S THESIS**

# An Analysis on US Subprime Mortgage Crisis – Expansion and the Burst of Bubble

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| Author | **Gorkem Yazicioglu** |
| Subject: | **E&F** |
| Academic Year: | **2009/2010** |
| Supervisor: | **Doc. Ing. Oldřich Dědek** |
| Date Submitted: | **21st May 2010** |

## Declaration

Hereby I declare that I compiled this diploma thesis independently, using only the listed literature and resources. I also consent to the publication of my Master´s Thesis on Institute of Economic Studies web sites and other places of Charles University in Prague.

Prague, 14th May 2010

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Gorkem Yazicioglu

## Acknowledgments

I owe great thanks to Doc. Ing. Oldřich Dědek from IES FSV UK for supervising my work and for continuous encouragement. I am also much indebted to PhDr. Jozef Baruník from IES FSV UK and ÚTIA AV ČR for insightful comments.

## Bibliographic Evidence Card

**Gorkem Yazicioglu:** *An Analysis of US Subprime Mortgage Crisis*, Charles University in Prague, Faculty of Social Sciences, Institute of Economic Studies, 2009, 100 pages, gorkem.yazicioglu@wallstreetsystems.com
**Supervisor:** Doc. Ing. Oldřich Dědek

## Abstract

This thesis provides an analysis on the expansion and the burst phases of the 2007 subprime mortgage bubble, in two interrelated sections. The first part - called “Expansion of the bubble”-, analyzes the speculative investor behaviour of the US Real Estate Crisis and exhibits that before the crisis the asset prices such as Commercial Mortgage Backed Securities and Standard and Poor’s 500 indexes showed statistical features which are not common among financial time series. Common stylized facts such volatility clustering and autocorrelation of absolute returns were tested for the years 1997-2010, using daily data on the index values. The results indicate that during bubble periods several of the stylized facts disappeared since investors did not respond as normal to the volatility events and price changes. Hence, I have suggested that irrational exuberance of investors during 2007 subprime mortgage crisis and other bubble periods can be detected by testing these stylized facts. In the second part,- called “The burst of Bubble” -, I showed that common macroeconomic risk factors can explain, to a very large extent, all types of mortgage’s credit risk and I concluded that individual mortgages are possessing significant default correlation, especially subprime mortgage classes. Finally this thesis ends with demonstrating the significance of correlated defaults on default distributions’ fat tails which explains the defaults of high tranche MBS products.

**Keywords:** Stylized facts, volatility clustering, irrational exuberance, default correlation, subprime mortgage crisis

**JEL Classification:** G01, G10, G12, G17

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## 1.0 Introduction

Due to financial market innovations; in US we have encountered significant development and wide usage of securitization and increasing use of financial derivatives such as mortgage backed securities (MBS), collateralized mortgage obligations (CMO), and collateralized debt obligations (CDO) during the last ten years. For instance, the value of subprime mortgages in US was predicted to be about 1.3 trillion USD by March 2007 according to associated press which was claimed as the main culprit of US subprime mortgage crisis. According to Bernanke, their delinquency rate had increased to 21 percent as of January 2008, and by May 2008 it was already 25 percent. Consequently, large-scale banks and other financial institutions that were possessing MBS or CDO wrote-off massive losses, since the value of the underlying mortgages plummeted. They have collectively faced approximately $379 billion by May 21, 2008, thus I was motivated to analyze the US sub-prime mortgage crisis.

The statistical regularities which are common on different financial instruments, various markets and time intervals are generally accepted as stylized empirical facts. For example, volatility events tend to cluster through time, and also autocorrelations of absolute returns decay slowly. The existing literature on stylized facts focus only on the econometric aspects of these stylized facts i.e. how they invalidate any financial model. However, there is no research on how some of these stylized facts can be used as a framework for speculative bubble detections. What are their economic meanings? Were there any empirical differences in terms of, for instance, volatility clustering or autocorrelations of absolute returns? Since none of these questions answered by the existing literature extensively, I've decided that the main aim of the first part of my thesis should be to test stylized facts by using daily Commercial Mortgage Backed Securities Index (CMBS) and a S&P 500 daily returns data to understand when and what are the irregularities in asset returns occur due to 2007 house bubble crisis and how they evolve. Additionally, other supplementary empirical indicators were included to have more complete framework. These tests would assist me to understand the investor behaviour during bubbles and its reflections on time series.

I have ventured that before the crisis during the expansion of bubble period these return characteristics are not observable. I have expected that investor's do not respond to volatility events, asset price decreases and other common events during the bubble periods as they do in other time intervals. I have also anticipated that if these investor behaviours can be documented by stylized facts, then these tests can be used to analyze further bubbles.

In order to investigate the possible unexpected characteristics of these indices, I have conducted econometric tests on common stylized facts. I have showed that several of these well observed stylized facts are not observable anymore during the expansion of the bubble period, I showed that volatility events did not cluster before the crisis, the autocorrelations of absolute returns did not decay slowly indeed they actually did not exist before the crisis, I also showed that volatility-return relation did not exist, return autocorrelations were weaker and kurtosis was lower before the crisis. However, the literature suggests that these are the common characteristics of many financial time series in the long term, Rama Cont (2000). My results demonstrated that before and after the crisis time series possess different characteristics as I expected, hence the results implicitly provide a set of indicators for the recognition of investors' irrational exuberance, a term which was used by Alan Greespan to explain asset price speculation.

The bubble continued to expand until the sudden burst occurred, like in every bubble. In the second part of my thesis I have analyzed this eventual burst of the bubble after expansion. In the course of 2007 credit crunch, comprehending the credit risk of these portfolios is of critical importance both to the lenders and to the regulators. This chapter analyzed how an MBS with a good credit rating can default. To show this, I have demonstrated that default correlations were embedded in an MBS and I argued that during the crisis default correlations played a vital role in high default rates of MBS products.

How is the credit risk of different individuals related? How important are credit risk correlations? Is it important enough to worry about them? What are the principal forces driving them? Having seen the magnitude of the sub-prime mortgage crisis, I would claim that we don't even grasp the significance of the correlation problem. The answers to these simple questions have substantial consequences for the valuation of MBS.

In order to test the existence of default correlations, I used time series analysis on delinquency and default rates. I showed that substantial amount of the default rates particularly for subprime default rates can be explained by a few common macroeconomic factors, unemployment (1), house prices (2) and disposable income (3), which in return cause a correlation among the mortgages. I have also exhibited the defaults of higher tranche mortgage backed securities are possible by pointing out the heavy tails of correlated default distributions. The fat tails appeared when I employed higher correlation values and default rates in my simulation. Therefore, it is concluded that identification of default correlation correctly has become a prerequisite for effective risk management since the default correlations of mortgages have a fatal influence on default risk.

The rest of the thesis is as following: The second chapter presents simplified economical backgrounds of the crisis and empirical analysis on common macroeconomical indicators. This chapter maintains the basis for model specification of further chapters by breaking down the credit crunch in three parts as it was explained by Allen and Gale. The next chapter is devoted to a literature review on asset price bubbles. The fourth chapter presents the stylized facts of 2007 mortgage bubble on CMBS and S&P 500 indices per separate time intervals and points them as empirical indicators for detection of irrational exuberance periods. The fifth chapter explains the default correlation and provides an analysis on the factors of mortgage delinquency and default rates to exhibit high default correlation among mortgages. Chapter six demonstrates the impact of correlated default probabilities on an MBS and the conclusion is provided in the seventh chapter.

# 2.0 The Expansion of the Bubble

Recent credit crunch has been discussed to be the biggest housing crisis since 1930's. Although, it is important for researchers to understand every aspects of this crisis, until now, existing literature focused on the strategic mistakes of banks and rating agencies based on wrong calculated default rates. The next section is a multipart analysis on the evolution of bubble and answers the questions like how the bubble happened, what were the indicators of bubble. The bubble was explained in a simplified by taking into account the macroeconomical factors which will be used further in the empirical analyses.

## 2.1 The Beginning of the Bubble

Federal Reserve System followed a loosened fiscal policy since the first quarter of 2001and started to decrease the short term Funds rate to 5.98% in January 2001. Furthermore, following the explosion of dotcom bubble and 9/11 attacks, Federal Reserve was frightened of deflation, afterwards carried on with decreasing the interest rate of short term funds until the level of 0.98% by December 2003. This was indeed the lowest short term interest rate ever seen in the last 45 years. Belke and Wiedmann, (2005) argued that one of the main reasons of this policy, - injecting more than sufficient liquidity into the financial markets - was to prevent the stagnation, the recession and the following panic of the financial markets. As it is known, expansionary monetary policies enhances the chances of asset price bubbles. Due to highly volatile stock markets, the portfolio managers remodel their portfolios with more real estate instead of equities. In addition to professional portfolio managers, with very low interest rates, households could easily pay for larger mortgage loan payments and consequently afford elevated house prices. Consequently, the house prices continued to increase with the enhanced demand.

### 2.1.1 Emergence of Subprime Mortgage

During Clinton period, starting from 1995, the number of sub-prime mortgage loans together with homeownership rate has started to increase significantly; particularly we have encountered an exponential rise of subprime mortgage market.

We can define the subprime mortgage as a loan given to a borrower who has a riskier credit profile; (s)he often lacks a long and clean credit history or has peculiarities which can be attributed generally to higher default risk class (Bernanke, 2007). According to the study of Dean Baker, subprime mortgage is also defined as the loan given to the people with impaired credit history. Subprime mortgage borrowers are the ones who do not have a regular job or who did not pay some credit debt in the past. Their debt service to income (DTI) together with loan to value (LTV) ratios has significant differences than other borrowers. They have DTI ratios which are over 55% or LTV ratios which are above 85%. These borrowers which are labelled as subprime borrowers pay on an average 1% higher interest payments than regular prime mortgage borrowers (Kiff and Mills, 2007).

Although subprime mortgage loans were introduced in 1982 with the emergence of adjustable rate mortgages[1], it was not until 1995 that the increase was worth to mention. Later on, in 1986, by the Tax Reform Act, the residential mortgage loan became the sole

[1] Interest rate of an ARM is fixed according to fixing rate or an index periodically, In US; it may be COFI, MTA, and LIBOR… etc. Hence the interest payments are changed periodically

consumer loan in the market which has tax-deductible interest payments. With this new tax reform, since there was high unrealized equity in the mortgage, homeowners were able to prefer mortgage loans for their individual expenditures and developments in their houses. In addition, automated underwriting[2] and technological advances have facilitated simpler credit scoring processes besides with the securitization[3] of MBS instruments, easy trading of mortgages is ensured through transferring the mortgage itself. Together with the new credit attitude of borrowers, all these advancements have assisted the emergence of the subprime mortgage market (Kiff and Mills, 2007).

#### 2.1.1.1 Homeownership Rate and Subprime Mortgages

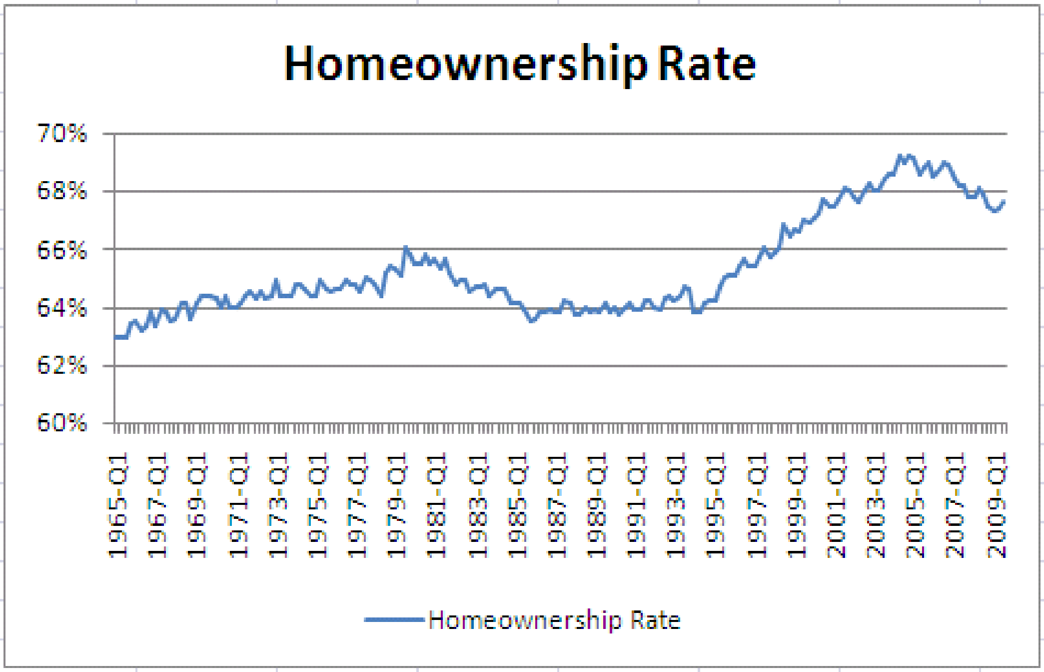
Hence, partly due to the emergence of subprime market, we have seen a peak in the homeownership rate[4] since the households started to have easier access to lower interest rate mortgages which was not available this easy earlier (Figure1). According to the Sewell, Clinton housing strategy ignited the rise of homeownership rate.

[2] Underwriting is done to determine risk-based price by looking at the credit history and LTVs of the borrowers.

[3] Securitization can be defined as the process of transforming illiquid assets, through financial engineering, into a security. By this process the original lender can be seperated from the borrower who is the ultimate bearer of credit riskthe ultimate bearer of credit risk.

[4] Number of owner-occupied housing units / number of occupied housing units or households

Figure 1: Quarterly Homeownership Rate in US (1965-2009)




Source: US Census Bureau

An increase in homeownership rate among the poor classes particularly for Hispanics and Blacks would have several social advantages; less crime rates, elevated sense for community, enhanced school performance...Relaxing the standards triggered the increase in subprime mortgage. Since 1995, Nonconforming subprime mortgages had a considerable increase in relative to the mortgages which are guaranteed by Federal Housing Administration (FHA) and traditionally have lower lending limits and less flexibility. In 2006, only 51% of mortgages were prime loans which generally require more detailed credit history. The rest of the mortgages were comprised of Alt-A[5], and subprime mortgages. Alt-A and subprime mortgages both have higher credit risks than prime mortgages.

[5] ALT A is actually an ambiguous category between prime and subprime mortgages

By the development of mortgage markets, a significant demand increase happened on real estates. On the other hand, construction industry, quite naturally, was not able to manage to respond to the elevated demands of the customers. Hence, house prices were primarily inflated by the unexpected demand of customers.

#### 2.1.2 The Indicators of the Bubble

In the following part, I have demonstrated basic indicators of real estate prices in order to show how the prices inflated before the crisis.

##### 2.1.2.1 The Affordability Index

The Housing Affordability Index is a monthly index which measures the level affordability of a house by a typical family. From the definition of Realtors, a trade organization for real estate agents, "Affordability Index[6] measures whether or not a typical median income family could qualify for a mortgage loan on a typical home." Since 2003, the affordability index was gradually decreasing. Between January 2003 and August 2007, in total it decreased 22%. To sum up, housing was becoming more expensive for a median income family between 2003 and 2007.

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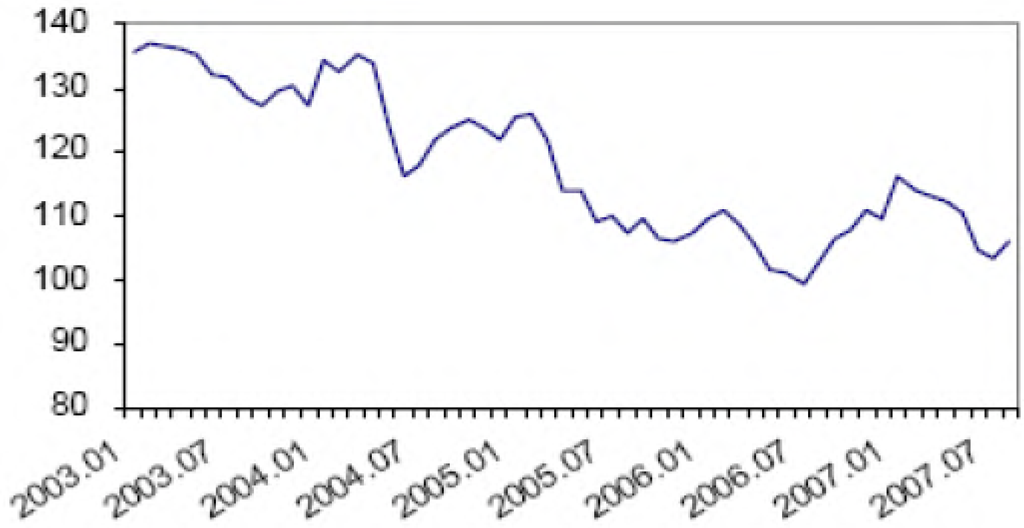
[6] The explanation from National Association of REALTORS of the index is this; The index takes both fixed-rate-mortgages and adjustable-rate-mortgages into account and calculates a composite mortgage payment. In order to interpret the index, consider this: A value of 100 means that a family with the median income has exactly enough income to qualify for a mortgage of a median-priced home. An index value over 100 means that families earning the median income have more than sufficient income to qualify for mortgage loans of a median priced house, with a 20 percent down payment. For instance, a composite House affodibility index of 120 signifies that families earning the median family income have 120 percent of the income required to qualify on a traditional loan covering 80% of a median priced existing single family house. An increase in the House affordibility index, then, means that the same family is now more able to afford a median priced house. This calculation always considers a down payment of 20% of the house value and it considers a qualifying ratio of 25%. That means the monthly payment can not exceed 25% of the median family monthly income.

Figure 2: Housing Affordability Index




Source: National Association of REALTORS

### 2.1.3 The Ratio of House Prices to Rents

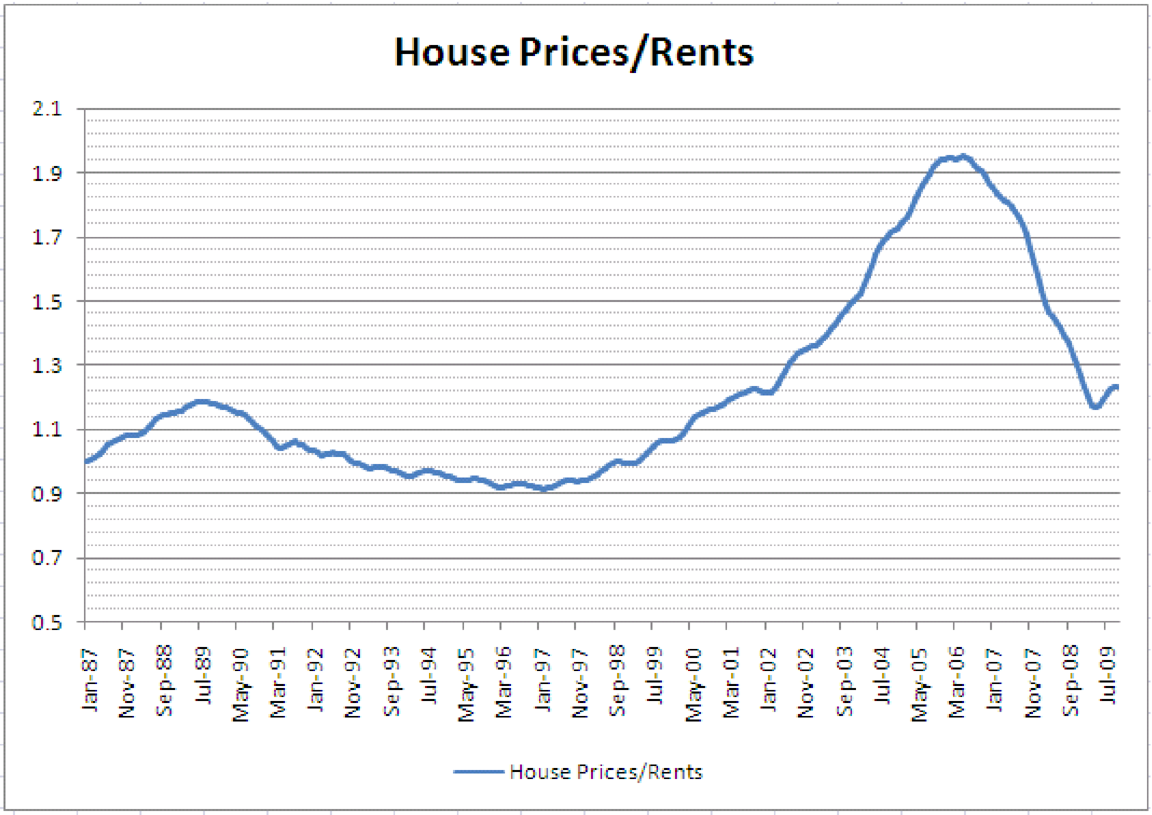
This ratio, in housing markets, is closely related to the price to earnings ratio in stocks, which is widely known as the Gordon Equation. We can define the Gordon equation as the risk adjusted discounted market value of an asset's expected income stream[7] which results as the market value of the asset. Hence, the intrinsic value of a real estate should be the projected income stream resulting from the ownership or the saved income by not paying a rent (The Economist, 2003). In the US this house prices-rental income relationship has deteriorated, in May 2006, as you can see below in Figure 3, the ratio has reached its peak. Quite clearly this indicates that individuals bought houses for speculative reasons. This is very similar to investors buying stocks for expected capital gains without economic reasoning. The

7 $\frac{P_t}{E_t} = \frac{\delta(1+g)}{i+\rho-g}$

Where Pt is the price of the asset, Et is the earnings, $\delta$ is percentage share of earnings, g is the growth rate of earnings, i is the risk-free interest rate and $\rho$ is the risk premium. This equation indicates that the price of an asset should rise as the risk-free interest rate and/or risk premium falls.
(IMF World Economic Outlook, 2000:78-79)

purchasing decisions were solely made based on future expected capital gain after the sale of the house, without taking into account the income streams from rents.

Figure 3: House Price Index to Rent of Primary Residence (Jan 1987- Oct 2009)

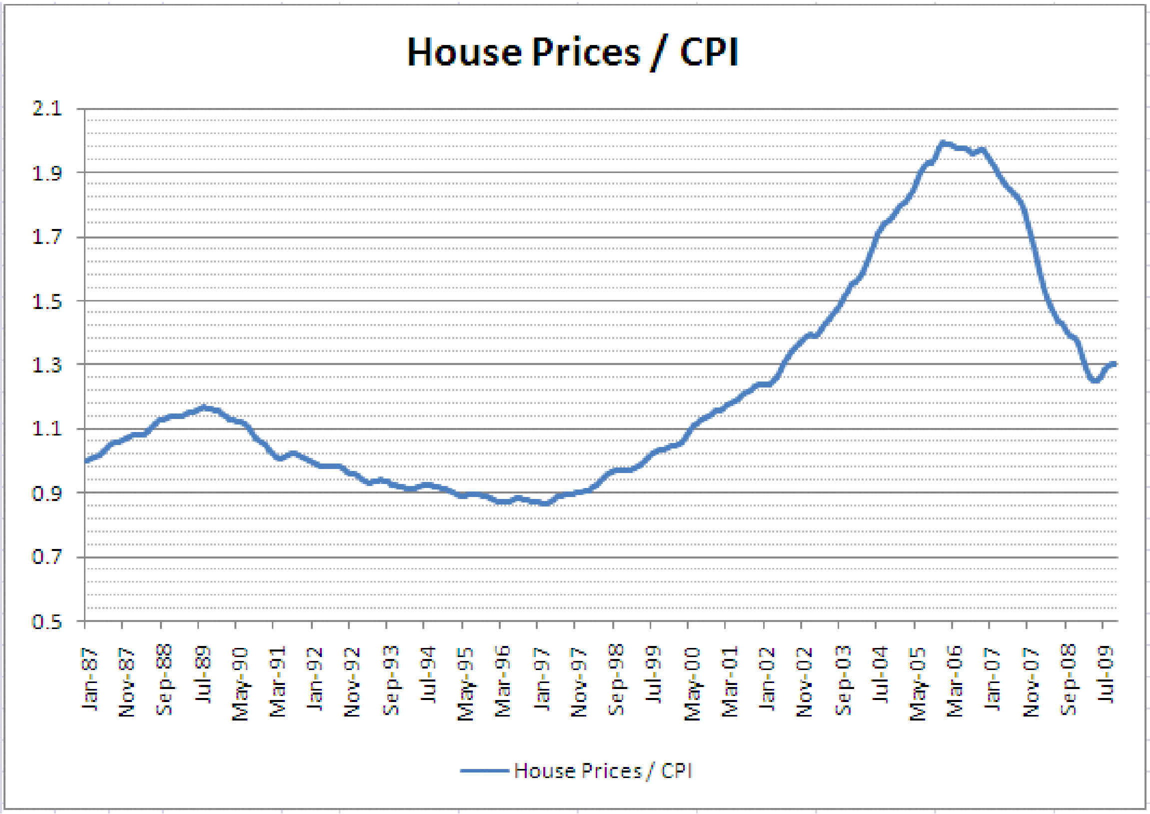



Sources: U.S. Bureau of Labour Statistics and Standard&Poor's

### 2.1.4 House Prices and Consumer Prices

The other relationship which is assumed to exist is between real estate prices and consumer price index (CPI). It has long been assumed that real estate prices and CPI have tendency to move together, and it did move together until just before the bubble. However, the striking boost of real estate prices has challenged this traditional relationship of house price and consumer price index. Below, in figure-4, the ratio of house prices to consumer prices was provided between 1987 to 2009. The prices of houses have raised twice more in comparison to the consumer prices before the crisis, and induced a bubble.

**Figure 4: House Price Index to CPI (January 1990-October 2009)**



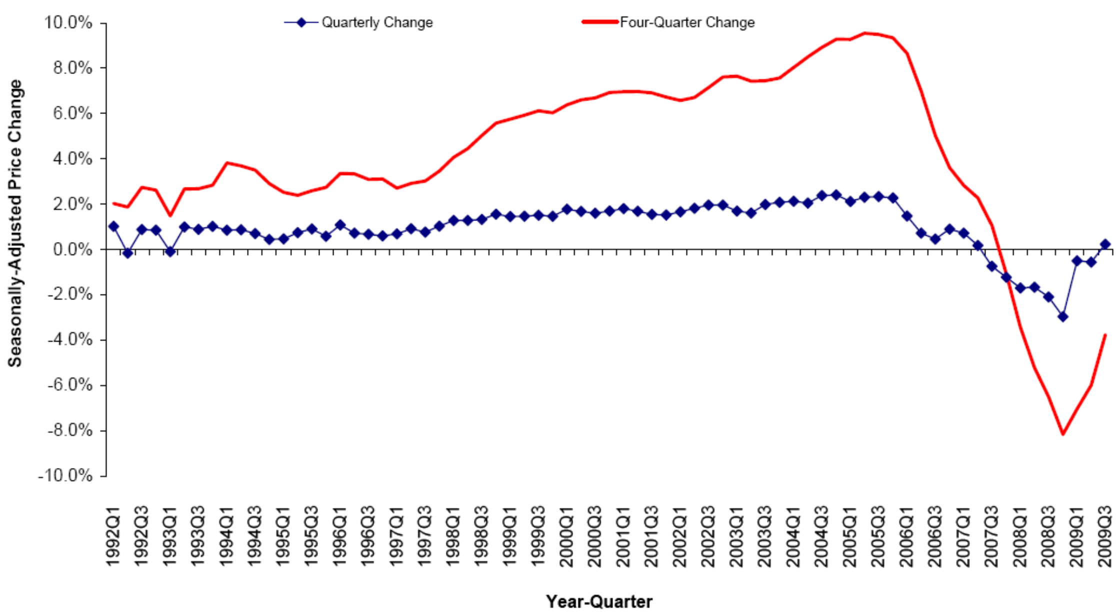

**Sources: and U.S. Bureau of Labour Statistics, consumer price index (all urban consumers) and Standard & Poor's**

Figure 4 shows the relationship of house price index to consumer price index, we can clearly see that, the rise of house price index greatly surpassed the rise of consumer price index (CPI) after 1999. In mid 2006, when the house price index came to its peak, in comparison with 1996 values, the house prices index was twice more than the CPI in value. These figures conclusively provide an indicator that the housing market had an explained increase in US in comparison with consumer prices and rent incomes.

## 2.2 Second Phase – The Start of the Crisis

Federal Housing Finance Agency (FHFA) administers several studies and surveys on real estate markets and prices. The agency also calculates quarterly US house price index[8]. The US real estate prices have soared slightly more than 50 percent from 2002 to 2007, according to the report of FHFA. The annual average real estate prices growth rate was around ten percent. In the second quarter of 2005, when the increase rate was at its peak level, the annual price growth was even more than fourteen percent. This rise of house prices encouraged homeowners to take more loans collateralized with the existing house which had a higher value than before.

**Figure 5: Federal Housing Finance Agency (FHFA) House Price Index History (1990 Q1-2009 Q3)**



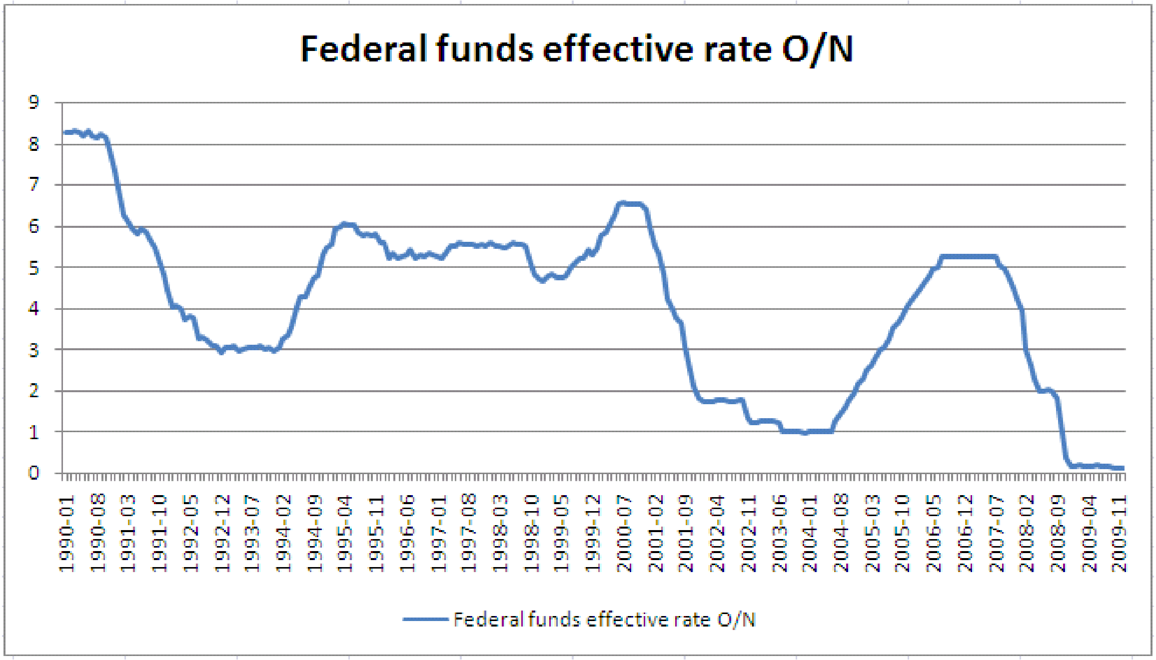

**Source: Federal Housing Finance Agency (FHFA)**

8 OFHEO and Case&Shiller have similar index construction methodology.

The number of new house constructions boosted from 1.4 million new houses per year in July 2000 to almost 2.28 million new housing structures in Jan 2006. For instance, in 2005 the house constructions industry produced 6.2% of Gross Domestic Product, excepting 1950's this was actually the highest rate ever seen for house constructions, (The Economist, 2007). This has also precipitated a rise in employment rates in the housing market. Housing related job categories including Residential housing constructions, according to Labour Department, jointly accounted to 6.6 percent of the total employment in US. From January 2001 to May 2006, the 46 percent of the new jobs created in US was actually related to housing and construction industry (Fox, 2007).

The rise in the house prices triggered to massive profits in the whole US economy until the sudden drops in all markets. The increase in consumer spending was simply backed with lowest interest rates ever seen recently and the wealth-soaring impacts of increasing real estate prices. Increase in asset prices not only boosted expenditures, but also permitted homeowners to get more loans on top of their existing loans since the equity of their mortgage was growing (The Economist, 2002). It is worth to mention that financial markets made huge gains from the real estate bubble. Between 2004 and 2006, according to Wall Street Journal, housing bubble led to a combined profit of one and a half trillion USD from the high risk sub-prime mortgages. Almost 3000 banks, credit unions, mortgage companies, thrifts benefited from the massive profits.

**Figure 6: Federal Reserve System Short Term Funds Rate (Jan1990-Nov2009)**

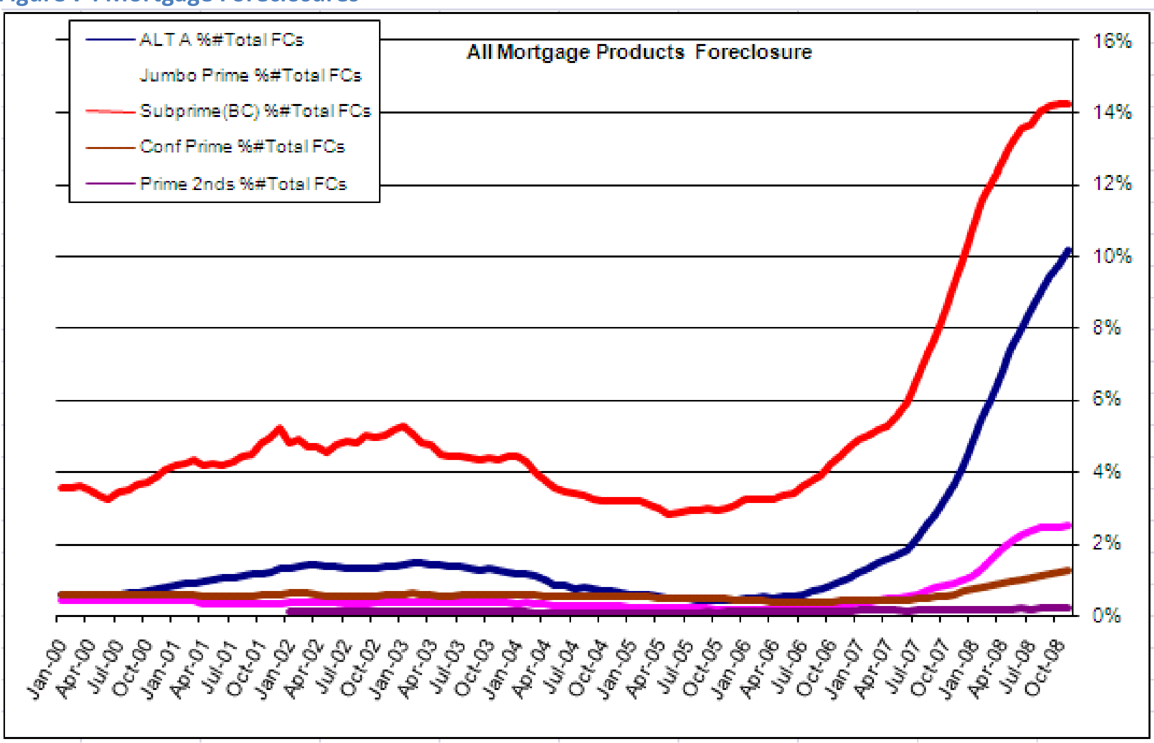



**Source: Federal Reserve System**

However, this wonderland story had an ugly end. Federal Reserve System (FED) commenced to raise the short term funds rate progressively. In 2004 the short term funds rate was one percent, by June 2006 it was already 5.25 percent, in less than 2 years the rate was increased more than 4 percent (see Figure 6). Robert Shiller in 2000 indicated that strict monetary policies after a loose one often lead to the bursts of the asset price bubbles.

With the drops in the real estate prices together with the rise of interest rates of mortgages, the mortgage loan portfolios originated in 2006 started to deteriorate (figure 9). Previously, increasing real estate prices together with low interest rates allowed easier prepayments for borrowers with payment difficulties since their collateral value or their mortgages was rising. To their surprise, when the economic conditions are reversed, rises in monthly payments and drops in house values have left the subprime borrowers no choice but to quit their mortgages by defaulting. The increase in late payments has caused the increases in the delinquencies and eventually defaults and foreclosures. For instance, just between October 2006 and December 2006, more than 310000 mortgage loans defaulted, although the previous two years average was 230000. Between January 2007 and April 2007, almost 3% of all total sub-prime mortgages started default proceedings (The Economist, 2007). Below, in figure 7, you can see that in just more than two years the foreclosure rates sub-prime mortgages quadrupled. The foreclosure in other mortgage loan classes also increased significantly, particularly ALT A.

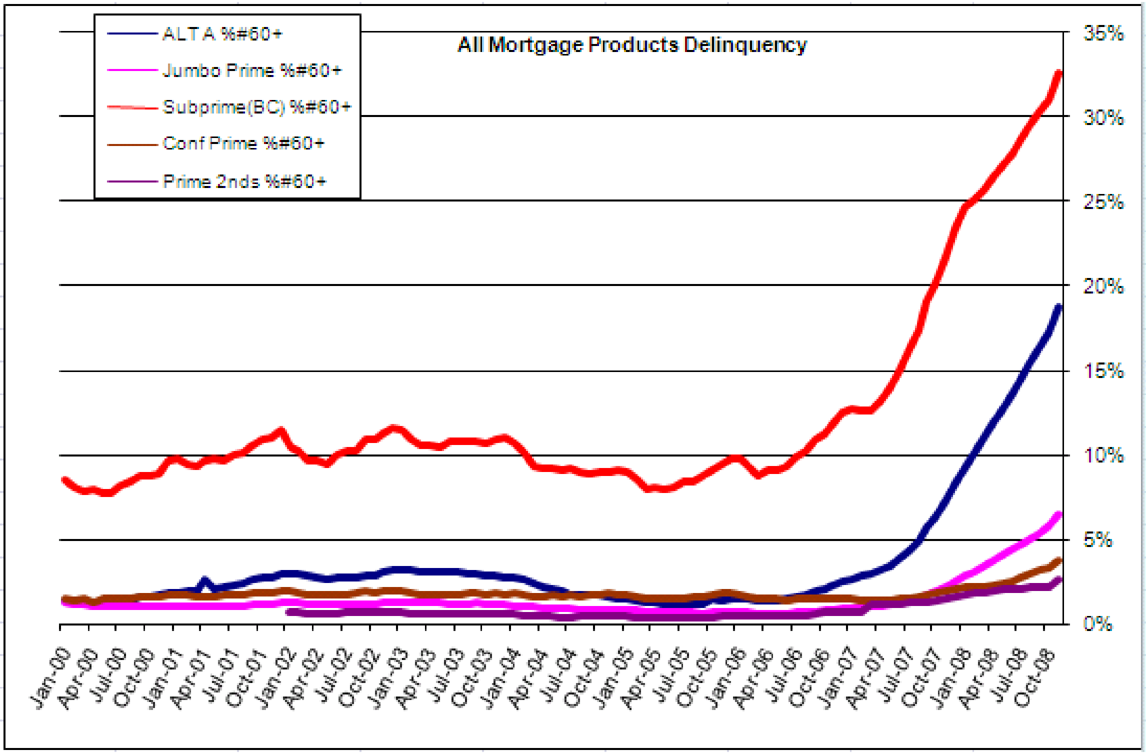
**Figure 7 : Mortgage Foreclosures**




**Source: Loan Performance Market Performance Report**

In figure 8, you can see the delinquency rates of all mortgages classes over time, quite naturally the rates skyrocketed and they are in line with the foreclosure rates.

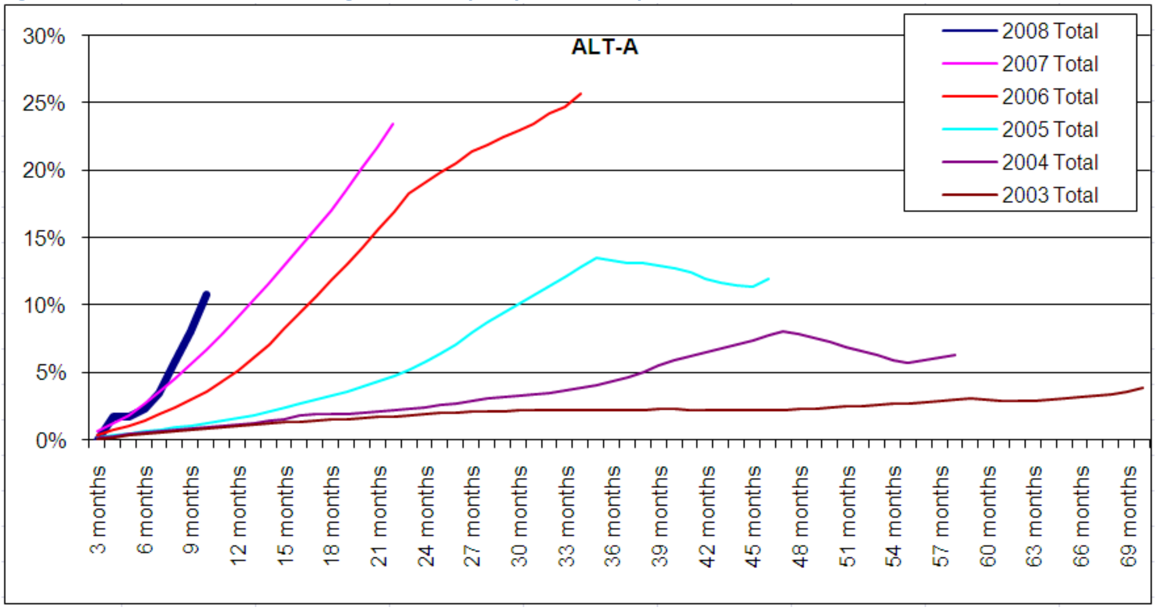
Figure 8: Mortgage Delinquency Rates




Source: Loan Performance Market Performance Report

Evidently, the new mortgage originations rate started to be lower, but since these securities had a higher yield with lower risk profile according to the credit risk ratings, the demand remained to be strong; some lenders chose to loosen their underwriting standards. They have resorted to high "risk-layering", which means joining together the poor credit records with other factors increased credit risk, for instance; low credit scores (FICO score lower than 600), no down payments, no secondary mortgage collaterals, no documentation of assets, no documentation on income, and a purchase-money mortgage, became more common. In the end, these new standards led the rise of "early payment defaults"(see Figure 9)—these are the defaults which are happening just in the very first months of the origination among sub-prime adjusted rate mortgages and interest only loans, this happened especially to those originated in 2006 (Bernanke, 2007). The originations of 2007 also were not getting better than the 2006 ones as you see in the figure below.

Figure 9: Percent of Outstanding loans sixty days or more past due

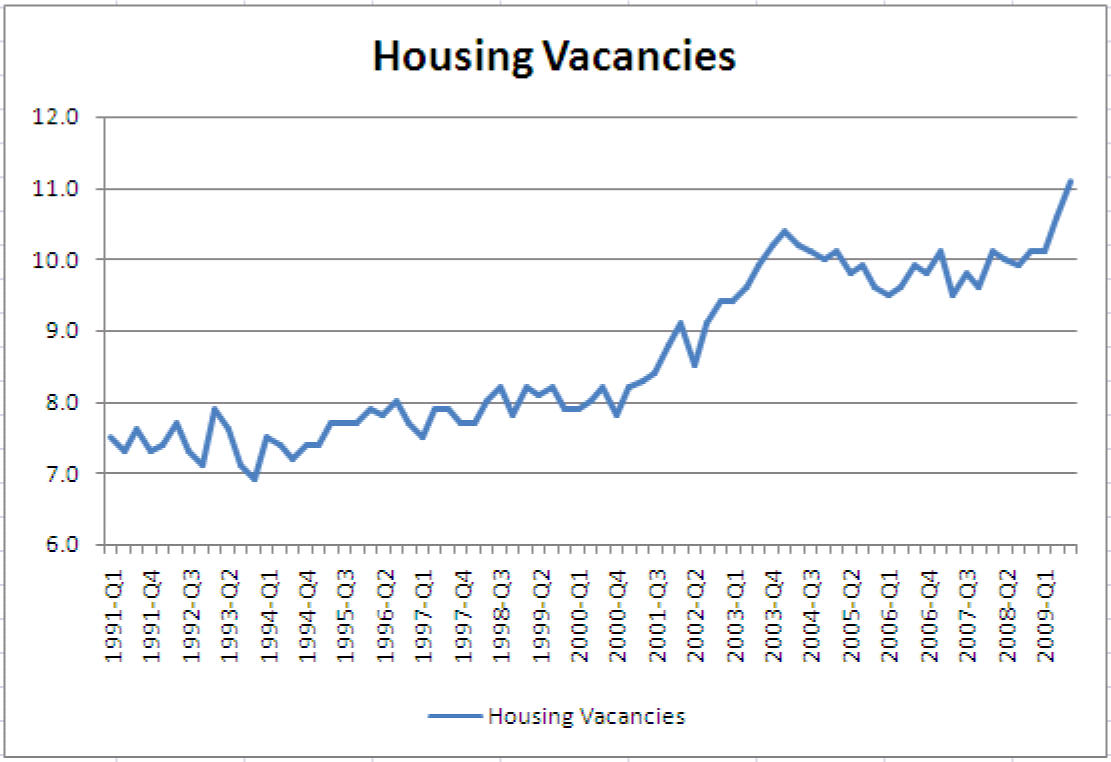



Source: Loan Performance Market Performance Report

The aforementioned developments of the mortgage industry have also assisted the sudden rise in delinquency rates. Securitization caused difficulties in assessing the credit risk, especially when different mortgages with various risk profiles were grouped in securities. Thus, investors ultimately bought MBS products without being aware of the credit risk they bear. The adversely affected companies were mostly the speciality finance companies and the banks which had subprime-specialist subsidiaries. Starting from mid-2006, some of the nondepository, low equity companies have either ceased operations, announced bankruptcies or have been saved by the government. (Kiff & Mills, 2007). These companies represent in total 40% of 2006 subprime originations (Kiff & Mills, 2007).

As you can see from Figure 10 below, the US housing vacancy rate has commenced to increase from 2000Q1, with the increase in new housing constructions figure 11. Besides, since 2006 third quarter, due to the increase in the foreclosure rates the housing vacancy rate did not decrease although the new housing constructions decreased to its lowest.

Figure 10: Housing Vacancy in US (1995 Q1-2009 Q3)

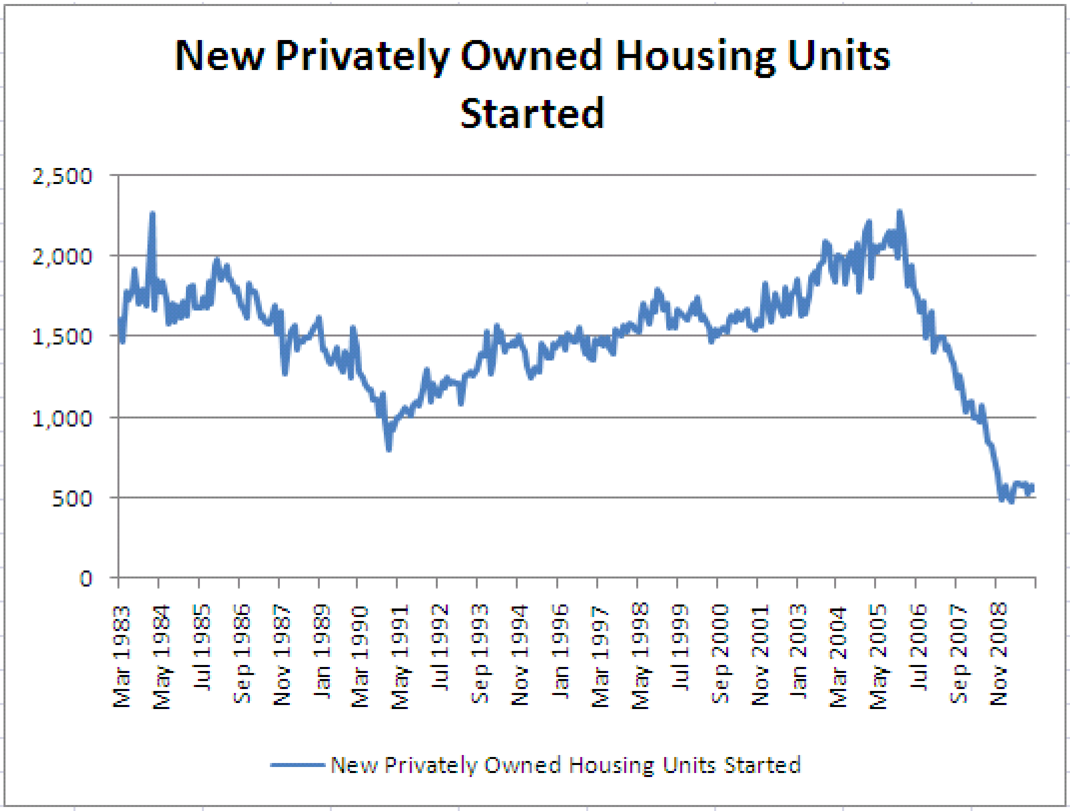



Source: U.S. Census Bureau

## 2.3 Third Phase: Economical Consequences

The strong demands on housing encouraged the construction companies to produce more houses. Until 2006, the number of new housing constructions increased gradually. While suddenly the demand was cut, constructions industry was working at its high speed. In Figure 11 we can see the monthly new housing units started.

Figure 11: New Privately Owned Housing Units Started (Jan 1983- Oct 2009)

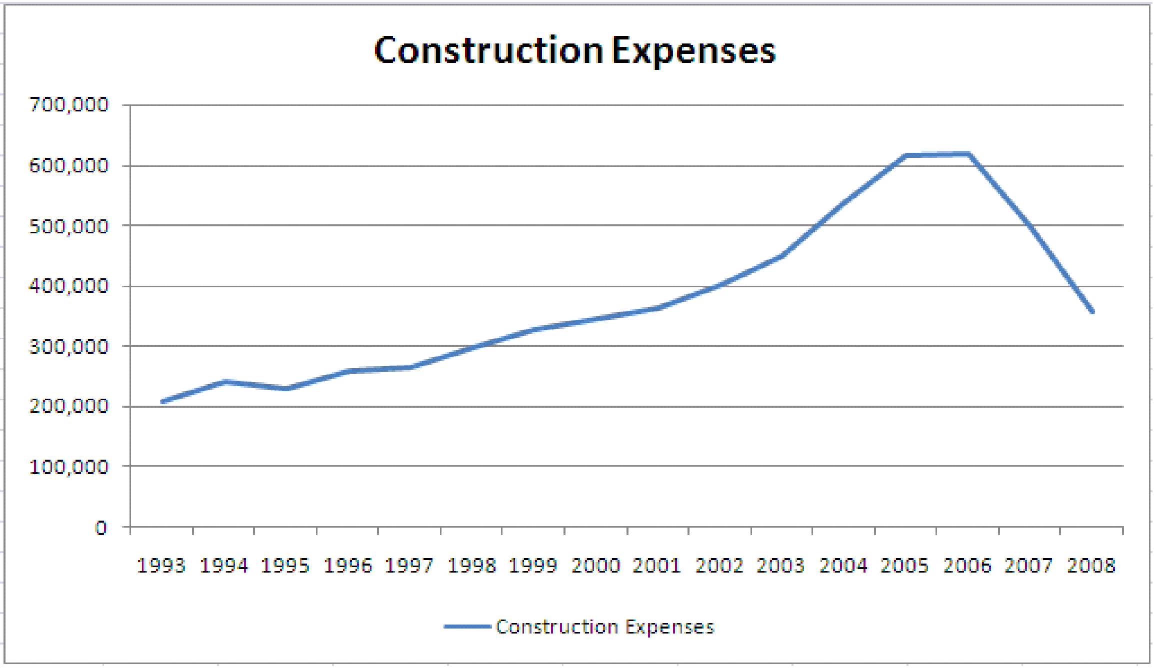



Source: U.S. Census Bureau

The number of new housing constructions didn't decrease until mid 2006, due the time lag in responsiveness of supply side to demand to side of the market.

In the figure below, annual construction expenses were graphed. Construction expenses started to decrease in 2007, (Figure 12).

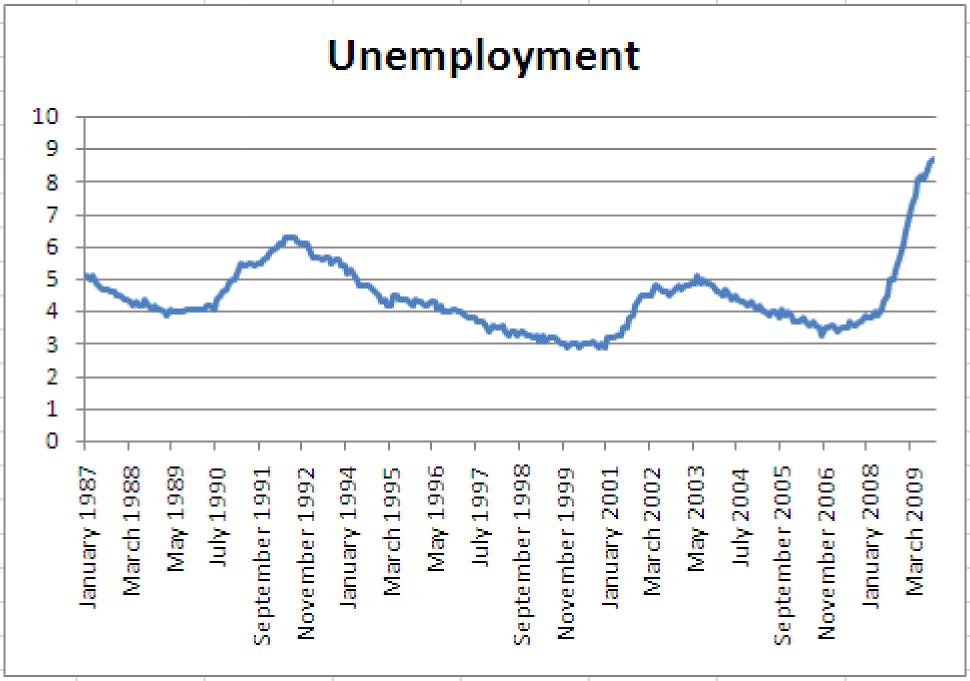
Figure 12: Construction Expenditures (1993-2009)




Source: U.S. Census Bureau

Labour market has also affected adversely with the slowing economy. According to the Labour Department, in 2007 unemployment started to rise and the number of new jobs had entered into declining path after four years of increase.

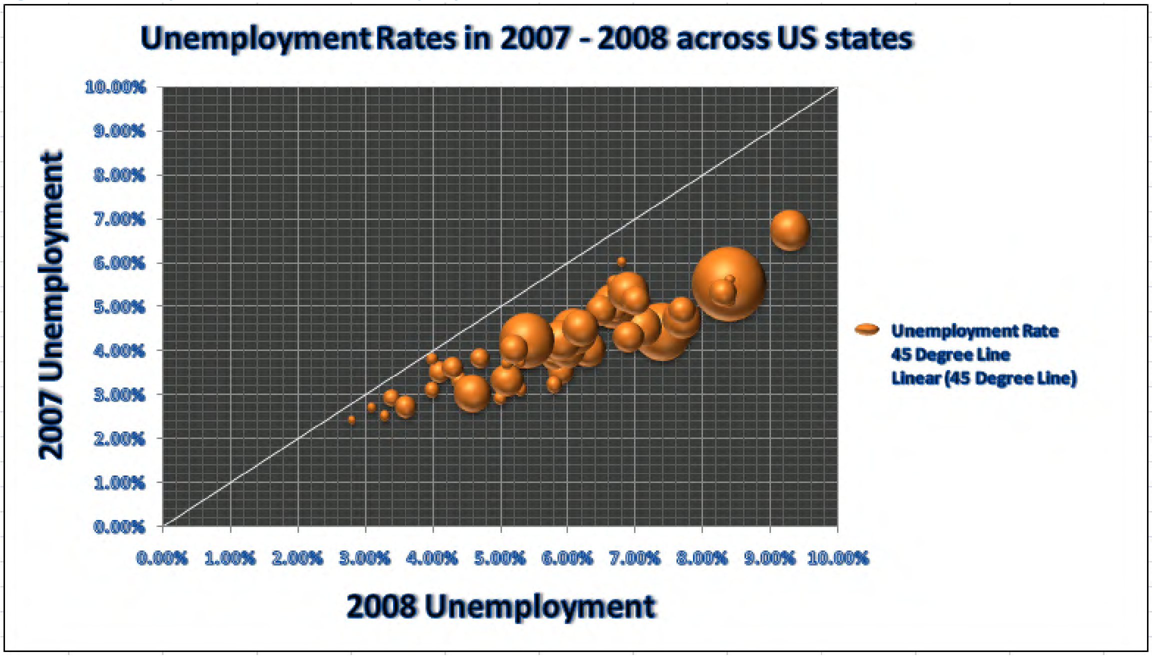
Figure 13 Seasonally Adjusted Unemployment rate (%)




Source: U.S. Census Bureau

In the following chart you will see the comparison of US states in terms of employment where the volume of the bubble represent the population of the state. Since all bubbles are below the 45 degree curve, we conclude that all the state to a certain extent had increase in unemployment.

Figure 14 : Comparison of State Unemployment Rates




Source: U.S. Census Bureau

The moment the late payment and foreclosure rates of the subprime mortgage loans have raised, the market confidence on the mortgage backed securities (MBS) have started to diminish. Thereafter, some primary lenders, for instance Countrywide Financial[9] Corporation, have started to tighten their loan terms and stopped all the sub-prime mortgage originations. In 2007 third quarter, swiftly the number of new subprime mortgages plummeted 50% in comparison with the previous quarter of 2007. Considerable number of institutions and companies were hit in the beginning of the crisis. Mortgage institutions and thrifts, i.e. Washington-Mutual wrote-off huge losses, even sizable banks like Citigroup Inc, brokerage companies like Merrill Lynch & Co., Inc.,[10] the mortgage-finance companies, Federal Home Loan Mortgage Corporation (FHLMC) and Federal National Mortgage

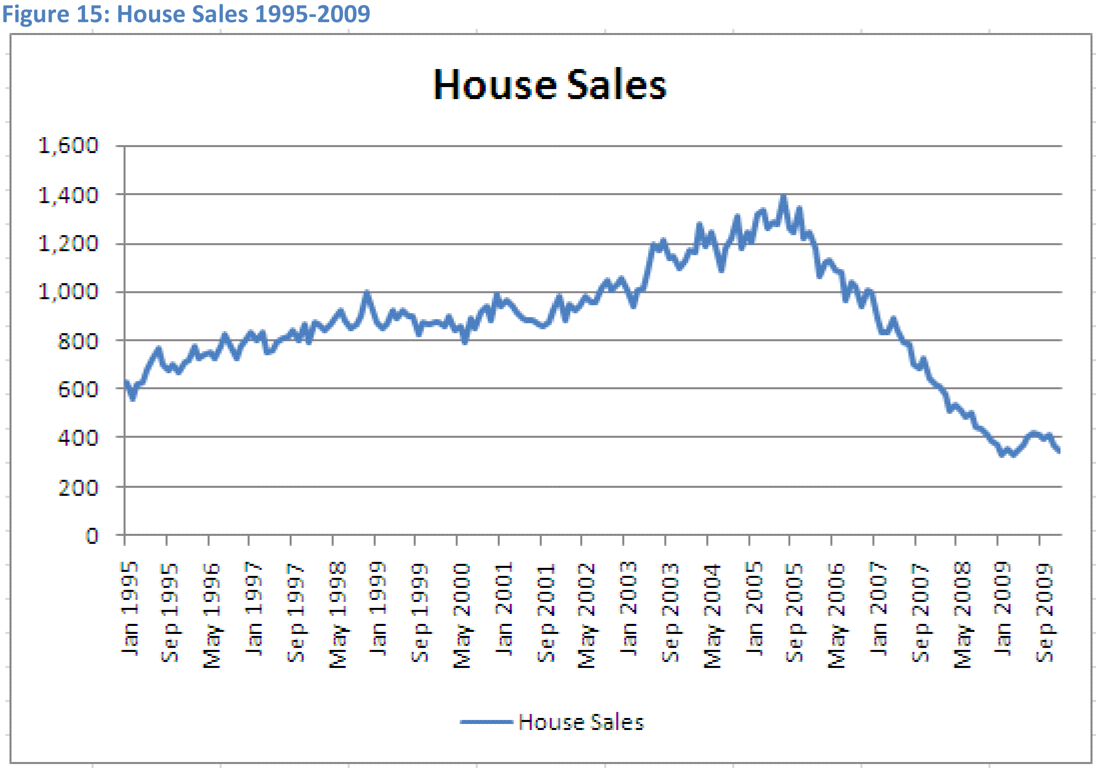
[9] Currently operating under the name Bank of America Home Loans

[10] Was acquired by BOA, and the name changed to Bank of America Merrill Lynch

Association (FNMA) have been seriously slammed by the sub-prime mortgage crisis (Isidore, 2007).

Since the market confidence plummeted, the 3 month interbank rates have been increased. During the credit crunch, market interest rates have also increased, thus it became more difficult for borrowers with payment difficulties to refinance. Federal Reserve System has started to inject ample liquidity to the financial markets to keep the system afloat. Furthermore, Federal Reserve has reduced the short-term funds rate to the level of 4.24% by end of 2007; by the end of 2008 it was 0.15%. The rapid decline of interest was aimed to alleviate the panic in the financial system and to prevent an even more severe downturn.

**Figure 15: House Sales 1995-2009**




**Source: U.S. Census Bureau**

However, the strict credit terms triggered a rather vast and prolonged housing downturn than it was expected initially. These tighter credit conditions also extended the fall of the residential investments and lead to a huge suppression on refinancing and consumer expenditures. As a result, the drops in household consumptions and construction expenditures together with the rise in the unemployment would further impair the GDP growth, which had grown in the 3rd quarter of 2007 only by only 0.9%. Besides, RealtyTrac[11] has projected that up to one and a half million homeowners have started foreclosure proceedings by the end of 2007; it is twice more than 2006's figure, and in addition to the foreclosures. Moreover, it estimated that, before the end of 2008, two and a half million ARMs[12] will receive higher rate fixings. (The Economist, 2007)

Increasing foreclosures of the subprime mortgages in United States infected a wide range of markets around the globe with a mind blowing speed.  Two banks, the Clemos and Northern Rock in UK were among the ones which were swept up during the sub-prime mortgage crisis. It was very surprising for so many people that some US local mortgages had such widescale effects on global economy. (Barr, 2007)The culprit of this epidemic is this; securitization has allowed the trading of mortgages so smooth in order to spread the risks all around the globe to hinder major financial institutions' collapses. In the meantime, it has also integrated distant markets to the US market. Consequently, a niche market crisis in one of distant countries could contaminate easily a lot of other markets which, at the first glance, had very little connection with each other. With the instant availability of information, these infections have a more potent scope which hasn't been seen before (Barr, 2007).

[11] a firm which tracks foreclosures in US
[12] Adjustable rate mortgages

The early overseas impacts of the crisis realized in July 2007, IKB Deutsche Industriebank AG, one German bank was rescued by the Kreditanstalt für Wiederaufbau (KFW) Group[13]. Spill over effects were immediate, German saving banks bailed out Landesbank Sachsen in October 2007. Besides the bailouts, Citigroup, HSBC Holding plc and ABN Ambro have provided massive funds to money market management which was struggling to cope with the mortgage crisis. The problems in United Kingdom was not different, Barclays PLC had to bail out its hedge fund Cairn Capital (Forbes 2007). HBOS PLC had to finance to its conduit fund Grampian Limited (Independent, 2007) and also the Bank of England had assisted Northern Rock to keep itself afloat with emergency loans. In a nutshell, the credit crunch had a global impact on financial markets.

[13] A government-owned development bank, meaning Reconstrution Credit Institute

# 3.0 Literature Review on Bubbles

One of the most notable researchers of property bubbles is Robert Shiller. He has built fame during 1980's with his argument that in the stock markets the prices do not show the intrinsic values of stocks, (Fox, 2007)[14]. The phrase, "Irrational Exuberance" is usually used to explain the elevated state of investors' speculative ambition during bubble periods. Schiller notes that, "Irrational Exuberance" was initially coined by Alan Greenspan[15].

With his speech[16], he pointed to the probability that; the increase in stock markets could be led by the irrational exuberance and he indicated that the possibility of an eventual decrease in stock markets afterwards. The stock markets' responded adversely to his speech. Almost instantaneously, Tokyo (Nikkei 225) and Hong Kong (HSI) stock exchanges fell by three percent. Frankfurt (DAX) and London (FTSE) stock exchanges followed them, with a four percent fall. S&P 500 also responded with two percent at the open of trade. Greenspan's at first sight innocent question caused strong reactions of the markets, consequently, the phrase "irrational exuberance" gained fame and popularity (Schiller, 2000).

[14] In March 2000, he discussed the stock market bubbles in his book "Irrational Exuberance"

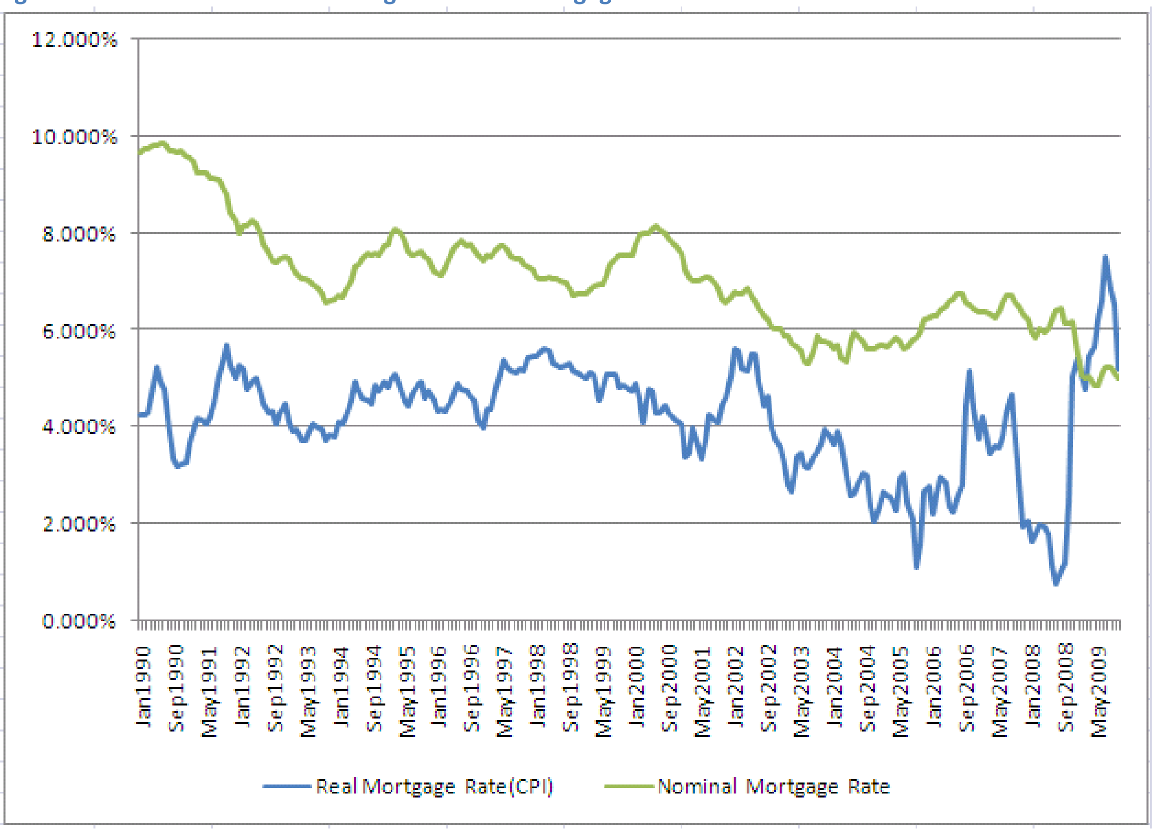
[15] An american economist who served as the Chairman of the Board of Governors of the Federal Reserve System from 1987 to 2006,

[16] During the "Annual Dinner and Francis Boyer Lecture of The American Enterprise Institute for Public Policy Research" in 1996, December 5, he said that, "...sustained low inflation implies less uncertainty about the future, and lower risk premiums imply higher prices of stocks and other earning assets. We can see that in the inverse relationship exhibited by price/earnings ratios and the rate of inflation in the past. But how do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade?"

Robert Shiller addresses the irrational elements which influence investors' judgements in 2000. He indicates that investors are not aware of the intrinsic value of the stocks, and most of the investors simply are not interested in finding the right price level of market (Schiller 2000). For instance, in the case of a bubble, house buyers consider that a house which they would ordinarily think overpriced has become a reasonable acquisition because they think that they are going to benefit from a considerable rise in the prices (Case and Shiller, 2003). This creates a self fulfilling prophecy or in another words a positive feedback loop. Besides, home buyers might only consider the nominal interest rates instead of real interest, which is explained as a "money illusion".

In sub-prime mortgage market we could also see these kinds of psychological elements. Case and Shiller in 2000 made a survey in United States on and detected curious patterns regarding the households' view on real estate market. They have found out that 90% of the home owners projected a rise of real estate prices in the coming days and homeowners thought that the market prices were not over inflated, even when the bubble is at its peak level. Furthermore, house owners shared a common view on housing, they indicated that investing on housing is far more secure than investments on stocks and consequently, they had invested on new houses rather than on stock market. They also pointed out that their investment decisions were based on the level of nominal interest rates instead of the real interest rates. This issue has actually been argued by the economists, they noted that since nominal interest rates and the inflation levels were jointly lower, mortgages' real interest rates indeed cannot be considered as low. Figure 15 demonstrates the movements of the real and the nominal mortgage rates between 1990 and 2009. The monthly values were calculated by discounting the effects of consumer price index. Furthermore, as their survey indicates while borrowing, homeowners look at only nominal interest rates. Even though, nominal mortgage rates have pursued a stable trajectory since 1993, the real interest rates were quite volatile and starting from 2005 the real mortgage rates have risen drastically between until 2007. Hence the "money illusion" quite clearly presented itself one more time.

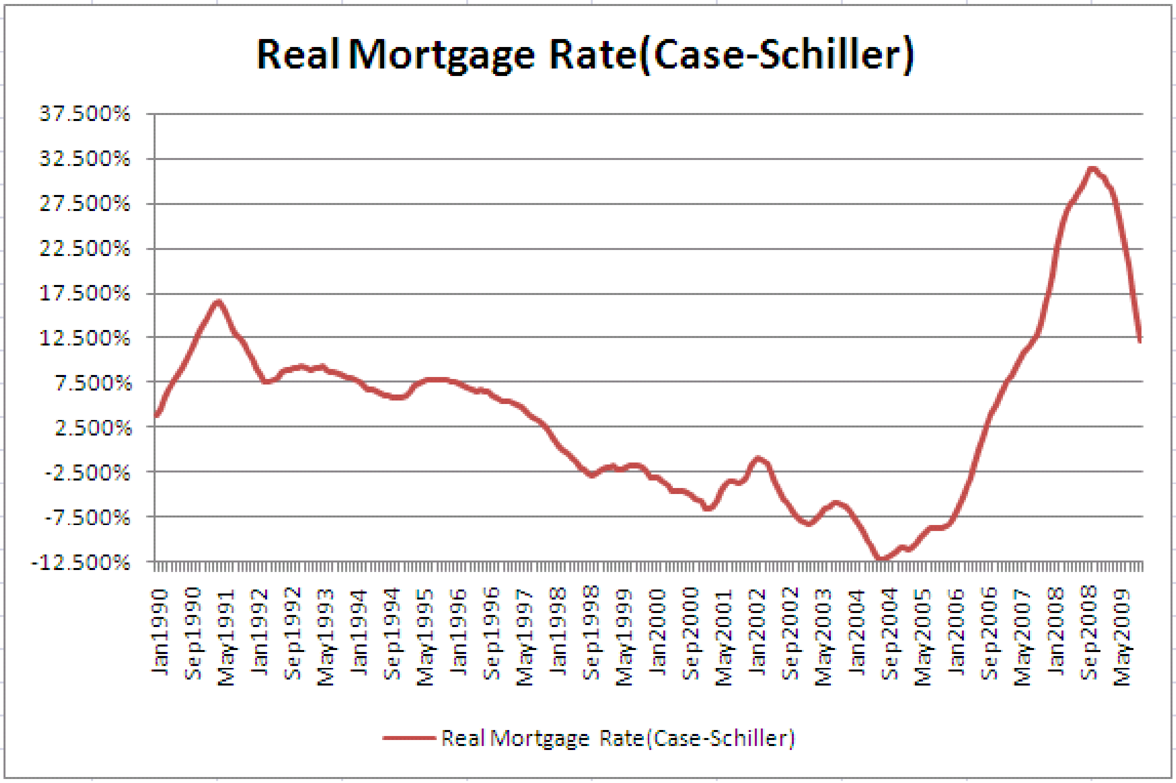
**Figure 16 : Nominal and Real Average Contract Mortgage Rates**




**Sources: Mortgage-X**

In Figure 16, you can see the real mortgage rate calculated against the home price increases, (instead of CPI). From 1998 until mid 2006, actually homeowners were making unrealized profits whilst they keep their houses. In the figure below, real mortgage rate is calculated after taking house price appreciation into account. Quite clearly, the interest rate they pay was much lower than their gains from real estate price increases, particularly between 2002 and 2006, a period which is marked as the real estate bubble. This also might create a vicious cycle in the minds of homeowners.

**Figure 17 : Real Mortgage Rate originated from House Price increases**




**Sources: Mortgage-X**

Other important researches in this field are also performed by Case and Shiller. In one of their article from 2001, Case, Quigley and Shiller investigated different relationships between boosts in housing and financial wealth together with consumer expenditure, by implementing panel data analysis with fourteen countries yearly data from 1975 until 1999. In addition, they also employed a panel survey for US states from 1982 until 1999 by using quarterly data. They have made a projection with expenditures related to income and wealth variables. Their findings proposed that primarily for US, the consumption behaviour of households in US can be explained better with the housing market rather than the stock market. In addition, wealth effects are more powerful in housing market. The result of this study is, when the house prices increase the household consumption also increases, however, the relation between household consumption and stock prices is not comparable with the previous one.

It has been also showed by Case and Shiller (2003) that real estate prices and income are related. In their empirical research they have attempted to identify the symptoms of house price bubble. With major economic indicators such as population, income per capita, employment, mortgage rate, number of new housing constructions, and unemployment rate, they tried to explain the house prices. For the period between 1985 and 2002, choosing states with volatile house prices, they have demonstrated that income variable is correlated with house prices most, and for the period between 2000 and 2002, and they could not say conclusively that a bubble didn't exist in the US states which they selected.

There were also researches claiming that there was actually no bubble in the house prices. For instance, McCarthy and Peach in 2005 argued that Federal Housing Finance Agency house price index[17] (FHFA) isn't a real "constant quality" house price index. They claimed the house price index has risen due to the rises quality of newly built houses. Hence, even though McCarthy and Peach have also noticed the rise of price to rents ratio, their conclusion was; FHFA house price index was indeed misconstructed, therefore misguiding. Moreover, they also claimed that the fall of interest rates caused the bubble of doubt. However, they didn't consider any other indicator such as the extraordinary rise in the mortgage lending, increase of real estate prices in main states, surge of construction consumption and

[17] ) Formerly known as Office of Federal Housing Enterprise Oversight (OFHEO) house price index

projections, in addition, by grounding on not well documented arguments, they have made a conclusion that that the real estate prices were not over inflated.

# 4.0 Stylized Facts: Detecting Bubbles

The theories which focus on explaining the behaviour of different traders and the obscurity about the value of the security are based on the availability of the information as well as the arrival time of the information. The behaviours of the traders have been analyzed empirically by the availability of the high-frequency data sets. After a decade of empirical analysis on asset returns, researchers concluded with some common statistical regularity. These regularities which are common over different instruments, different financial markets and time periods are often called as stylized facts (Cont, 2001). Cont (2001) lists eleven stylized facts of asset returns as "(1)absence of autocorrelations; (2)heavy tails; (3)gain/loss asymmetry; (4)aggregational gaussianity; (5)intermittency; (6)volatility clustering; (7)conditional heavy tails; (8)slow decay of autocorrelation in absolute returns; (9)leverage effect; (10)volume/volatility correlation and (11)asymmetry in time scales".

Due to lack of intraday data of CMBS index, in this thesis, I have tested six of these stylized facts. This part includes a short literature review for each of the stylized facts which are tested in this thesis.

### 4.1.1 Absence of autocorrelations:

Predicting the future price of the stock through past history of a common stock's price has been a great interest for business and academic areas. Fama (1971), analyses the stock return autocorrelation through theory of random walk. The theory of random walk includes two distinct hypotheses regarding stock returns: (1) successive stock returns are independent of each other and (2) the stock returns correspond to some probability distribution. He mentioned that the independence of stocj price changes is the outcome of noisy price mechanism. By noisy price mechanism, Fama (1971) refers to uncertainty on the true value of the security due to new information which has arrived or about to arrive in the market. If information arise independently over time and if the uncertainty concerning the present value of the stock does not follow any consisting patterns then the future price returns will be independent. Fama also comments on "superior traders", even if prices were dependent, which means every new buyer could influence the others to have the same investing behaviour, there would be still superior traders who will detect the abnormalities in the market and will correct them by buying or selling. If there are enough such traders, then the price will tend to stabilize around its intrinsic value, reducing risks of bubbles or crashes.

Atchisoz *et al.* (1987), tested autocorrelation by using the Scholes and Williams model of nonsynchronous trading. Their empirical results were far above the model estimation results. They concluded that model couldn't explain the high observed autocorrelation of the indices examined. After their paper, empirical literature suggests that observed autocorrelation in stock returns cannot be explained due to lack of large enough time variation of expected returns. (Conrad & Kaul, 1988, 1989; Lo &MacKinlay, 1988, 1990; Ogden, 1997).

### 4.1.2 Heavy Tails

The statistical distribution of asset returns is very important to test pricing theories, as the distribution determines the assumptions on the behaviour of market prices. Early financial research in financial econometrics assumed Gaussian distribution for asset returns (Markowitz, 1952; Tobin,1958; Sharpe,1964). Then Mandelbort (1963a, 1963b; Mandelbort and Taylor,1967) showed that in financial markets stock returns do not fit well to Gaussian distribution instead they possess excess kurtosis. They found that Gaussian distribution is more likely to overlook the heftiness of the extreme returns in both tails of the distribution.

The supporting literature suggests many different distributions of asset returns. Mandelbrot indicates that in financial markets stock price changes follow non-Gaussian stable Levy processes. Press (1967) claims that heavy tails in distributions are formed due to the compound Poisson processes for the variance parameter of normal distributions. Following, Clark (1973) argues that financial market stock price changes exhibit finite-variance distributions, for instance lognormal–normal distribution, rather than any other stable regime. He explained this different evolution of price series with the availability of varying information. We know that when there is no information trading is slower than the days when information is available. And when new information arrives to market, the price evolves to its new value much faster. Consequently, stock price changes follow this different regime.

One of the simplest measures of the distribution is kurtosis. In statistics, kurtosis is explained as the measure of "peakness" of the distribution. A high kurtosis figure indicates the distribution has more weight in the peak and in the tails, since the extreme variations from the mean are infrequent. Generally, it has been assumed that financial time series possess excess kurtosis.

Underestimating the risk of kurtosis leads any financial model to underestimate the risk of the variables which are affected by high kurtosis. One famous incident is, Long-Term Capital

Management (LTCM), a US hedge fund which was cofounded by Nobel laureate Myron Scholes, once overlooked the kurtosis risk. In the end of four successful years, in the late 90s, Long-Term Capital Management had to be saved by the primary investment banks since LTCM underestimated the high kurtosis of several financial assets which are the underlyings of the fund's own trading positions, Krugman (1998).

The results of Mandelbrot has been endorsed by more recent researches by Bouchaud and Potters (2001), Con (1997,2000), Dacorogna et al. (2001), Plerou et al. (1999), Rachev (2003) and Voit (2003). There have been many other suggestions on the form of the distributions but no general consensus has been reached on the exact form of the tails so far.

### 4.1.3 Volatility clustering

Another well-established stylized fact among the financial markets is volatility clustering. Mandelbrot (1963) defines volatility clustering as "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes." Cont (2005) summarizes the emprical observation of this stylized fact as following; while stock price changes themselves are independent, any measure of volatility of the returns exhibit a positive, significant and slowly decaying autocorrelation function. Volatility clustering is explained simply with the following phrase “periods of tranquillity interrupted by periods of turbulence”. Neither the volatility nor the tranquillity in the market does last short. The switch from one of these extreme regimes to another one is indeed a very gentle process therefore large returns slowly decay until the next tranquil state is reached (Plerou et al., 1999; Cont, 1997, 2001; Mantegna and Stanley, 2000; Bouchaud and Potters, 2001; Voit, 2003).

Volatility clustering implies that asset returns are not independent across time contrary to Fama's random walk theory; however another stylized fact "the absence of linear autocorrelation" indicates that asset returns are independent. Observations of volatility clustering led to proposed models of ARCH (Engle, 1982), GARCH (Bollerslev,1986) and FIGARCH to replicate volatility clustering in financial time series. These models formulate the notion that price volatility has a tendency to rebound to a mean instead of staying steady or around a stable value. One of most popular one GARCH assumes that today's price volatility is dependent on the long term variance, expected variance of yesterday and real variance of yesterday.

### 4.1.4 Slow decay of autocorrelation in absolute returns

According to Cont (2001), the autocorrelation function[18] of absolute returns decays/decreases slowly as a function of the time lag, roughly as a power law with an exponent $\beta \in [0.2, 0.4]$. This is sometimes considered as a sign of long-range dependence

Dependence of price changes, for instance autocorrelation of absolute and squared price changes, are areas of recent academic researches. Harvey, Ruiz, and Shepard (1994) together with Melino and Turnball (1990) used stochastic volatility models to explain the patterns of squared stock price changes. Both of their econometric models are based on stock volatilities having a correlation structure which declines like a short range dependent process. One of the more recent works on squared returns is the model Breidt, Crato, and de Lima (1998)

---

[18] Autocorrelation function (ACF) is the correlation of the process with itself. More formally, from Eichner et al 2007, let X= ($X_t$, t = 0, 1, ..., T) be a stationary stochastic process with standard deviation, α, and mean, μ. Then the ACF,δ(k) dependent of number of k lags is;

$$\delta(k) = \frac{\sum_{t=0}^{T-k}(X_t - \mu)(X_{t+k} - \mu)}{\alpha^2\,(T-k)}$$

prepared. They employed a long-memory stochastic volatility (LMSV) model which explains the persistent dependence of squared price changes.

### 4.1.5 Leverage effect

Another important stylized fact is 'Leverage effect', which was first observed by Black (1976). Black (1976) presented that the volatility of shocks have a tendency to rise when the price decreases. In other words, bad news (negative shocks) affects the volatility more than the good news (positive shocks); as a result volatility gains a tendency to be higher in a decreasing market than in an increasing market. Consequently, it is an observation that volatility and return possess negative correlation.

Black (1976), referred to this effect leverage due to the reason that negative shocks tend to decrease the stock price, hence raising the debt-equity ratio of the firm, as a result stock prices exhibits high volatility. In other words, the leverage effect is explained as the asymmetric impact of shocks on volatility.

This effect is especially important for option markets, Backus et al. (1997). First of all leverage effect implies that after price decreases, money volatilities tend to increase. Secondly, when the leverage effect is strong, volatility smile appears (Backus et al., 1997).This happens when at-the-money options[19] have lower volatilities than in-or out-of-the money options (Backus et al., 1997).This skew reflects that a negative correlation return and volatility encourages in the distribution of stock price changes, a negative skew.

[19] At the money option's strike price is equal to the price of the underlying asset.

### 4.1.6 Volume/volatility correlation

In financial markets, the correlation of the volume of a trade and the volatility of share prices has been a great interest. The major motivation was the observation of “market breaks” of 1987 and 1989, when high price volatility coupled with heavy trading volume in equity markets. Another motivation of this interest is the highly developed theoretical literature that examines asymmetric information in the stock markets; the interaction between market makers and speculative traders. Many of these models have distinct implications for the relation between trading volume and price volatility.

Clark (1973) in his paper argues that unobserved mixing variable which he comments as “information arrival process” causes the joint distribution between price changes and volume. Lamoureux and Lastrapes (1990) using individual stocks develops the variance function by adding the level of trading volume. They found out that GARCH effects have been disappeared with this variance function. They argue that the same factors produce volatility and volume correlation and they also suggest that volume is a proxy for information arrival.

Bollerslev and Jubinski (1998), using 100 equities investigated long run dependencies of trading volume and price volatility relationship. For a detailed survey of earlier literature of this topic, one could check Karpoff (1987) who presented a broad review of previous researches on this relationship.

The primary stylized fact mentioned in the literature is this; the volume of daily trade is positively correlated with the contemporaneous price volatility. Both volatility and trading volume has exhibit patterns of strong serial dependence. In addition, returns and trading volume display patterns of conditional heteroskedasticity as well.

## 4.2 Data and Methodology

In order to test the stylized facts, I have collected daily Commercial Mortgage Backed Securities Index (CMBS) and also S&P 500 daily returns data to present as benchmark. CMBS index is prepared daily by the Merrill Lynch. According to their definition, "it includes 3,017 constituents which has total capitalization of more $367 billion. All qualifying bonds have a fixed rate coupon and must be rated investment grade based on an average of Moody's, S&P and Fitch credit ratings. Floaters, inverse floaters, IOs, re-securitized, and agency deals are excluded from the Index". Although it doesn't represent perfectly the mortgage market, it can be used as a good proxy. As a benchmark I've selected S&P 500, which I assume will have the same characteristics. Both index values are log differenced to create stationary return series.

Due to the data limitations not all the stylized facts were tested for before and after the crisis. Although intraday S&P 500 index is available and an analysis of it, is academically stimulating, since this thesis is on mortgage backed securities, S&P 500 is just used a benchmark of CMBS. The facts which I tested from the list of Rama Cont (2000) are the following.

1. "Absence of autocorrelations: (linear) autocorrelations of asset returns are often insignificant, except for very small intraday time scales (20 minutes) for which microstructure effects come into play."

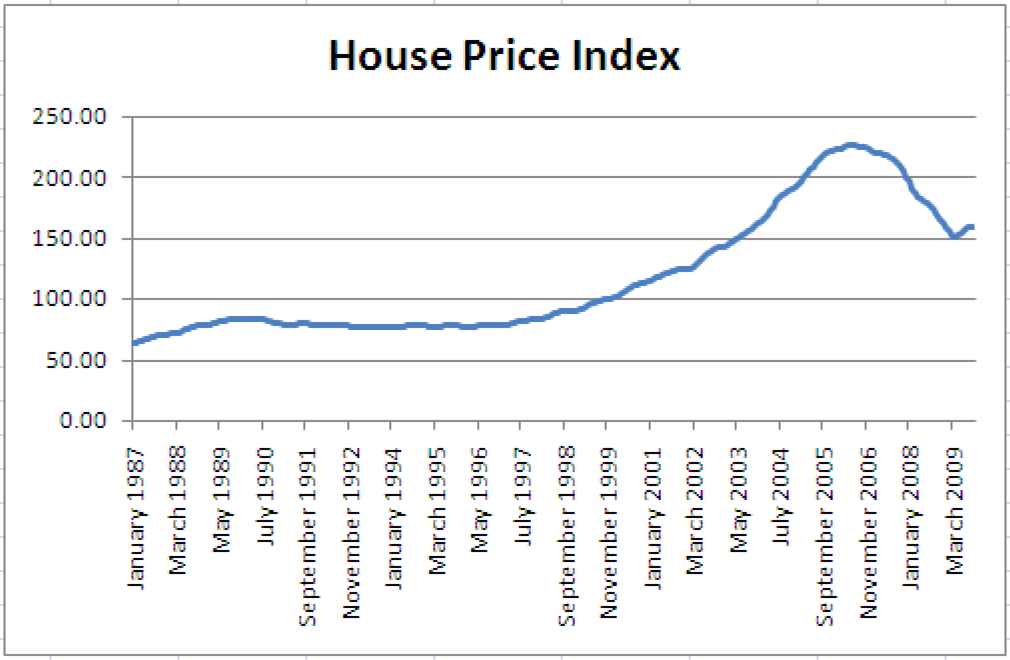
2. “Heavy tails: the (unconditional) distribution of returns seems to display a pareto-like distribution. The excess kurtosis of normal distributions is zero. However, still the precise form of the tails is difficult to determine.”
3. “Volatility clustering: different measures of volatility display a positive autocorrelation over several days, which quantify the fact that high-volatility events tend to cluster in time.”
4. “Slow decay of autocorrelation in absolute returns: the autocorrelation function of absolute returns decays slowly as a function of the time lag, roughly as a power law with an exponent $\beta \in [0.2, 0.4]$. This is sometimes interpreted as a sign of long-range dependence.”
5. “Leverage effect: most measures of volatility of an asset are negatively correlated with the returns of that asset.”
6. “Volume/volatility correlation: trading volume is correlated with all measures of volatility.”

## 4.3 Results and Findings

### 4.3.1 Commercial Mortgage Backed Securities Index

I have decided on the dates based on the time plot of the real estate prices.

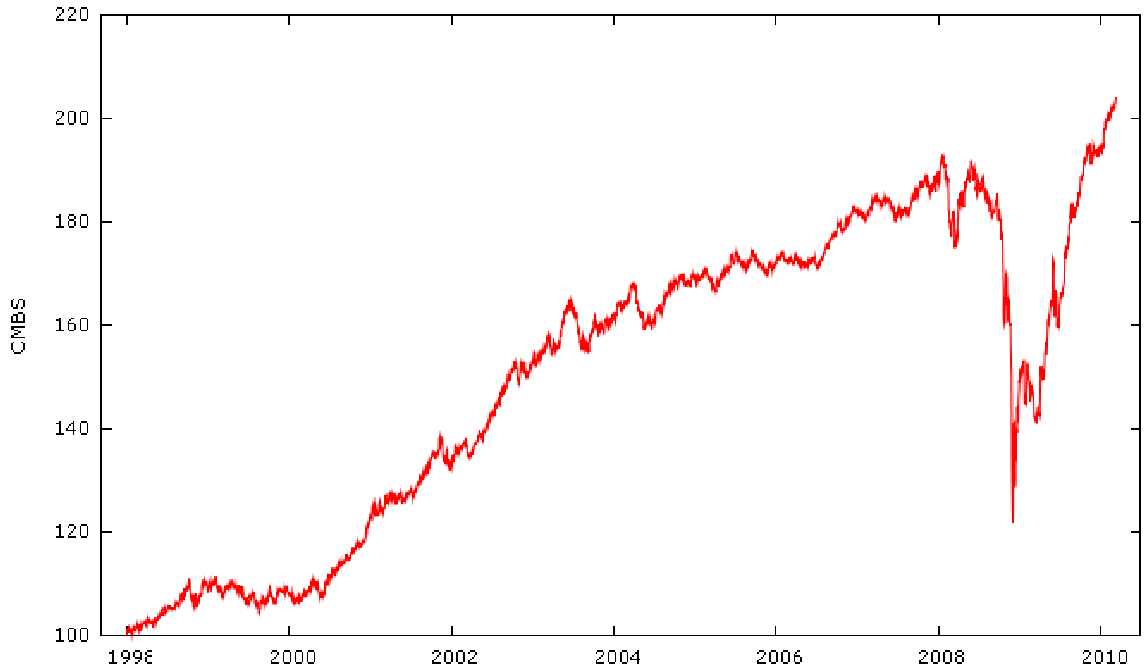
Figure 18 Case–Shiller National Home Price index




Source : Standard & Poors

September 2006 was the date when the real estate prices started to decline considerably. I've selected 7/13/07 as the start date of the crisis. On this date the S&P 500 has reached its peak after a long bull period. Although there is no consensus on the date it's now widely accepted that it's in the months of June or July 2007. In figure below you see the CMBS index time plot. It's in line with the real estate prices.

Figure 19 Time Plot of CMBS Index




Source: Merrill Lynch

#### 4.3.1.1 Kurtosis

Kurtosis is the measure of peakness of a distribution. Excess Kurtosis is zero for normal distributions and common financial series possess kurtosis values way above 3.

In the long run CMBS index returns have excess kurtosis with a value 64.09, however before the crisis we can see that the return distribution is not very different from normal distribution with a value 2.34. After 7/13/07 the kurtosis was 25.796

Figure 20 CMBS Data Description & Kurtosis

| CMBS | 12/31/1997-3/11/2010 | 6/3/2002 - 8/29/2006 | 7/13/2007-3/11/2010 |
|---|---|---|---|
| Mean | 0.00022404 | 0.00021216 | 0.00016834 |
| Median | 0.00028334 | 0.0002433 | 0.00043146 |
| Minimum | -0.094016 | -0.015975 | -0.094016 |
| Maximum | 0.048116 | 0.015058 | 0.048116 |
| St deviation | 0.0053283 | 0.003037 | 0.0096174 |
| C.V. | 23.783 | 14.315 | 57.132 |
| Skewness | -2.8697 | -0.057383 | -2.235 |
| Ex. Kurtosis | 64.094 | 2.3473 | 25.796 |

#### 4.3.1.2 Absence of autocorrelations:

It has been stated that "linear autocorrelations of asset returns are often insignificant, except for very small intraday time scales (20 minutes) for which microstructure effects come into play".(Cont, 2001)

Below you can see the autocorrelations of returns in the first 10 lags. The ones marked with red are significant. As opposed to other stylized facts, before the crisis the number of significant autocorrelations is much lower than any other time.

Figure 21 CMBS Autocorrelations

| **CMBS - ACF** | **12/31/1997-3/11/2010** | **6/3/2002 - 8/29/2006** | **7/17/2007-3/11/2010** |
|---|---|---|---|
| **1** | 0.1536 | 0.0247 | 0.1928 |
| **2** | 0.0642 | -0.1165 | 0.1371 |
| **3** | -0.0728 | -0.0467 | -0.0765 |
| **4** | -0.0337 | 0.023 | -0.0463 |
| **5** | -0.0997 | 0.0078 | -0.133 |
| **6** | -0.0634 | 0.008 | -0.0851 |
| **7** | -0.0869 | 0.0472 | -0.1337 |
| **8** | 0.1233 | 0.0093 | 0.1594 |
| **9** | 0.0327 | 0.024 | 0.04 |
| **10** | 0.1082 | 0.0107 | 0.1541 |

#### 4.3.1.3 Volatility clustering

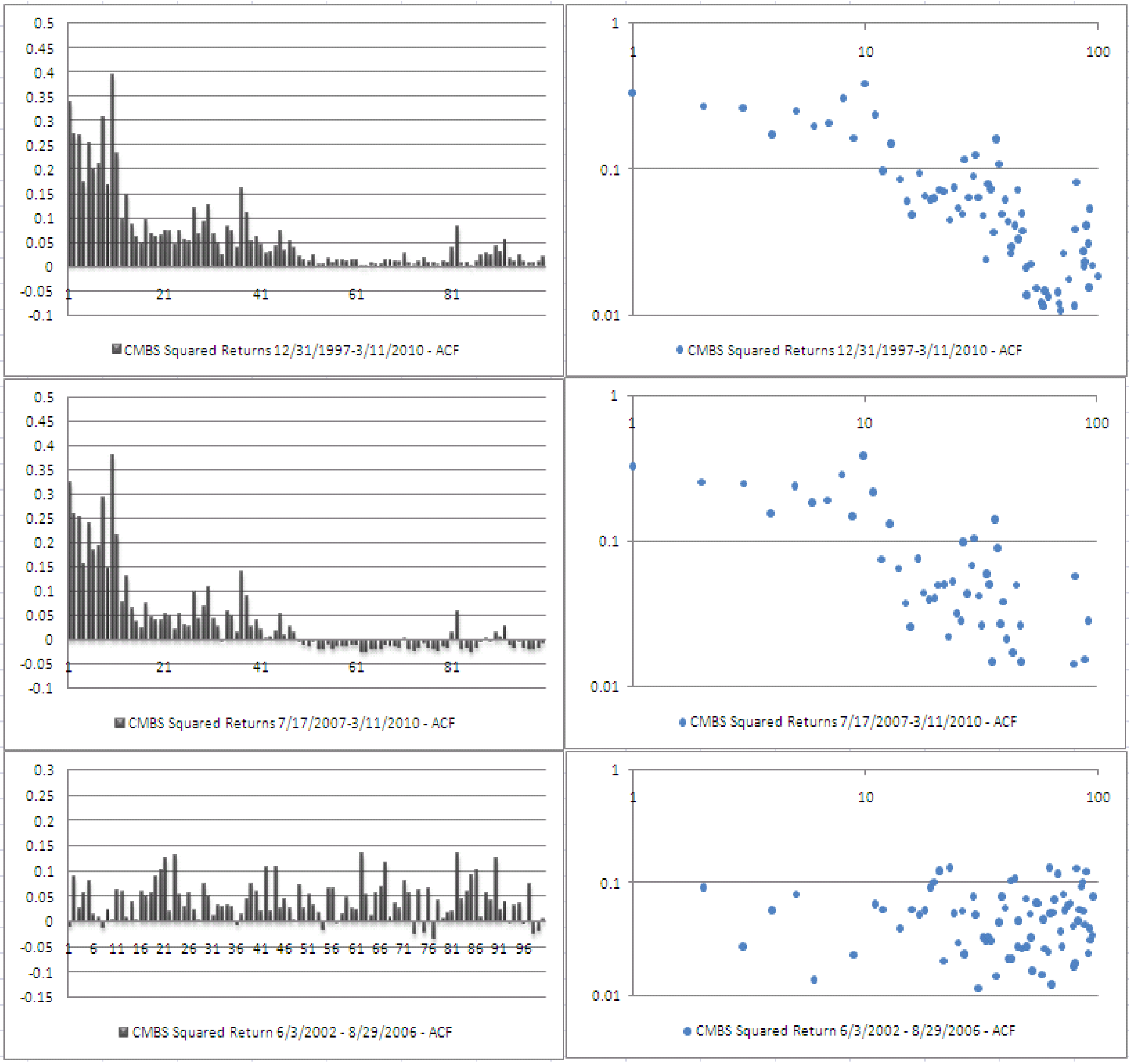
"Different measures of volatility display a positive autocorrelation over several days, which quantify the fact that high-volatility events tend to cluster in time"(Cont, 2001). In order to measure the volatility clustering I have defined the volatility by the following formula;

$$\text{Volatility} = (\text{Return - Mean (Return)})^2$$

Then I have calculated autocorrelations of these volatility values. If autocorrelations exist and decay slowly, it indicates that volatility events cluster through time. In the following figures, you can see clearly that volatility did not cluster in time before the crisis as it did after the crisis or in the long term. The log-log plots[20] also confirming that slow decayance did not happen therefore volatility events didn't cluster.

[20] In log-log plots, both X and Y axis are in logarithmic scale and negative values do not appear

Figure 22 CMBS Volatility Autocorrelations and Log-Log Plots



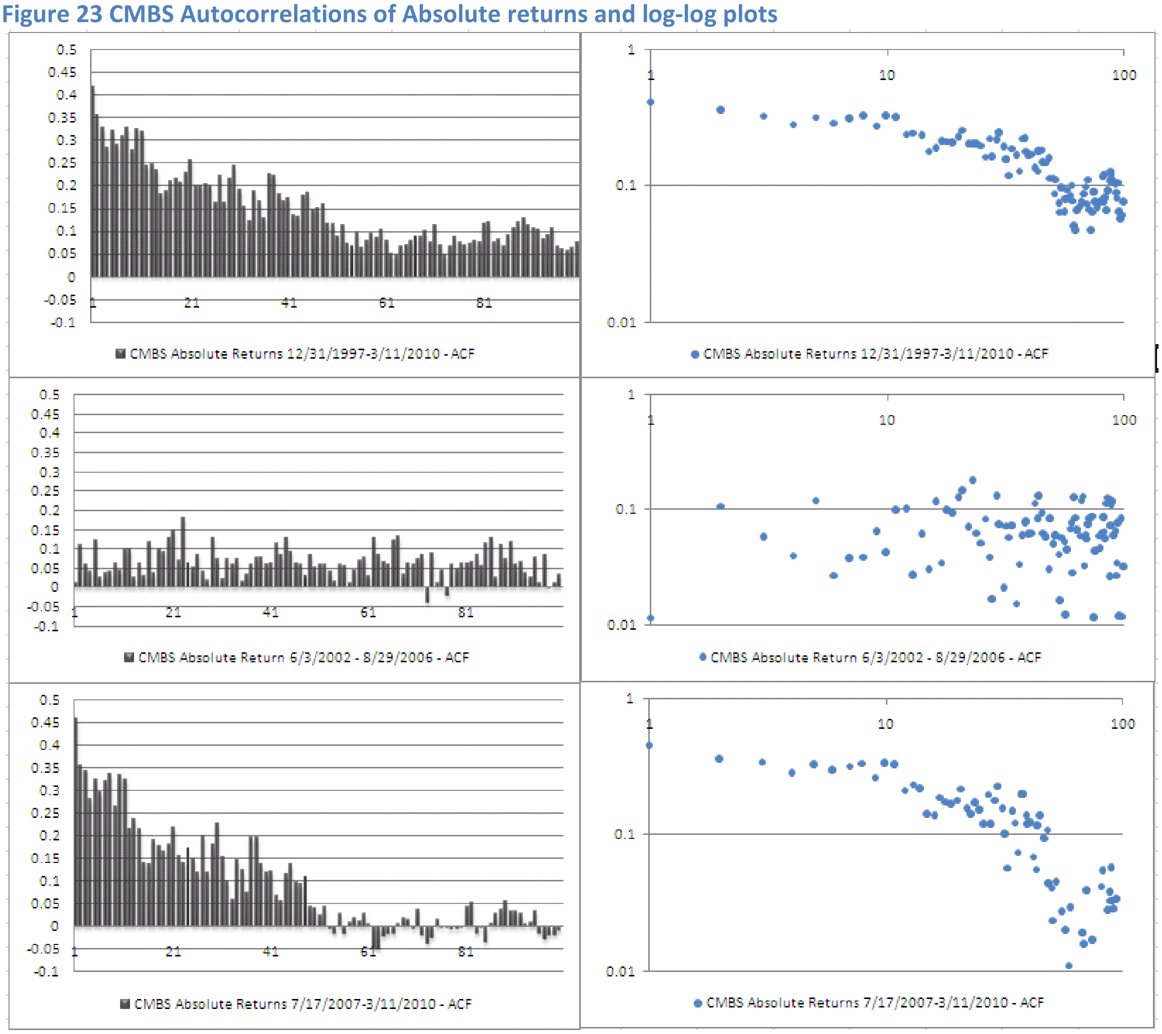

## 4.3.1.4 Slow decay of autocorrelation in absolute returns

"The autocorrelation function of absolute returns decays slowly as a function of the time lag, roughly as a power law with an exponent $\beta \in [0.2, 0.4]$". (Cont 2001)

In order to calculate the beta values, I have estimated the absolute values of the return index and following this autocorrelations are calculated. Then, beta exponent was calculated by regressing the log values of autocorrelations against the log values of lags. This is sometimes interpreted as a sign of long-range dependence.

The autocorrelations in absolute returns decay slowly as the literature indicates, except for the period before the crisis. In the figure below, the slow decay of autocorrelations were demonstrated by the values of autocorrelations per lag and their log-log plots.

**Figure 23 CMBS Autocorrelations of Absolute returns and log-log plots**



The β values for different values are presented in the following figure. Before the crisis it simply doesn't exists but after the crisis and in the long term the values are around (0.2-0.4) as the theory indicates. Hence, these results can be interpreted as an indicator of investor's unresponsiveness towards asset price changes. One possible explanation is as follows, normally, investors are eager to buy when asset prices decrease and vice versa, and this causes a slow decay of autocorrelations in absolute returns. But this behaviour can disappear sometimes for instance during bubbles.

Figure 24 CMBS Beta Values of Absolute Return ACF

| CMBS | 12/31/1997-3/11/2010 | 6/3/2002 - 8/29/2006 | 7/13/2007-3/11/2010 |
|---|---|---|---|
| β coefficient | 0.297824766 | -0.140311605 | 0.495818551 |
| p-value | [0.0000] | [0.1556] | [0.0000] |
| R square-adj | 71.40% | 2.16% | 62.18% |

#### 4.3.1.5 Leverage effect

It is said that, "most measures of volatility of an asset are negatively correlated with the returns of that asset." (Cont 2001)

In the following table you can see that volatility and return are negatively correlated during crisis and in the long run. However, there is no relationship before the crisis.

Figure 25 CMBS Volatility - Return Correlation

| CMBS | 12/31/1997-3/11/2010 | 6/3/2002 - 8/29/2006 | 7/13/2007-3/11/2010 |
|---|---|---|---|
| corr(volatility,ret) | -0.35298843 | -0.02752167 | -0.42391577 |
| p-value | 0.000000 | 0.36030 | 0.000000 |

### 4.3.2 S&P 500

I have also conducted the same study on another asset price index, S&P 500. As expected the results are quite similar to the ones we had seen in CMBS index. I have included the previous dotcom crisis which happened after 3/24/2000 and divided the time before the subprime mortgage crisis into two time periods 3/14/03-10/22/04 and 10/29/04-7/13/07.

Figure 26 Time Plot of S&P 500


#### 4.3.2.1 Kurtosis

Excess kurtosis values S&P 500 are higher in the first and last columns.

Figure 27 S&P 500 Data Description & Kurtosis

| | 1/6/97-3/5/10 | 10/9/98-3/24/00 | 3/24/00-3/14/03 | 3/14/03-10/22/04 | 10/29/04-7/13/07 | 7/13/07-3/5/10 |
|---|---|---|---|---|---|---|
| Mean | 0.00012683 | 0.0012636 | -0.00081333 | 0.00067684 | 0.00047047 | -0.00046009 |
| Median | 0.00067803 | 0.0013555 | -0.0014219 | 0.0012549 | 0.00080497 | 0.00089762 |
| Minimum | -0.094695 | -0.039099 | -0.060045 | -0.035867 | -0.035343 | -0.094695 |
| Maximum | 0.10957 | 0.046546 | 0.055744 | 0.034814 | 0.021336 | 0.10957 |
| St deviation | 0.013515 | 0.012247 | 0.01455 | 0.0084663 | 0.0065393 | 0.020001 |
| C.V. | 106.56 | 9.6922 | 17.889 | 12.509 | 13.9 | 43.471 |
| Skewness | -0.17225 | 0.027372 | 0.22309 | 0.013338 | -0.26742 | -0.14022 |
| Ex. Kurtosis | 7.2792 | 0.56312 | 1.1211 | 1.1352 | 1.6062 | 5.1424 |

#### 4.3.2.2 Absence of Autocorrelations

This is not very obvious but the last and the first columns have significant values in the first two/three lags.

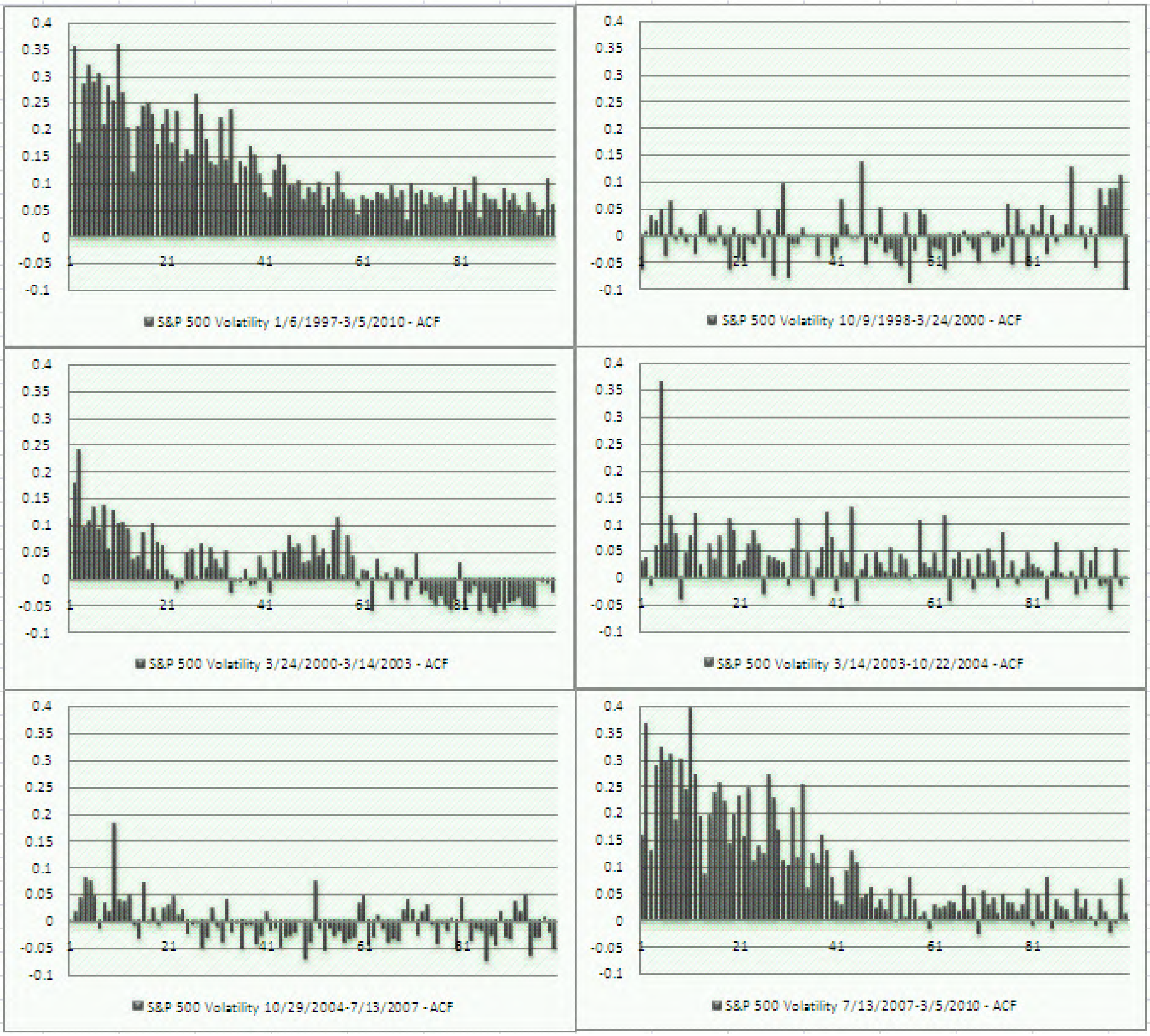
Figure 28 S&P 500 Autocorrelations

| S&P 500 - ACF | 1/6/97-3/5/10 | 10/9/98-3/24/00 | 3/24/00-3/14/03 | 3/14/03-10/22/04 | 10/29/04-7/13/07 | 7/13/07-3/5/10 |
|---|---|---|---|---|---|---|
| 1 | -0.0736 | 0.0207 | -0.0149 | -0.1224 | -0.0491 | -0.1394 |
| 2 | -0.0669 | 0.0219 | -0.0531 | 0.0032 | -0.0723 | -0.1097 |
| 3 | 0.0291 | -0.1035 | -0.0158 | 0.0109 | 0.0183 | 0.0943 |
| 4 | -0.0107 | 0.0123 | 0.0008 | 0.0216 | -0.0093 | -0.0342 |
| 5 | -0.0347 | -0.0376 | -0.0386 | -0.0813 | -0.0597 | -0.0214 |
| 6 | 0.0059 | 0.0172 | -0.0412 | -0.0199 | -0.0404 | 0.0354 |
| 7 | -0.0423 | -0.0826 | -0.0053 | -0.077 | -0.0265 | -0.0449 |
| 8 | 0.0235 | 0.0024 | 0.0116 | 0.0304 | -0.0321 | 0.04 |
| 9 | 0.0023 | -0.0397 | 0.0092 | 0.0024 | -0.0128 | -0.0044 |
| 10 | 0.0315 | 0.0676 | -0.0185 | -0.0805 | 0.0691 | 0.0428 |

#### 4.3.2.3 Volatility Clustering

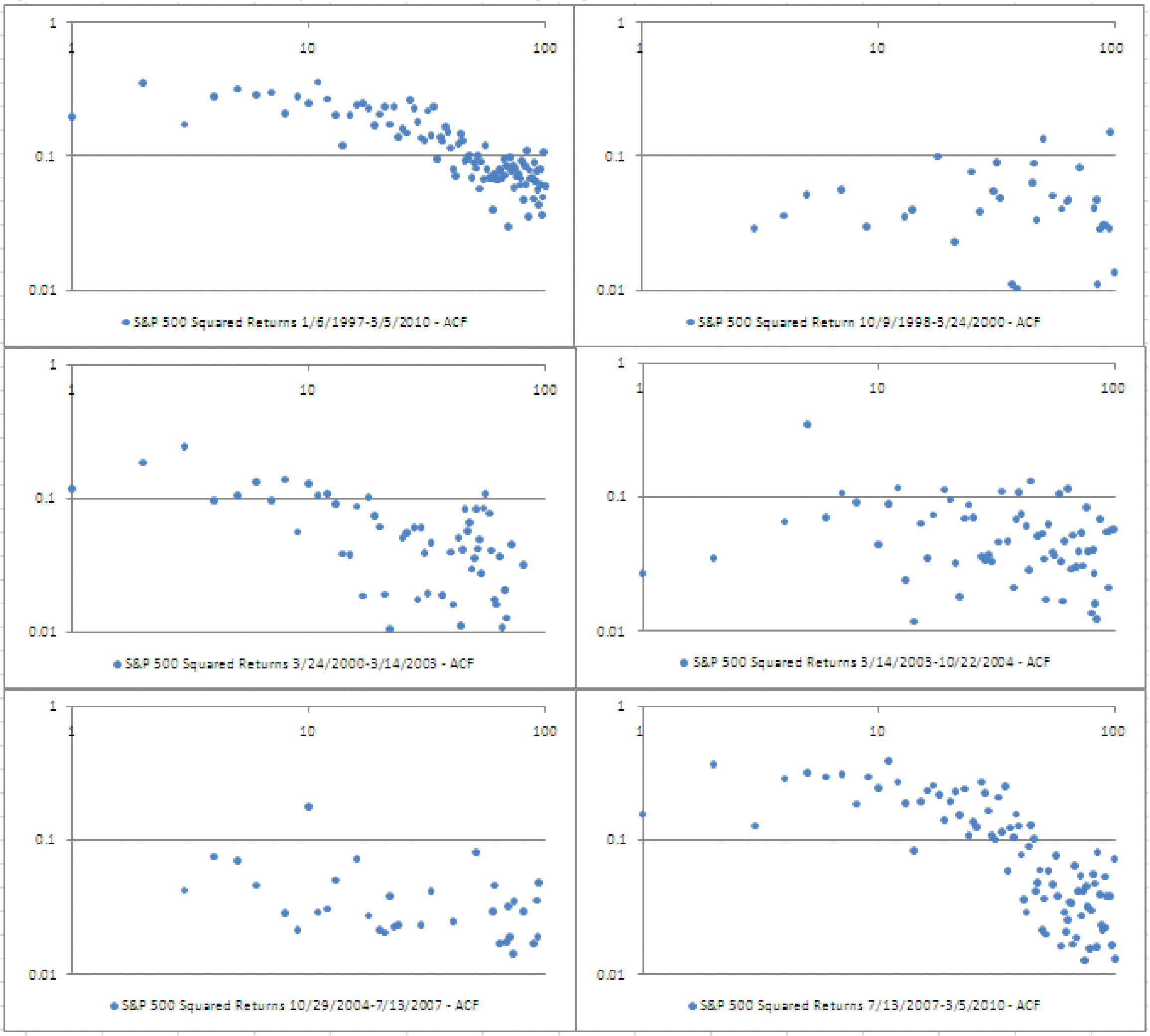
We can see lower volatility clustering prior to the crisis. The volatility clusterings are slightly more obvious in the period between two crises. Particularly just before the sub-prime mortgage crisis they almost disappear completely.

Figure 29 S&P 500 Volatility Autocorrelations




This can be explained as; investors did not respond to any volatility event extremely, for instance after a high volatility day with a strong increase, there was a more silent day without significant increase or decrease. The non-existence of slow decayance of volatility autocorrelations can be seen below in log-log pots. Please note that in log-log plots the negative values are not visible.

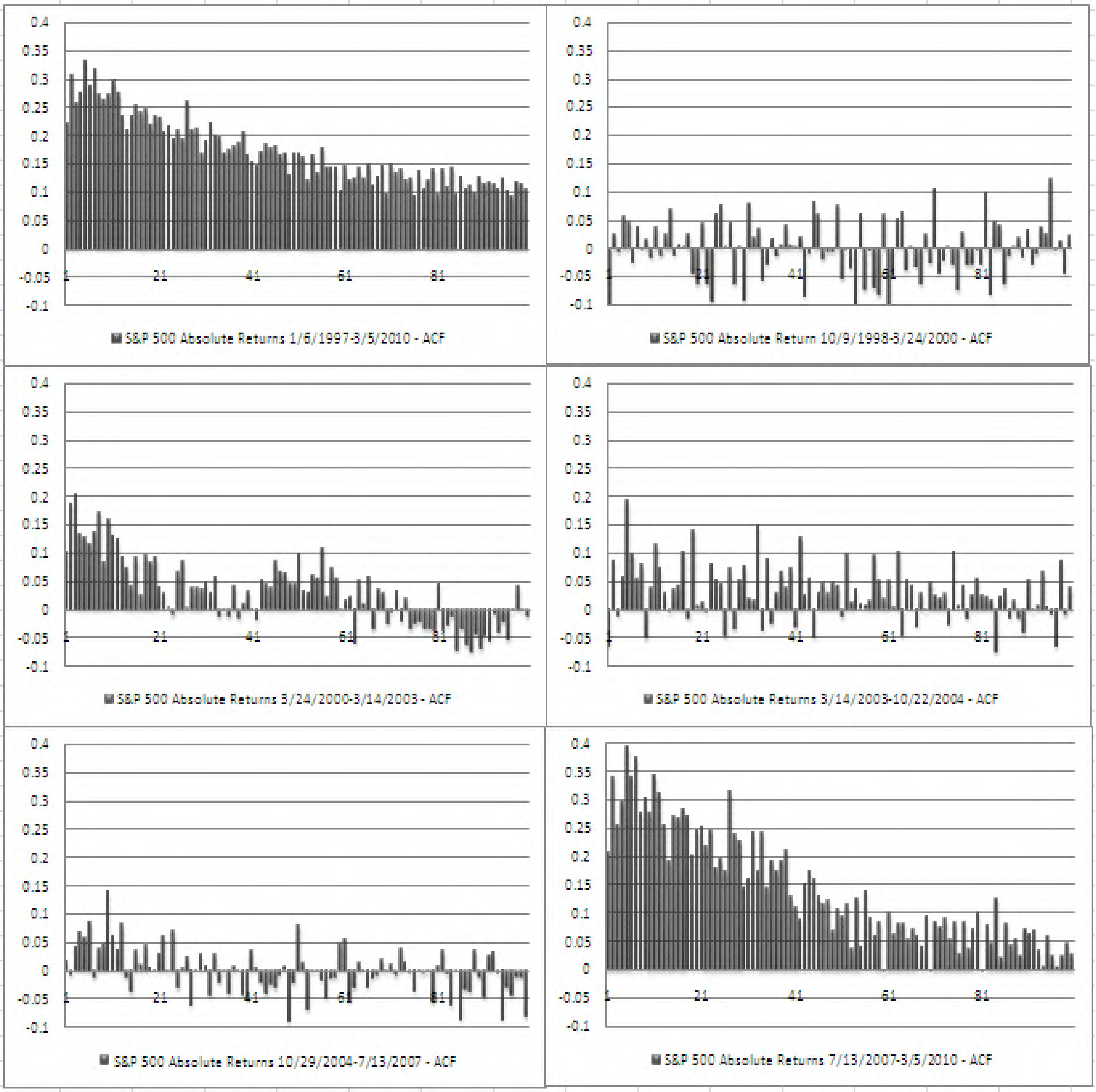
Figure 30: S&P 500 Volatility Autocorrelations Log-Log Plots



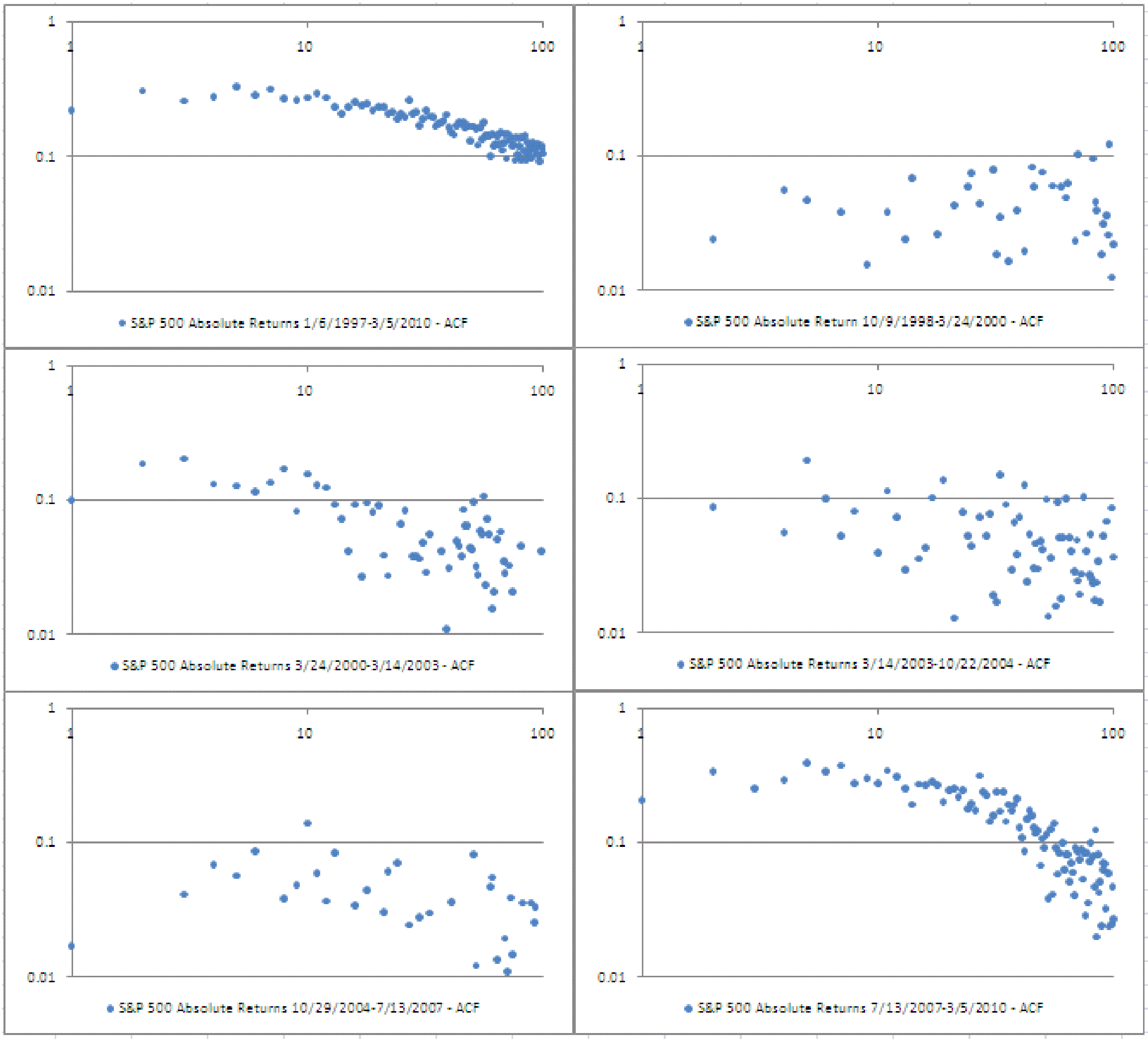

### 4.3.2.4 Slow decay of Autocorrelations in Absolute Returns

The results are similar to the volatility clustering. Particularly just before the crisis autocorrelations disappear again.

**Figure 31 S&P 500 Absolute Return Autocorrelations**




Below you can see the log-log plot of the autocorrelations. The autocorrelations after the crisis is significantly different the bubble periods.




In the table below we can see the beta values for the ACF results. I have calculated the Beta values of these autocorrelations by regressing absolute log values of autocorrelation coefficients against log values of lag numbers. In general, it is said that financial time series exhibit a beta coefficient around 0.2 to 0.4In the long term and after the crisis and the dates between two crisis have more significant beta values than the other dates (prior to crisis). This can be regarded as another indication of positive feedback loop.

**Figure 32 S&P 500 Absolute Return Beta Values**

| | 1/6/97-3/5/10 | 10/9/98-3/24/00 | 3/24/00-3/14/03 | 3/14/03-10/22/04 | 10/29/04-7/13/07 | 7/13/07-3/5/10 |
|---|---|---|---|---|---|---|
| β coefficient | 0.185723079 | 0.251088791 | 0.660224246 | 0.246474118 | 0.574111663 | 0.289209302 |
| p-value | [0.0000] | [0.4399] | [0.0005] | [0.0973] | [0.0190] | [0.0000] |
| R square-adj | 57.08% | 2.23% | 23.26% | 4.88% | 15.72% | 42.26% |

#### 4.3.2.5 Volume-Return Relationship

The results differ from the CMBS results. Volume and return had no significant correlation in any period.

**Figure 33 S&P 500 Volume-Return Correlations**

| | 1/6/97-3/5/10 | 10/9/98-3/24/00 | 3/24/00-3/14/03 | 3/14/03-10/22/04 | 10/29/04-7/13/07 | 7/13/07-3/5/10 |
|---|---|---|---|---|---|---|
| corr(Volume,ret) | -0.01880275 | 0.06617833 | 0.02784206 | -0.00982768 | -0.01062314 | -0.0016453 |
| p-value | 0.2793 | 0.2053 | 0.448 | 0.8433 | 0.7822 | 0.9662 |

#### 4.3.2.6 Volatility-Return Relationship

The results do not provide meaningful interpretation. We can also not say conclusively that there is no negative correlation prior to crisis.

**Figure 34 S&P 500 Volatility-Return Correlations**

| | 1/6/97-3/5/10 | 10/9/98-3/24/00 | 3/24/00-3/14/03 | 3/14/03-10/22/04 | 10/29/04-7/13/07 | 7/13/07-3/5/10 |
|---|---|---|---|---|---|---|
| corr(volatility,ret) | -0.05654652 | 0.01709685 | 0.12627467 | 0.00753259 | -0.14082141 | -0.05246634 |
| p-value | 0.0011 | 0.7438 | 0.0006 | 0.8796 | 0.0002 | 0.1759 |

### 4.4 Conclusion

In the section above, I showed that return autocorrelations are very weak and kurtosis is lower before the crisis. I've demonstrated that volatility events did not cluster before the crisis. In addition the autocorrelations of absolute returns did not decay slowly, more importantly; they did not exist before the crisis. Lastly, I also presented that volatility-return relation do not exist in CMBS but it exists for S&P 500.

The results signify that before the crisis investor behaviour was different than the one after the crisis or any other time. The return distribution was different and autocorrelations diminished. The return volatility relationship disappeared together with volatility clusters and slow decay of autocorrelations in absolute returns. In a nutshell, this indicates that during the bubble periods, particularly in the 2007 sub-prime mortgage bubble, investors did not respond to bad news, price changes and volatility events as they responded in other times.

For instance, a price decrease is usually followed by a decrease, due to investors' adjustment and vice-versa. However, when there is an irrational exuberance, an increase was not followed by a decrease; instead the prices remained the same. I suggest that together with other indicators these stylized fact differences can be used as framework to detect these bubble periods.

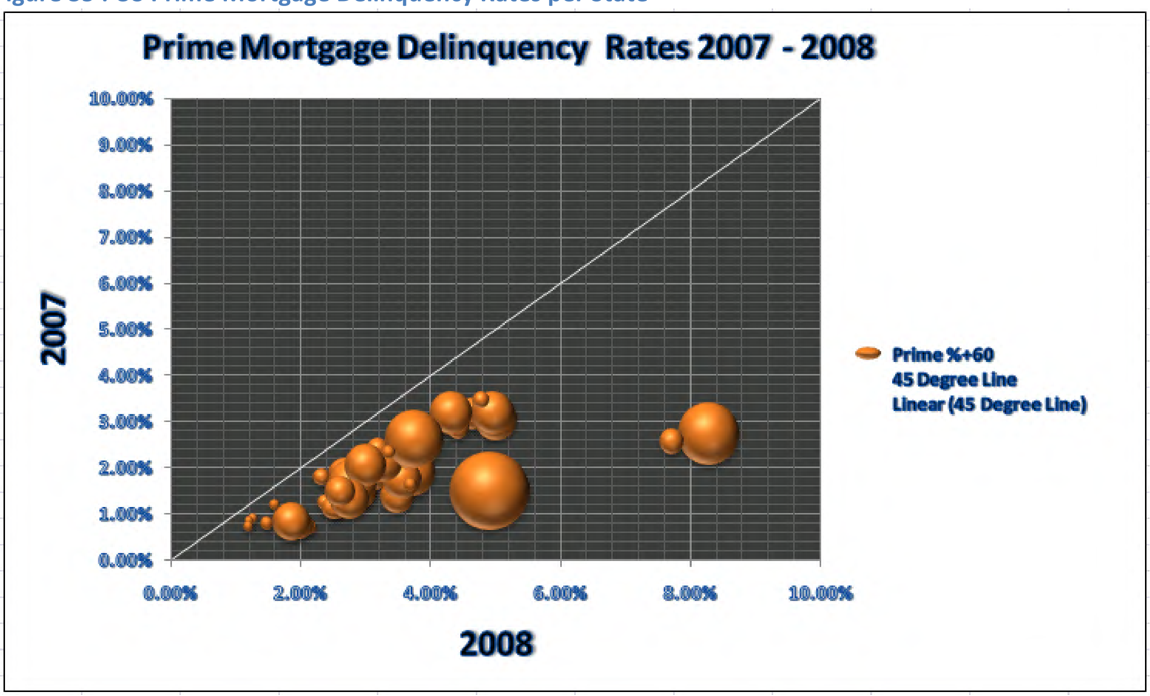
# 5.0 Analysis on Burst of Bubble: Mortgage Defaults

Burst is the destiny of the bubble, the moment demand becomes weaker, the money illusion spell and the positive feedback loop are broken. In this chapter,

## 5.1 Literature Review

In the following figures you will see the comparison of mortgage delinquency rates of 2007 and 2008 per US states. The volume of the bubbles represents the population of the state. W can see that, in all states %+60 day delinquency rates were increased significantly.

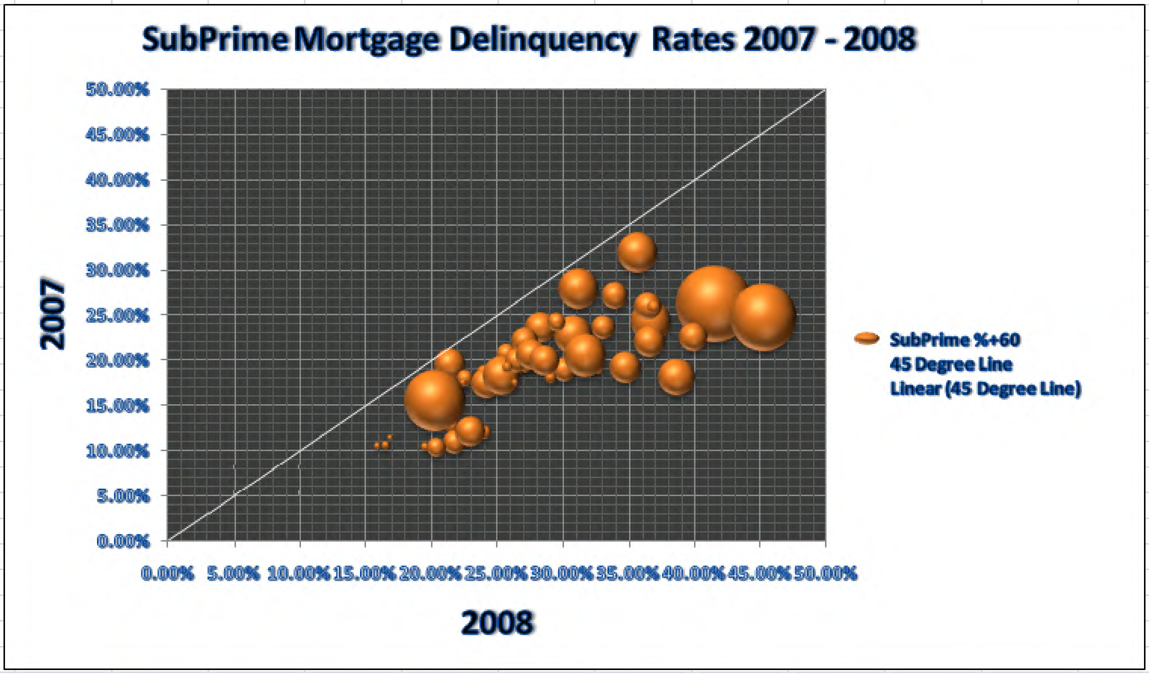
Figure 35 : US Prime Mortgage Delinquency Rates per State




Source: LoanPerformance

In the figure below, we can also see that for sub-prime mortgages the increase was even stronger. There was simply no state where delinquency rates were not increased.

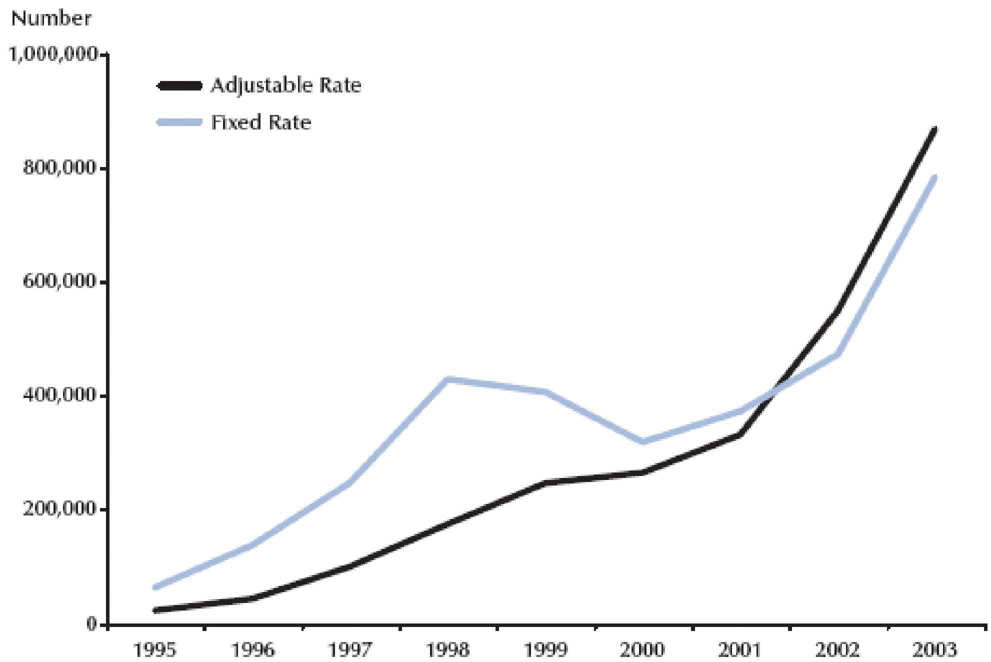
Figure 36 : US Sub-prime Mortgage Delinquency Rates per State




Source: LoanPerformance

Researchers were encouraged by these sky-rocket rises in subprime mortgage delinquency rates to decompose the factors that caused this problem. Considerable amount of researchers speculated that relaxed lending standards of mortgage issuers, for instance underwriting mortgages with no documentation of income, high debt-to-income ratios (DTI) and with high loan-to-value (LTV) ratios was one of the main reasons of subprime crisis. In figure below, you can see that subprime mortgage lending increased dramatically after 1995.

Figure 37: Number of Loans Originated




Source: LoanPerformance subprime mortgage database

Figure 38 : Number of Loans Originated

Anyone can assume that this increase also caused a rise in the riskiness of subprime mortgage pools, since originators decreased their standards in order to originate more loans. The increase of riskiness in the subprime mortgage was indicated by many researchers including Danis and Cross.

Although it is not surprising to see an increase in delinquencies and foreclosures due to elevated riskiness, several researchers such as Doms, Furlong and Krainer have suggested the mentioned increases in the riskiness of subprime mortgage pool can't be the main reason of mortgage delinquency increase. Instead they- together with Cross 2006-, suggested that the general economic variables are more likely the reason of delinquencies

The other more important factors which contributed to the default rates were the economic conditions. For example, in a city with a higher unemployment rate, see figure 14, and stagnation in income growth; higher delinquency and foreclosure rates can be expected. Since unemployed people won't be able to pay their mortgage payments, they will be forced to sell their properties and prepay their mortgages. However, as you can see in figure 15, the demand for houses plummeted during the crisis. Naturally, the house prices were also decreasing, see figure 18, therefore capital gains was not also possible for homeowners. As a result, the mortgage borrowers exercised their options of default. But if they had increasing prices, then they would have more options such as refinancing or selling the house and prepaying the mortgage. Moreover, if they expect a house price increase in the future, they would try more to keep the mortgage and not to default. Additionally, house price decrease also lowered the future expectations on the possibility of a house price recovery. We would not be too wrong to expect that a market inconfidence caused by house price decreases also deepened the crisis. Collectively, all the mentioned developments visible in real estate prices constituted the primary culprit of the rapid increase in delinquency and default rates.

### 5.1.1 Default Correlation

Default correlation can be defined as the magnitude of dependence of different credit risks. Default correlation is one of the necessary factors in the calculation of the mortgage portfolio value together with recovery, prepayment and default rates. It represents the concept that systemic events such as macroeconomic variables trigger the default events to realize at the same time. To explain, default of borrowers might be caused by systemic underlying variables. In the context of residential mortgages, macroeconomic events such as, house price changes, interest rate and curve changes and changes in unemployment rate and disposable income might be among the common factors...

### 5.1.2 Default Correlation Modelling

We would not be too wrong to think that default correlation on residential portfolios is very low. However, when it comes to lower credit quality mortgages, comprehending default correlation is of critical importance among subprime mortgages. Generally speaking, credit quality of loans and significance of default correlation can be negatively correlated. There are already several researches on the correlation between the credit quality and default correlation of commercial portfolios. For instance, it has been demonstrated by Zhou (1997) that implied default correlations[21] are very low for companies with high credit ratings but they are very high for companies with lower credit ratings even for the very short run. Employing portfolios with bonds and loans of firms, Lucas et al. (2001) demonstrates that with a constant correlation level, a lower portfolio quality increases extreme credit loss quartiles increases when the credit quality of portfolio decreases. Similar to this one, Loffler (2003) indicates that the uncertainty of default correlation is a much more important factor for portfolios with B-rated in comparison to portfolios with BBB-rated when uncertainty is one percent Value at risk (VAR).

In the default probability literature most of the researches made are focusing on the individual firm level indicators to explain the probability of default. In this paper my focus is to test the relationship between the credit risk and the macroeconomic indicators. In recent years researchers have been investigating the probability of default through macroeconomic

[21] according to the Z-values (a risk score)

conditions. One of these studies is  Foote et al. (2008) where they show that negative equity is necessary but not a sufficient condition for default. In the event of an idiosyncratic (individual) shock such as unemployment, illness, divorce, borrowers with the positive equity can sell the house and refinance the mortgage, so they rarely default. While, Vandell (1995) also shows that under the same shocks borrower with negative equity may default as selling the house is dependent on being able to raise cash to cover the difference between mortgage balance and the cost of a sale or cost of getting a new mortgage. In the same paper, Vandell also shows that probability of default for the barrowers with negative equity are smaller that the simple models would predict.

Schwartz and Torous (1993) argue that prepayment option defines the default option regardless of the barrower's equity. Based on this argument, there have been models developed in the literature where they account for the value of the option to sell the house (refinance the mortgage). One of them is Deng et al. (2000), where they estimate a duration model which includes the competing risks of prepayment and default. Their model embraces the effect of the value of the mortgage backed security.

Other part of literature focuses on the effect of different macroeconomic shocks on the default rate. Helwege and Kleiman (1996) model growth in Gross Domestic Product (GDP) as a dummy variable to study the relationship between recession and the actual rate of default, where GDP is 1 when the growth is greater than 1.5% per year and 0 when it falls below that percentage. Friedson et al (1997) use the same model by including the effect of interest rate and test it empirically by using the data of corporate bond between the years

1971-1995. Their results show that as real interest rates increase, asset value decreases and as a result the estimate of probability of default increases. More general, they found that probability of default is correlated with macroeconomic conditions.

Out of this literature one of the recent studies, more relevant to my study is Qu (2008) where he uses the data of post financial crisis period (2000-2005), to investigate the relationship between business cycle and the probability of default. Their model is a multifactor systematic risk model. They use expected default frequency as a proxy for probability of default; calculated through their monthly data by using firm profile and market information. Their data includes six European countries, US and some other industrial countries but their analyses are mostly on Swedish data. In this paper their main focus is how much the probability of default can be influenced if certain macro factors increase or decrease. They also search for industry differences as well as country differences. They found that there is a correlation between the probability of default and the macroeconomic factors such as industrial production, interest rate spread, exchange rate and share price. They also conclude that these results are different for other countries. Lastly, they check the impact of these factors; based on the author's result; spread has the least impact while better the company, there is a less effect of macro changes on the probability of default.

Earlier researches emphasized that the significance of systemic events determining companies' credit risk in the stock market. Nickell et al. (2000), De Servigny & Renault (2002), Lucas & Monteiro (2005), Das et al. (2005), and Koopman, indicated the significant affect of common macroeconomic factors on default risk. Naturally, next question is should we expect default correlations in a mortgage portfolio. If it exists, what is the magnitude of correlation?

The objective of this chapter is to verify the relationship between macroeconomic conditions and the probability of default by using the Case-Shiller National Home Price Index for the period between April 1997 and November 2008. This chapter can answer questions like, what are the macro factors which affect the probability of default. And how much of the credit risk can be explained by these factors?

Even though earlier studies on default correlation relate with commercial loans and bonds, - since the main characteristics of non-investment grade bond portfolios and the subprime mortgage portfolios are similar - I expect to see that, the significance of default correlation is higher for lower quality mortgage portfolios.

## 5.2 Data and Methodology

In this part, the delinquency and the foreclosure rates of Prime, ALT_A and sub-prime mortgages are analyzed with the following base regression.

$$\%\Delta Dependent_t = \beta_0 + \beta_1 \%\Delta HPI_t + \beta_2 UnemploymentRate_t + \beta_3 DisposibleIncome_t + \varepsilon_t$$

The analysis is performed on monthly data for the period between 1997/04 until 2008/11. Case–Shiller National Home Price index was log differenced. Unemployment data is obtained from Bureau of Labour Statistics on all population with 25 years or older; I used this 25 year age limitation since people younger than 25 are not likely to enter mortgage contracts. Disposable Income was also obtained from Bureau of Labour Statistics and it was log differenced. The dependent variables are foreclosure (FC) rates, 90 day+ delinquency rates and 60 day+ delinquency rates of nation-wide Prime, Alt A, and sub-prime mortgage pools. They are obtained from LoanPerformance.com and the sole difference of these mortgage pools are their credit ratings. Initially I have tested other independent variables, CPI, real mortgage rates, house sales and number of new houses. Since none was significant, hence, they were all dropped.

Based on the White test and Durbin-Watson statistics, it's easy to conclude that there is autoregressive conditional heteroskedasticity. In a time series analysis, correct model specification in accordance with the economic theory is crucial in coping with autocorrelated error terms, therefore, practically the first resort to mitigate the autocorrelation problem is to include lags of the relevant variables. It's widely accepted that HAC (heteroskedasticity and autocorrelation consistent standard errors) estimation is last option in dealing with autocorrelation and heteroskedasticity. However, our case was also resistant to any kind of structural changes, log transformation, lagged variables. Therefore, for all the regressions I run, I had to use the HAC (heteroskedasticity and autocorrelation consistent) standard errors with bandwidth 3, Bartlett kernel, as Newey-West (1987) suggests. This estimation is an obviously an extension of White's (1980) heteroskedasticity-consistent covariance matrix (HCCME) estimator to the case of autocorrelated errors. The technical details of the estimation methodology are out of scope of this thesis.

## 5.3 Results and Findings

### 5.3.1 Prime Mortgages

Below is the table for the three regressions which run on house price index, unemployment and disposible income. HPI is significant in all cases, unemployment only significant for delinquency rates, and disposible income is not significant in any case. R square is sufficient enough for a meaningful analysis.

| | Prime_FC | Prime_90 | Prime_60 |
|---|---|---|---|
| Constant | 0.00149318 | -0.00242756 | 0.00166811 |
| | 0.6352 | 0.3907 | 0.8073 |
| House Price Index | -0.308504 | -0.255055 | -0.634338 |
| | 0.0001 | 0.0000 | 0.0001 |
| Unemployment | 0.00150954 | 0.00200915 | 0.00422897 |
| | 0.1098 | 0.0187 | 0.0372 |
| Disposible Income | -0.0662143 | -0.0665391 | -0.173141 |
| | 0.3115 | 0.1512 | 0.1963 |
| Adj R squared | 0.44357 | 0.58274 | 0.50068 |

In the table below I have dropped the insignificant variables and rerun the regression with restricted models.

| | Prime_FC | Prime_90 | Prime_60 |
|---|---|---|---|
| Constant | 0.00709422 | -0.00304482 | 6.19462E-05 |
| | 1.16E-10 | 0.3168 | 0.9933 |
| House Price Index | -0.313991 | -0.25889 | -0.644317 |
| | 0.0007 | 0.0001 | 0.0001 |
| Unemployment | | 0.00210347 | 0.00447441 |
| | | 0.0190 | 0.0359 |
| Disposible Income | | | |
| Adj R squared | 0.4055 | 0.57961 | 0.49815 |

The results indicate that prime mortgages are affected by the macroeconomic variables to a certain extent. It seems like house prices are the main independent variable, and unemployment is the secondary one. The signs are correct, higher unemployment increases the default and delinquency rates and decreases in house prices also increase default and delinquency rates.

### 5.3.2 ALT A Mortgage Pool

Alt A is an actually ill-defined mortgage pool which practically stays somewhere between the prime and sub-prime mortgages.

| | ALT_A_FC | ALT_A_90 | ALT_A_60 |
|---|---|---|---|
| **Constant** | -0.0220191 | -0.0163288 | -0.0393141 |
| | 0.0422 | 0.0366 | 0.0302 |
| **House Price Index** | -1.35193 | -0.834249 | -2.39062 |
| | 0.0000 | 0.0000 | 0.0000 |
| **Unemployment** | 0.0118483 | 0.00803857 | 0.0220027 |
| | 0.0003 | 0.0006 | 0.0000 |
| **Disposible Income** | -0.233736 | -0.365703 | -0.645246 |
| | 0.1151 | 0.1273 | 0.1099 |
| **Adj R squared** | 0.66374 | 0.66644 | 0.70618 |

The comments for prime mortgages are also valid for these regressions, house prices and unemployment are the primary variables and disposable income is not significant in any regression.

| | ALT_A_FC | ALT_A_90 | ALT_A_60 |
|---|---|---|---|
| **Constant** | -0.0184971 | -0.0254116 | -0.0452998 |
| | 0.0264 | 0.0272 | 0.018 |
| **House Price Index** | -0.84772 | -1.37301 | -2.42781 |
| | 0.0000 | 0.0000 | 0.0000 |
| **Unemployment** | 0.0083699 | 0.0123667 | 0.0229173 |
| | 0.0007 | 0.0003 | 0.0001 |
| **Disposible Income** | | | |
| **Adj R squared** | 0.66152 | 0.65902 | 0.70092 |

You can see above the restricted models where disposable income is dropped. The $R^2$ has increased in comparison with the prime mortgages as the credit rating of an Alt A mortgage is lower than a prime mortgage.

### 5.3.3 Sub-prime Mortgages

In the table below, I have run the three regressions on sub-prime mortgages. The $R^2$ has increased even more, indicating that sub-prime mortgages are proportionally less affected by individual factors. Signs have correct economic meanings again where an increase in disposable income decreases the default and delinquency rates.

| | SUB_FC | SUB_90 | SUB_60 |
|---|---|---|---|
| **Constant** | -0.0047477 | 0.00302049 | 0.00449536 |
| | -0.7147 | -0.8377 | 0.8591 |
| **House Price Index** | -2.04315 | -2.24629 | -4.44702 |
| | 0.0000 | 0.0000 | 0.0000 |
| **Unemployment** | 0.0165482 | 0.0134731 | 0.0341629 |
| | 0.0000 | 0.0015 | 0.0000 |
| **Disposible Income** | -0.575295 | -0.59143 | -0.0341629 |
| | 0.0941 | 0.0873 | 0.1040 |
| **Adj R squared** | 0.72138 | 0.72511 | 0.75254 |

It seems like sub-prime mortgage default risk is prone to the macroeconomic variables. In a nutshell, sub-prime mortgage default rates should be highly correlated with each other since we can explain most of the delinquency and the foreclosure rates by simple macroeconomic variables which affect the whole pool of sub-prime mortgages.

## 5.4 Conclusion

In this part, I attempted to show that mortgages credit risk is subject to macroeconomic risks. To preview the results, I have showed that house price decrease is main culprit of the delinquencies and defaults. Unemployment and disposable income also correlate with sub-prime mortgage defaults and delinquencies.

To explain in more detail, it is showed that when house prices decline sub-prime mortgage default risks increase, simply because people are now more willing to hand in their property keys to the creditors. The results point that, house price appreciation is the primary predictor of the delinquencies and defaults. It might be also due to psychological effects of house price decreases. For instance, when prices are stagnating or decreasing, homeowners might think that the price decreases will continue even more.

We can also expect rises in default rates when there is an increase in unemployment rates and a slowdown in disposable income growth rates. There can be one drawback to the model, since interest rate changes are not used. However, the dataset included both fixed rate and floating rate mortgages, and therefore it was impossible to use interest rate changes in the models. In addition house price index is sensitive to interest rates and the effect can be assumed to be included. It can be also expected that some of the default rate increase can be attributed to the rise in the riskiness of the mortgage portfolio. However, due to the lack of data, the magnitude of this effect was not tested.

To sum up, I have demonstrated that significant amount of credit risk can be explained by common macroeconomic factors; namely, house price index, unemployment and disposable income. Additionally, we can see clearly that the magnitude of the correlations depend on the R square of the regressions. Hence, the results are also supporting the hypothesis that a decrease in the credit rating increases the default correlation. Because a prime mortgage default is correlated with individual factors more than subprime mortgages.

Obviously the next question is how important are these default correlations for MBS products?

# 6.0 The impact of Correlated Default Probabilities on an MBS

## 6.1 Introduction

According to, Nagpal and Bahar (2001), the mathematical relationship between joint default probabilities and default probabilities can be called as the default correlation. They demonstrated that, the notion that credit events are correlated by an empirical analysis with historical default rates. Without taking into account the default correlation, credit loss distributions will not calculated correctly, especially the tails of the distribution will be underestimated. Hence, if the impacts of default correlations to the investment portfolios are not identified properly, the degree of risk and the adequate capital required to manage the estimated risk will be biased. Therefore, it is very probable that any ignorance of default correlations in the forming of credit risk models particularly for subprime portfolios - as explained in the previous chapter - would cause significant error in the financial model. In this part I will present impact of correlation on the default distribution.

Virtually credit risk impacts every financial contract, it also affects MBS. The default probabilities of prime mortgages, in normal situations are quite extraordinarily low according to loanperformance data; frequently they were less than 0.003%/year. Under normal circumstances, assuming no crisis, these probabilities of default in a Mortgage Backed Security are independent of each other, since defaults simply occur due to individual bad luck such as being unemployed, which in return causes delinquencies and foreclosures. As a result of this, since each probability is independent of each other, at any given time, the aggregate default probability of an MBS is really low. This is particularly valid for senior trenches which are not impacted from defaults of mortgages until all the capital of lower level junior trenches are exhausted. As explained the credit risks of MBS senior trenches are practically close to zero under normal conditions. Actually, this is also what the investors thought about MBS defaults earlier.

However, when house prices drop, especially during a crisis when real interest rates rise and unemployment rate increases; default risk probabilities also soar dramatically. As I demonstrated in the previous chapter, since these are the macroeconomic variables impacting virtually all sub-prime mortgages, it has been concluded that sub-prime mortgage defaults are correlated with each other. When a mortgage defaults, it’s more likely that the other one will default as well, since the reason of default was same. As a result, the default risk of these senior trenches, which under usual circumstances is practically zero, can begin to be different than zero. In this section I will try to demonstrate this fact, the impact of a default correlation on default distributions.

I start with the assumption that the underlying instruments of an MBS are correlated and identical. With the presence of correlation I will show that the high probability of multiple defaults in an MBS with fat-tailed distributions. This fact has particularly important consequences in the sub-prime mortgage crisis where we have seen high delinquency rates even in the senior tranches.

## 6.2 Specification

Default correlation is the notion that the likelihood of one debtor defaulting on its mortgage or any kind of debt is affected by whether or not another debtor/mortgagor has defaulted on its debts. Now let's define it with a simple example, if one company is the creditor of another: let's say, if Credit A defaults on its obligations, then it's more sensible to think that Credit B is also more likely to be unable to pay its own obligations if there is a positive default correlation. Simply, the default of one mortgage makes it more likely the other mortgage will default as well.

Now, let's define this more formally.

I assume that there are n number of assets of k number of defaults where $k \leq n$.

***Assumption (1)***: Each asset has default probability p.

***Assumption (2)***: Each pair of assets has default correlation ρ between them.

$p_j = E\,(\,x_j \mid x_1 = 1\,,\, x_2 = 1, \,\dots\,,\, x_{j-1} = 1)$ for $j = 1, \dots, n$

The probability that k out of k assets default is

$$\prod_{j=1}^{k} p_j = E\left[\prod_{j=1}^{k} x_j\right] \quad for\ k = 1, \dots, n \qquad (1)$$

***Assumption (3):*** The default correlation between asset j+1 and asset j+2 remains equal to ρ regardless of the number of known defaults among the other j assets.

$\rho = Corr\,(x_{j+1}, x_{j+2} \mid x_1 = 1, x_2 = 1, \dots, x_j = 1)$ for $j = 1, \dots, n$

$p_{j+1} = p_j + (1 - p_j)\,\rho$ for $j = 1, \dots, n-1$ (2)

And in closed form equivalent

$p_{j+1} = 1 - (1 - p)(1 - \rho)^{J-1}$

This increasing default probability when given other defaults is one aspect of the fatter tails of the Correlated Binomial distribution.

## 6.3 Calculation of Correlated Default Probabilities with Bernoulli Trials

In the literature there are three general methods to simulate correlated binomial distributions. All three imposes different consistent relations on the joint probability function. Namely, they are beta binomial distribution, mixed Bernoulli models and the last one, Moody's binomial distribution. Hisakado et al (2006) indicated that one can use these models in an exchangeable manner. I will be using the latter.

The probability calculation of k defaults out of n assets can be done with the formula.[22]

$$C(n,k)E\left[\left(\prod_{j=1}^{k} x_j\right)\left(\prod_{j=1}^{k}(1 - x_j)\right)\right] = C(n,k)\sum_{j=0}^{n-k}(-1)^j\,C(\mathrm{n-k,j})\prod_{i=1}^{j+k} p_i \qquad (3)$$

$$C(n,k) = \frac{n!}{(n-k)!\,k!} \qquad Combination\ function \qquad (4)$$

The probability of zero defaults and n survivals will be then

$$E\left[\left(\prod_{j=1}^{k}(1 - x_j)\right)\right] = 1 + \sum_{j=1}^{n}(-1)^j\,C(\mathrm{n,j})\prod_{i=1}^{j} p_i \qquad (5)$$

[22] However in VBA code I use a different algorithm

The details of calculation is given in appendices

## 6.4 Findings

In case of default correlation, the higher trenches in an MBS do not have zero default probability anymore. This fact, which can be noticed even with very low correlation rates, becomes very obvious when we increase the correlation rate. As we know correlated defaults constitute the particular phenomenon of the subprime mortgage crisis, namely the fat tails.

The following simulation provides the default correlation of multiple assets, i.e. mortgages. We can see better that the tails are becoming heavier. The simulation supposes that there are 10 identical assets with 5% default rate with three different correlation rates with each other 0%, 2.5% and 5%. As the correlation increases, the tails start to get fatter. Simply the probability of 0 defaults and 7 defaults start to increase simultaneously.

**Figure 39 : Correlated Default Probabilities with three different low level correlations**

| | Correlation | | |
|---|---|---|---|
| Defaults | **0** | **0.025** | **0.05** |
| 0 | 0.5987 | 0.6307 | 0.6570 |
| 1 | 0.3151 | 0.2674 | 0.2319 |
| 2 | 0.0746 | 0.0786 | 0.0774 |
| 3 | 0.0105 | 0.0188 | 0.0242 |
| 4 | 0.0010 | 0.0038 | 0.0070 |
| 5 | 0.0001 | 0.0007 | 0.0018 |
| 6 | 0.0000 | 0.0001 | 0.0004 |
| 7 | 0.0000 | 0.0000 | 0.0001 |
| 8 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 0.0000 | 0.0000 | 0.0000 |

For visual demonstration, I used a case with 50 assets, 10% average default probability. I employed five different correlations from 0 to 0.5. The probability distributions for different level of correlations are below.

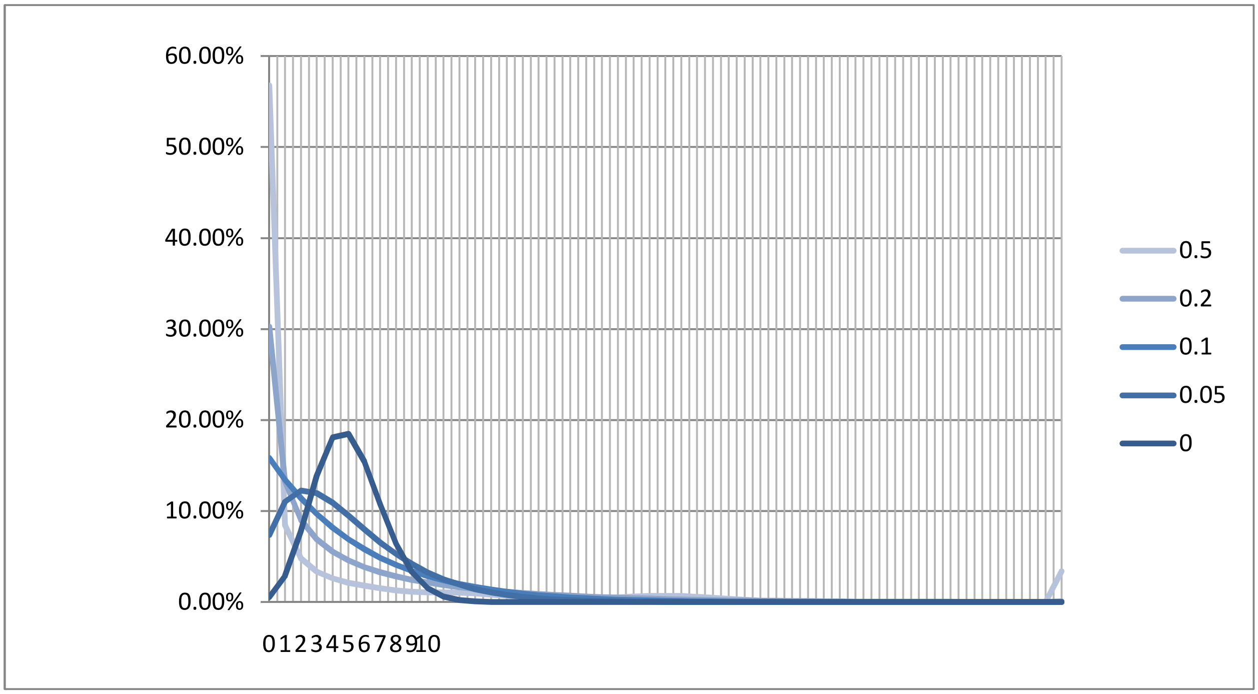



Figure 40 Correlated default probabilities with 5 different correlation levels

As, the distribution moves away from the dark blue to the light blue one, the peak in the right and left tails become visible. Now if the default correlation is 50% with 10% average default rate the default of all 50 assets at the same time is slightly higher than 3%.

To summarize, correlated defaults changes the default distribution, the tails get fatter, thus one should worry if there is high default correlation. These findings imply that default risk correlations are fundamental and one should take into account them in default risk modelling if there is a possibility of default correlation. Therefore any financial model which does not take into account the default correlation would be invalid and produce unexpected results.

# 7.0 Conclusion

This thesis investigated the time series characteristics of the 2007 subprime mortgage bubble and analyzed the eventual consequences of the burst of this bubble.

Price increases always get investor attention. In the market not all the investors work out the figures to find the intrinsic value of the asset, even if they are interested in the real value of the house or stock they frequently deviate from their initial perception when they are under the affect of other investors, competitors and peers. For some investors increasing prices (itself) is the indicator of purchase. They might invest based on this indicator and eventually these additional investments further increase the value of the asset thus accomplishing a full positive feedback loop. Kindleberger (1987) defines the asset price bubble as following;

> *"a sharp increase in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers - generally speculators interested in profits from trading in the asset rather than its use or earning capacity"*.

In dynamic systems the equilibrium value changes continuously, resulting in price volatility. Normally, when there is a price increase, self-adjustment mechanisms come into the play and the price decreases since there are more investors prepared to sell and fewer prepared to buy at the inflated price. This negative feedback limits the increase or decrease and puts restrictions on bubbles. As a result, some stylized facts appear in financial time series such as, slow decayance of absolute return autocorrelations and volatility autocorrelations. However, like we had seen in the 2007 subprime mortgage bubble, the asset prices and the whole economy might enter into positive feedback loop, a state of disequilibrium of increasing prices in all markets. Consequently, in case of a financial bubble, partly due to positive feedback loop, some of these well observed characteristics may disappear temporarily. In these cases we might encounter no volatility clustering, absence of autocorrelations, fast decayance of absolute return autocorrelations, no volatility-return relationship, and more normal distribution than the previous heavy tails. These all were observed in CMBS and S&P 500 indices where asset prices had risen far beyond the levels of rational intrinsic values. Consequently, as explained in the earlier chapters, these stylized facts then can be used to identify the irrational exuberance periods.

And like every bubble, the prices increased only to fall rapidly afterwards. The house prices decreased, unemployment increased and disposable income stagnated simultaneously. Under these circumstances, we can also expect that the probability of mortgage defaults increases. In addition to this, these macroeconomic changes also collectively indicate one more important element which was overlooked before this crisis, the default correlation. Since all these three independent factors can explain the significant amount of default probability, we can conclude that the defaults of these mortgages are correlated to a certain extent even though we can't calculate exactly the degree of correlation.

The results also showed that default correlation might increase with decrease of mortgage quality. For instance, the default of sub-prime mortgage is expected to be much more correlated than the default of a prime mortgage, since its default is affected more by the macroeconomic factors rather than individual factors.

Lastly, I showed that when the defaults are correlated the default distributions are altered, the tails become fatter. Hence, then it becomes clear that higher tranches of subprime MBS products can also suffer from defaults when there is high correlation among the individual mortgages

# 8.0 References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# 9.0 Appendices

## 9.1 Computational Algorithm of Correlated Defaults

There are several reasons for the need of a high precision calculation, firstly, any round up, at each iterative process, causes an ill condition. Particularly the multiplication of small numbers complicates the matters even further. Consequently, the mathematical operations with very small numbers require utmost precision. In order to alleviate this problem high precision arithmetic code is used. The detailed comments on the code can be seen inside the following code.

The top level routine is based on the following cases.

$p_j = E( x_j \mid x_1 = 1 , x_2 = 1, \ldots , x_{j-1} = 1)$ for $j = 1, \ldots, n$

### 9.1.1 Two asset case

$p_1 = p$ whereas $p_2 = p + (1-p)\rho$

Then for $P(x_1=1 \text{ and } x_2=1) = p(p + (1-p)\rho)$

I know that $P(x_1=1 \text{ and } x_2=1) + P(x_1=1 \text{ and } x_2=0) = P(x_1=1)$

Therefore, I also know that probability of one default and one survival is

$P(x_1=1 \text{ and } x_2=0) = P(x_1=1) - P(x_1=1 \text{ and } x_2=1)$.

Naturally, the probability of no defaults out of two assets is

$P(x_1=0 \text{ and } x_2=0) = P(x_1=0) - P(x_1=0 \text{ and } x_2=1)$

As the assets are identical the probabilities of one survival and one default is the same

$P(x_1=1\ x_2=0) = P(x_1=0\ x_2=1)$ Hence,

$P(x_1=0\ x_2=0) = P(x_1=0) - P(x_1=1\ x_2=0)$

$P_{j,n}$ is representing the probability of j defaults of n assets only in one specific order.

For example, $P_{1,2} = P(x_1=1 \text{ and } x_2=0) = P(x_1=0 \text{ and } x_2=1)$. Similarly,

$P_{1,2} = P_{1,1} - P_{2,2}$ and $P_{0,2} = P_{0,1} - P_{1,2}$

### 9.1.2 A.2. For general case

$p_{j+1} = p_j + (1 - p_j)\rho$ for $j = 1, \ldots, n-1$

$P(x_1=1, x_2=1, \ldots x_n=1) = \prod_{j=1}^{k} p_j$

$P_{j,n}$ is representing the probability of j defaults of n assets only in one specific order.

$P_{k,k} = \prod_{j=1}^{k} p_j$

$P_{j-1,k} = P_{j-1,k-1} - P_{j,k}$

$P_{1,1}$ and $p_1$ are known. Other values of $P_{j,k}$ will be found from the following[23]

$P_{0,1} = 1 - P_{1,1}$

For k = 2 to n

$p_k = p_{k-1}(1 - \rho) + \rho$

$P_{k,k} = p_k \cdot P_{k-1,k-1}$

For j = k-1 to 0 step -1

$P_{j,k} = P_{j,k-1} - P_{j+1,k}$

Next j

Next k

In order to get the probabilities with any order the results need to be multiplied with

$$C(j,k) = \frac{n!}{(n-j)!\,j!} \qquad \textit{Combination function}$$

### 9.1.3 Code

The code is in the excel sheet provided

[23] These are very similar to the top level algorithm