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A Study of Be Stars' Atmospheres

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I hereby confirm that this thesis is a result of my own independent work, with the use of the cited literature exclusively. I agree with the loaning of the thesis.

In Prague, 23^{th} Dec, 2005

Júlia Sokolovičová

Here I would like to thank my supervisor for his neverending patience and equally vast supply of literature, as well as the staff at the Astronomical Institute in Ondřejov who provided me with much needed advice and personal space, thanks to which I was able to put this thesis to paper.

My further gratitude goes to my parents for their, though long-distance, support in everything I do, and to all of my dear friends who graciously put up with me during this time - two times at that.

Contents

Title: A Study of Be Stars' Atmospheres
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Abstract.
Be stars are a group of hot, massive, emission-line stars characterized by intense radiatively-driven stellar wind and extremely rapid rotation. The typical emission in the Balmer series arises in a thin equatorial disc around the star. The detailed structure and most of all origin of the disc are still unknown. The aim of this thesis is to give a thorough review of the main directions of research into the field. We describe observational features that set Be stars apart from the rest of the B spectral class: Balmer emission, linear polarization, infrared excess and superionized resonance lines in the far UV range; also main physical characteristics of the underlying star, and variability. We discuss the so-called Be phenomenon - the unknown mechanism creating the emission-producing circumstellar disc in Be stars, and providine an overview of the most successful models attempting to explain the Be phenomenon. Finally we discuss the rotation law of the disc and its importance in placing a physical constraint on possible models for Be stars, and apply one of the examined rotation parameter finding methods.

Keywords: stars: emission-line, Be; circumstellar matter; rotation

Název práce: Studium atmosfér Be hvězd.
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Abstrakt.
Be hvězdy jsou skupinou horkých, hmotný hvězd s emisí ve spektru, které se vyznačují intenzivním, zářením hnaným hvězdným větrem a velice rychlou rotací. Jejich typická emise v Balmerově sérii vzniká v tenkém ekvatoriálním disku kolem hvězdy. Přesna struktura a především pøuvod disku jsou stále neznámé. Cílem této práce je podat døukladný přehled o hlavních směrech výzkumu v této oblasti. Popíšeme pozorované charakteristiky, které odlišují Be hvězdy od zbytku spektralní třídy B, jako je emise v čarách Balmerovy série, lineární polarizace, infračervený exces a superionizované rezonanční čáry v daleké ultrafialové oblasti; fyzikální charakteristiky samotné hvězdy a proměnnost. Rozebereme tzv. Be fenomén - neznámý mechanismus, který kolem Be hvězd vytvaří disk produkující emisi, a podáme souhrn nejúspěšnějších modeløu. Na zavěr prodiskutujeme rotační zákon disku a jeho význam jako fyzikálního omezení modeløu Be hvězd,a aplikujeme jednu z metod jeho zjištění v praxi.

Chapter 1

Introduction into the Be Problem

1.1 A Short History

The Be phenomenon has first been reported by Padre Angelo Secchi in 1867. He noted that the spectrum of γ Cassiopei displayed a bright emission line in the hydrogen line of Hβ, where only dark absorption lines should have been. Today we know that the absorption in the spectrum of a star arises in it's atmosphere, a gaseous envelope around the star. Thus the emission hinted at a peculiarity further out *around* the star and beyond the atmosphere.

In the first half of the 20th century, since 1913, great progress in the study of these B-class emission-line stars has been made by Paul W. Merrill. His observations resulted in hundreds of newly discovered Be stars, as well as the Mount Wilson Catalog of Be stars that he coauthored.

The first theoretical explanation of the Be phenomenon, one that in its basics remains valid to the present day, was attempted by Otto Struve in 1931 [1]:

> *" The suggestion is now offered that rapidly rotating single stars of spectral class B are unstable, and form lens-shaped bodies which eject matter at the equator, thus forming a nebulous ring which revolves around the star and gives rise to emission lines."*

1.2 Definition

Be stars are dwarfs and sub-giants of the spectral class B that exhibit, or at sometime in their observational history have exhibited emission superimposed over absorption lines in their spectra, most commonly in the H_α line of the

Balmer series. They can be single stars, or members of multiple systems. Be stars fall under the broader class of emission-line stars that stretches across the O, B and A spectral classes. Other distinct types of stars with emission found in their spectra comparable to, and sometimes classified as Be stars are Herbig Ae/Be stars, or HAEBE (pre-main-sequence objects associated with a nebula), and Algol systems (mass-transfering binaries).

We classify a Be star immediately as soon as an emission has been reported. This is because the emission may completely disappear and then reappear in the course of years or even dozens of years, and this event seems to be quite a common occurrence. The generally accepted idea is that the emission is caused by the presence of a circumstellar envelope that may be either ellipsoidal or have the form of a thin disc. The latter seems to be most probable, on observational and theoretical grounds.

We exclude supergiants from the definition above since in them the emission may be caused by a different mechanism in the sense that they already would have an envelope due to their evolutionary status, capable of causing the observed emission. B-type supergiants with emission belong to the B[e] group, which includes also pre-main sequence and main-sequence objects, and protoplanetary nebulae . Aside from emission they typically display an array of forbidden low-excitation absorption lines, which are thought to be caused by the star rotating very close to the stability threshold.

1.3 Observational and Physical Characteristics

What are the differences between the spectrum of a 'normal' B-class star and a Be star?

First of all, stellar flux and photospheric absorption indicate effective temperature ($T_{eff} \sim$ 10000 to 30000 K), mass (M $\sim$ 3 to 20 $M_{\odot}$), surface gravity and abundance of a standard main-sequence B star, rotating at a velocity generally above the average of the rest of the spectral class.

1.3.1 Emission

Emission in one or more hydrogen lines in the optical range is what identifies a Be star, but it may also be present in the lines of neutral He, and sometimes Fe II, Mg II and Si II. A typical emission in Hα is double-peaked, with the violet and red peaks not necessarily symmetrical.

According to how the double-peaked lines appear we may divide them into emission lines, in which the emission is greater than photospheric absorption (thus the central absorption does not dip below the continuum), and shell lines, in which the absorption is greater (thus it dips below the continuum).

Such sorting, however, is merely formal (e.g. [3]), as it is important to note that the emission we observe is merely apparent and not an actual physical occurrence (in the sense that the envelope is a source of radiation itself) , and therefore is dependent on the inclination angle i (the angle between the stars rotational axis and the observer's line of sight), as depicted in Figure 1.1. From this point of view, we could interpret emission-lines and shell-lines defined as above as lines from an envelope we are viewing nearly pole-on and equator-on respectively. This explanation comes originally from Struve [1], as well.

Figure 1.1: Production of emission lines in a rotating system of a star with an equatorial gaseous extended envelope. (Slettebak, 1988 [3])

So-called shell stars (e.g. 48 Lib) are a Be subclass that in addition to displaying shell-lines as defined above also show narrow absorption cores and very broad photospheric absorption lines (broader than classical Be stars). On the other hand, interferometric measurements of Quirrenbach et al. [6] have shown that while γ Cas and η Tau have been classified as shell stars, they are not viewed equator-on. This would mean that the so-called shell stars are *not* simply classical Be stars viewed at large angles, but actually a subclass that is physically different. The observed transitions between a classical Be phase and Be-shell phase in some Be stars confirms this conclusion.

1.3.2 Infrared excess

Infrared excess in a star's continuum is emission in the infrared region greater than what one would expect from a blackbody. Usually it indicates the presence of dust particles in the star's atmosphere, heated up by absorbing radiation of shorter wavelengths. In Be stars, however, it has been observationally supported and concluded [7] that it comes rather from free-free and free-bound emission in a disc-like region of plasma. According to Waters [8] this emitting region is ionized, extended up to $10R_*$, with a typical high electron density of $\sim 10^{11}$ to 10^{12} cm^{-3}.

1.3.3 Polarization

In almost all known Be stars intrinsic linear polarization of continuous light is found (e.g. Bjorkman 2000b). The rate of polarization may reach up to 2%. The magnitude in individual stars may vary with time and seems to tie in with emission-line strength and the violet-to-red ratio, but the intrinsic position angle always remains constant. Poeckert and Marlborough in [9] have shown that intrinsic polarization is also proportional to $v \sin i$: stars with low $v \sin i$ have low polarization rates.

Linear polarization is caused by the scattering of continuous light on free electrons in a structure around the Be star, thus in a way it measures the geometry of the scattering region. The measured polarimetric position angles (see [6])strongly indicate an axisymmetric equatorial structure of the Be star.

On the other hand, near to no rate of polarization has been found in lines [10], which limits the intensity of a possible magnetic field to 10^3 Gauss.

1.3.4 Balmer Discontinuities

Balmer discontinuity (or Balmer jump) is the well-known decrease in continuum intensity at the egde of the Balmer series. During the stages when emission or shell features in a Be stars's spectrum are especially pronounced, it can display two Balmer discontinuities (see [11, 42]. Whereas the first one is found in the usual wavelength location (3646 $\mathring{A}$, as in a normal B star), the second Balmer discontinuity is found at shorter wavelengths, implying that it was formed at a pressure is lower than that in the photosphere of the star. This feature is correlated to the spectral type and Balmer emission line strength [12].

1.3.5 UV Observations

While observations in the visible and infrared light, and polarimetry all suggest a rotationally flattened disc around the star, ultraviolet observations indicate a more complicated structure of the envelope - in addition to a cooler slowly-rotating dense disc there is a component of hot, thin, fast, radiatively-driven stellar wind spread over the rest of the star. The evidence is found in the resonance lines of Si III, Si IV and C IV. This is called superionization, as the existence of these ionization stages should not be possible in the later, cooler B-subclasses. Resonance lines often have asymmetrical profiles reminiscent of P Cygni-profiles [13]. If we apply the comparison then the blue-shifted wing of the asymmetric profile provides an upper limit for the stellar wind velocity, which may be up to 10^3 km s^{-1} , and imply mass-loss rates by wind outflow of 10^{-11} to 3.10^{-9} $M_{\odot}$ per year according to Snow in [13].

In UV resonance lines have also been detected [14] the variable so-called discreet absorption components (DACs) that shift in the line bluewards cyclically, on a timescale of days usually, and vary in intensity and radial velocity. They do not occur in normal B stars, but are known to be present in O-class stellar winds.

1.3.6 Rotation

Rapid rotation of the star underlying the circumstellar envelope implied by the broadening of absorption lines is definitely the feature most important to the evolution of a Be star. This had already been noted by Otto Struve. From their spectra we know that Be stars as a group are the fastest rotators in the B class.

Figure 1.2: Normalized distribution functions of normal Be stars (top) and Be-shell stars (bottom) as a function of $v \sin i$. (Porter, 1996 [21])

Do they actually rotate at threshold velocities (where gravity equals centrifugal force and the star becomes unstable)? To examine the dynamics of the star-cicumstellar disc system we need to know to what extent rotation alone is sufficient in the creation of an envelope.

The problem is that through rotational broadening we can only derive the value of the *projected* rotational velocity $v \sin i$, where i is the inclination angle, and v is the equatorial rotational velocity of the star. This means the values obtained depend on the way we view the star in question, and as we usually do not know the position of the rotational axis, it is difficult to determine whether they are truly fast rotators, if they rotate at threshold velocities, or if they are simply observed at a large angle, or, in other words, nearly equator-on.

Various attempts have been made to solve this problem in order to extract the true rotational velocities. For example Balona [15] assumed the rotational axis of a set of Be stars to be randomly orientated, and obtained a nearly gaussian velocity distribution with the mean value of 265 km s^{-1}.

The usual widths of photospheric absorption lines are hundreds of km s^{-1}, where the velocity is actually $v \sin i$. We usually consider the $v \sin i$ obtained from photospheric absorption lines to be the actual $v \sin i$ of the star. It may, however, actually be a value subject to underestimation and systematical error: for star rotating at over 90% of their critical velocity, the error in determining $v \sin i$ may reach 50% when viewed equator-on [16]. Also, for stars rotating at over 80% of V_{crit} gravitational darkening caused by rotation becomes prominent enough so that we would generally receive less photons from around the equator.

Therefore we would measure rotational broadening in lines in which the most of it comes from higher latitudes on the surface of the star, and obtain a lower value of $v \sin i$. Already the earliest calculations taking gravitational darkening into account [17] have however yielded results that were higher but still below the break-up threshold. If the value had been underestimated, some Be stars actually may be rotating at their critical velocities and the basic principle of Struve's model - rotational instability responsible for the creation of an envelope - may be applicable. Another possibility [15] would be an extremely differentially rotating star, where only the uppermost layer were unstable. This possible conclusion does not explain the existence of the rest of Be stars, though, especially when there seem to be intrinsically slow rotators among them (e.g ϕ And, ω CMa, HR 2309, [4] and other sources; see Figure 1.2).

According to [40] these slow rotators differ significantly from the rest of Be stars - if the typical value of $V \sin i < 150$ km/s, their continua have much a smaller intrinsic polarization of about 0.5%, and they *do not* display the resonance lines of C IV, Si IV and Si III that are the evidence of strong, fast stellar wind.

Moreover, several authors, for example recently Yudin [2] in a statistical analysis, have concluded that there are significant differences in mean projected rotational velocities between spectral subclasses of Be stars: later spectral type Be stars (luminosity class V and later) generally rotate faster than the rest, and closer to their break-up velocity.

Most authors choose 70 - 80 % of the critical rotational velocity as a typical value used in calculations.

1.3.7 Statistical Distribution

According to [33] the distribution of Be stars among normal B-class stars depends on the spectral type as seen in Figure 1.2. This seems to be a generally accepted fact that many Be models attempt to incorporate.

Figure 1.3: Distribution of Be stars among B stars from a sample of 4035 stars in the Galaxy, in percents (Balona, 2000 [33])

On the other hand, [34] have considered possible effects causing error in the Be/B ratio (error in determination of spectral type, difference in absolute magnitude between Be and B stars due to an extended envelope present, changes in luminosity), and presented a distribution with a maximum of 34% at spectral subtype B1e.

1.4 Variability

Another important Be feature is variability.Most Be stars are spectral and photometric variables on timescales that may range from minutes to years. The Be stars in which very little or no variations have been observed as of yet are suspected to be variable on much longer timescales, with them being observed in a quiet phase presently. Lastly, the variations may be periodic or episodical, and more types of them may occur superimposed at the same time. This is what makes modelling of processes in Be stars so difficult, and why many questions have not been resolved yet.

Periodicity of phenomena in Be stars is hard to define, as the variations reach from fast to very slow, in fact so slow, that a repetition of the occurrence hasn't been observed yet, thus it is impossible to determine if the phenomenon is periodic or aperiodic.

1.4.1 V/R Variations

The so-called violet-to-red ratio variability is among the most prominent in Be stars, and easily observable.

In the double-peaked emission lines that are so typical for Be stars, the blue peak (designated V, and closer to the violet end of the spectrum) and the red peak (designated R, closer to the red end of the spectrum) do not necessarily have to be symmetrical. The violet-to-red peak intensity ratio is an important characteristics of the emission line, and is also a highly variable one. The V/R variation tends to proceed from an asymmetric line, where either the red or the violet peak are prevalent, towards a symmetrical line with both peaks equally high, and later back to the asymmetric line with the V/R ratio reversed. The peaks' heights respective to the level of the continuum of course indicate the strength of emission superposed on the absorption line.

These variations happen on timescales of days to years. That is a very wide range. However, there seem to be two different kinds of V/R variations that differ in their physical cause. Slow V/R variations that take place on the timescale of years seem to be well explained by the theory of global one-armed disc oscillations (GDOs) by Okazaki and Kato [18], essentially a density wave that circles in the disc around the star, that we describe in detail in the following chapter. Rapid V/R variations that take place in the course of days, have not been sufficiently explained as of now. They might be connected to the general line-profile variations found in Be stars (on the grounds of their orbital period) or even be actually variations of the underlying absorption line only. They are usually found around the time of outbursts (see Aperiodic Variations) in some Be stars.

1.4.2 Balmer Discontinuity Variations

The variability of circumstellar Balmer jump has so far only been studied in one star, η Centauri [19], in which the Balmer discontinuity variability possibly displays the stellar period. It is considered to have a connection to the as of now not entirely explained rapid V/R variations.

1.4.3 The Be to B to Be-shell transition

Probably the strongest and most interesting among Be variations is the tendency of emission lines - the defining Be characteristics - to appear and disappear cyclically. A Be star in which the emission has vanished, appears just like a B star of normal properties, only rotating slightly more rapidly. This is also the reason why even a single emission-line appearance means a star will be denoted as Be.

It should be noted that the arisen B star, while otherwise normal, may have slightly altered latitudinal chemical abundancies due to rapid rotation, in the sense that the rotation would have caused the convection zone of the star to expand and chemical reactions to proceed differently as has been suggested by Porter [20]. Accordingly, it may be possible to tell apart a possibly previously emission-line B star, and a B star that never has or never will display emission, a truly normal B star, although no research has been conducted in the area up to now.

The emission appearance/disappearance process ties in with the gradual decrease (thus a variation as well) in emission strength until the emission peaks disappear completely, and only photospheric absorption remains. A reverse process takes place during the build-up of emission.

There are observable indicators of an emission build-up. In classical Be stars there is a brightening in the Paschen continuum in V of up to 0.5 mag, the $B-V$ index shifts towards red and the $U-B$ index towards blue. In shell stars their Paschen continuum dims by up to 0.3 mag, and both the $B-V$ and the $U-B$ indexes redden [23]. According to [55] this may be a consequence of scattering in the slowly forming disc.

It is not clear whether this happens in all Be stars. Also, we cannot say if a Be star that has lost the emission ever will exhibit emission lines again, or if a normal B type star is prone to exhibit emission at some time in the future. It is therefore almost impossible to decide if this phenomenon is actually periodic or not (or in which Be stars it is periodic and in which not) . It should be safe to say that in some Be stars the gradual transition between a B star and a Be star, sometimes including a Be-shell phase, has been observed to take decades (γ Cas, X Per, 59 Cyg, Pleione, 88 Her and others, generally most known Be stars) therefore if periodic then it is an extremely slow variation.

Because the apparent emission in the spectra of Be stars is caused by the presence of a circumstellar disc, this actually means that the disc itself disintegrated, thus the star becoming an average B star, and later developed again. Also, this may happen more than once during the Be star's lifetime.

The fact that a transition between a classical Be spectrum and a shell spectrum is possible as well, indicates further that Be and shell stars are not merely inclinationally differing variations of a general emission-line star (it is highly improbable that a star ould change the orientation of its rotational axis towards the observer in such a short time).

1.4.4 DACs

The so-called discrete absorption components(DACs) are long and narrow absorption troughs found in UV resonance lines that are typical for strong O-star winds, however not for normal B stars [25]. These components shift in the respective resonance line from its red edge towards blue cyclically, on a timescale ranging from days to one year [26]. The DAC variability probably reflects stellar wind structures varying in density, or density variations that may have been caused by the variability of the star itself .

1.4.5 Line Profile Variations

All known Be stars with the exception of a few later spectral subtypes display line profile variability (*lpv*) [27]. The *lpv* includes a diverse array of features, for example distinct red-to-blue moving features, similar to the DACs in UV resonance lines; narrow spikes occurring periodically at $\pm$ 0.7 $v \sin i$ from the line center.

The best-known and used explanation for *lpv* is the theory of non-radial pulsations (see e.g. [28]). It has also been suggested as the actual cause of the Be phenomenon. Since numerous Be stars belong in pulsational instability strips (for example Be stars β Cep or ζ Oph), it is expected that some of them do pulsate. At least one Be star has been observed to become a β Cep-type pulsator (27 CMa, [31]).

Nowadays it is usual to use common period-finding codes to search a Be star's variations for multiperiodic NRPs, which may obscure longer single periods. An alternative explanation of *lpv* was provided by Balona [32], [33]. They suggest that only short-period variations may be explained by a pulsational mechanism. The line profile variation with a longer period may be caused by cooler, co-rotating structures (equatorial, but also polar) in the atmosphere of the star. They find reasonable evidence for this theory in γ Cas [35], η Cen and ζ Tau, but do not provide an explanation of the strustures' origin.

1.4.6 Photometric Variations and Outbursts

Photometric variations of a Be star may be cyclic or irregular, simply periodic or multiperiodic, and of very different magnitudes. They also seem to be present in most early-type Be stars (for a large survey of photometric variability in Be stars see [46]).

Periodic photometric variations in the V magnitude with amplitudes of roughly 10^{-2} mag taking place on the timescale of days are usually interpreted as being caused by nonradial pulsations, although thay may also be explained by corotating photospheric features according to Baade and Balona [36]. There are periodic photometric variations with Δ V of about 10^{-2} to 10^{-1} mag taking place on timescales of days to 10^2 days found exclusively in Be binaries (Harmanec&Kriz, 1976). Quasi-periodic or irregular variations are usually greater in Δ V (0.01 to 1.2 mag according to various authors, e.g. [37]; [38]) and take longer (months to decades according to [41]). They are accompanied by other spectrophotometric changes, like for example the appearance of a second Balmer discontinuity in the spectrum, a positive correlation with emission strength of the Balmer lines, and the reddening of the Paschen continuum.

There seems to be a group of Be stars found (by the MACHO project) in the Large Magellanic Cloud (but not in our Galaxy), for which regular long-term light variations in the form of a rapid brightening, followed by a gradual dimming to the original level , are typical [45].

More than a half of the early known Be stars (up to B5), but only a fraction of later-type Be stars(B6 - B7), are so-called δ Eri stars, displaying periodic light variations incidental with the star's rotational period[33]. Furthermore, about a half of the δ Eri stars have double-wave light curves that can be explained by the presence of a dipole magnetic field that would enable for photospheric gas structures or clouds to form and co-rotate with the star. The light curve of such a Be star may vary from double-wave to single-wave repetitively on a timescale of months [43]. In most cases, the photometric period is the same as in line profile variations that have been mentioned above.

Very slow or apparently irregular photometric changes between continuum flux excess ($\Delta V > 0$) and deficiency ($\Delta V < 0$) show a change in Be 'type': classical Be star $\rightleftharpoons$ Be-shell star (as defined above), where $\Delta V = 0$ would indicate a normal B star phase. Of course, these would also be coupled with changes in appearance of absorption and emission lines.

Hubert and Floquet [44] have conducted a review of the HIPPARCOS data concerning outbursts (in this case temporary brightenings that are irregular, or with so far no discernible period) in Be stars, that shows that short-lived outbursts are mostly confined to early-type Be stars. Also, an inverse occurrence in the form of temporary fadings can be found in the most rapidly rotating Be stars. An outburst again will be accompanied by more general spectrophotometric changes. For example, in μ Cen an outburst was preceded by a decrease in emission line height and the widening of wings, then a rapid increase in emission line strength (the outburst itself) followed, coupled with rapid V/R variability, the appearance of high-velocity absorption lines, and increasingly widening separation between the violet and red peaks in emission lines [87].

Balona in [33] suggests that these occurrences be understood in terms of a weak dipolar magnetic field present in the Be star. Disconnection and reconnection of magnetic field lines would cause flaring and ejection of material from the star. In some cases the gas would be caught in closed loops, giving rise to the already mentioned co-rotating features and causing regular photometric variability, while in others mere irregular outbursts similar to solar flares would be created.

Chapter 2

Be Discs and Their Origins

2.1 General Picture of a Be Disc

In an ideal case, a Be star disc model should be capable of reproducing all of the observed features and variabilities described above. In reality, until now such a model does not exist, and therefore disc and disc origin modeling in Be stars remains an open problem.

A disc model may be empirical, thus based on a limited set of observational data, usually including many variable parameters and *ad hoc* assumed dependencies (density law, temperature law), and therefore capable of simulating only a limited set of Be-distinguishing features. Or, it may be theoretical, based upon the kinematics and dynamics of a stable thin disc, and the theory of radiation transfer, attempting to emulate as many observational features as possible. We will offer an example of each.

There are some factors one must consider to simplify the model of a Be disc:

2.1.1 Underlying Star

We usually assume that aside from the as of now still unknown process, or combination of processes, that causes a B star to develop emission, the star underlying the extended envelope is normal in terms of mass, size, luminosity, effective temperature and chemical abundance relative to its spectral subtype, and is spherically symmetric. We already mentioned that this does not necessarily have to be true - for example, the slightly more rapid average rotation of Be stars will probably cause changes in internal structure (convection zones) during the lifetime of these stars, thus a deviation from what we consider a typical evolution. Because close to no research has been made in this area, we cannot say how great of an impact this effect could have.

Also, a rapidly rotating star would become rotationally distorted proportionally to the rotation rate (a well known example is that of a solid body rotating at break-up velocity, in which the polar radius $r_p = (2/3)r_e$). Some earlier models, like the detailed model for γ Cas mentioned below, even directly state the assumption of a spherically symmetric star rotating at its critical velocity. Even if we assume that Be stars generally do not rotate at break-up velocities, but close to them, the effect clearly cannot be dismissed. Unfortunately, the detailed quantitative stellar evolution models available do not include the influence of rotation so far.

2.1.2 Symmetry

One of the basic assumptions as well is that of the envelope's symmetry along the rotational axis. This is what enables us to describe the star + disc system in cylindrical coordinates and use the well developed theory of thin, or thick, accretion discs. However, it seems that only a violation of axial symmetry, as is the case of one-armed disc oscillations, can explain some kinds of Be variability.

2.1.3 Observationally Supported Facts

The observed emission unambiguously supports the existence of an oblate (rotationally distorted) equatorial envelope around the star. What does the envelope look like according to the only direct measurements - spectrophotometry and polarimetry?

Spectrometry (e.g. [5] and others) suggests that the size of the H_α emitting region ranges generally from 3 to 10 R_*. This has quite recently been confirmed by the more precise interferometric measurements of Quirrenbach et al. [6], who give values of 3 to 12 R_* in four Be stars (very well consistent with all available Be disc models) . The lower boundary of three stellar radii remains questionable,as it would still be possible for the disc to be actually touching the surface of the star.

The rate of ionization in the equatorial disc suggests electron densities of 10^{11} to 10^{12} cm^{-3}.

The question remains if this equatorial envelope is rather thick (with a large opening angle), or if it has the form of a thin disc. The usually used value for an opening angle is $\sim 5°$, which fulfills the standard requirement of a thin accretion disc. For a more exact proof of this assumption, Wood et al.[47] present two solutions to radiation transfer in the Be star ζ Tau modelled after optical continuum spectropolarimetry, which imply an emitting region with a half-opening angle of either 52°, or 2.5°, the thin disc being able to reproduce infrared excess better. A further interferometric study by Quirrenbach et al.[6] too supports the thin disc in ζ Tau by being able to place an upper limit of 20° on the opening angle.

Figure 2.1: The two component picture of a Be star: fast, tenuous stellar wind and a cool, slower-rotating, dense equatorial disc. (Slettebak, 1988 [3])

Most of the characteristic observational Be features mentioned above - emission, linear polarization of the continuum and IR excess - indicate the existence of an equatorial disc-like envelope around the star. However, the measurements in the UV range complicate the final picture of the Be star + envelope system. We imagine that aside from the slower, cooler, dense, and optically thick equatorial disc, there is a fast, hot, tenuous stellar wind component over the rest of the star (Figure 2.1). Since the theory of stellar wind has been developed in detail for the longer known of O-star winds, the equatorial disc and its origin are the more important subjects to current research. While there have been many qualitative and quantitative approaches to the description of the disc's structure over the last forty years, some of them described below, there is virtually only one that truly attempts to simulate the creation of a Be disc.

2.1.4 Radiatively Driven Stellar Wind

The strong radiatively driven stellar winds present in Be stars are, together with extremely rapid rotation, generally seen as the key elements in the formation of a circumstellar disc.

Stellar wind is the continuous supersonic outflow of mass from a star. The stellar winds from hot, massive stars, such as the Be stars, differ from the stellar winds of cooler stars in such a way that the star's own radiation plays a very important role in driving the outflow. The star is very hot, which means it has a very high surface luminosity that imparts momentum to the atoms that take part in light scattering through absorption in spectral lines. Therefore in regions of the star's atmosphere where the radiative force exceeds gravity, material will start to move 'upward', away from the star. This is called radiative driving. It is so effective in hot stars, because a great part of the radiation they emit is in the UV, where their atmospheres have many absorption lines, and their opacity is greater (up to 10^6 times) than the opacity in the continuum [91].

Another aspect of radiative driving is that it is more effective, the faster the medium (stellar wind) moves (or, more accurately, the larger the velocity gradient). The transfer of momentum comes mostly from line scattering, where an electron in the atom is moved from one bound level of the atom to another. While in a static medium the amount of energy transmitted would have to have an discrete value for the electron to be shuffled in between levels of the atom, in a moving medium the Doppler effect shifts the the value of accepted energy toward longer wavelengths, thus enabling resonance from a broader interval of energies, and increasing the radiative driving of matter. In this sense the radiative force is a function of the velocity gradient - the larger the velocity gradient the more effective becomes the radiative driving.

A photon can be absorbed in a line only in a narrow (geometrically) region, the width of which is given by the velocity gradient of the wind and the frequency width of the line's profile function. If the interaction region is very narrow, the problem of radiative transfer is simplified. The so-called Sobolev approximation treats the line profile function $\phi(\Delta\nu)$ as a δ-function to achieve this. In effect, the interaction region is reduced to a point - the Sobolev point - along the path of the photon. The optical depth at the Sobolev point is then sufficiently described by the conditions in the Sobolev point only.

In reality, the line profile function has a finite width because of thermal and turbulent motions in the wind (the Gaussian velocity $v_G > 0$). If we define the Sobolev length as [91]

$$L_S(r,\mu) = \frac{v_G}{d\mu v(r)/dl}$$

where μ is the cosine of an angle formed by the direction of propagation of radiation, and the radial direction in place r, a volume defined by three Sobolev length vectors from the point r is the interaction region for absorption of a photon by the line.

The Sobolev approximation is valid if the particle densities and the velocity gradient of the wind vary only negligibly over a length of cca $3L_S$.

The total radiative acceleration trough absorption in an ensemble of assorted lines is given by [91]:

$$g_L(r) = \frac{2\pi}{c}\sum_l \kappa_l \int_{\mu_*}^{1} I_{\nu_l}\frac{1-e^{-\tau_{\nu_l}}}{\tau_{\nu_l}}\mu d\mu$$

I_{ν_l} is the intensity of the incoming radiation at the line frequency ν_l; κ_l is the absorptin koefficient in the line; and τ_{ν_l} is the optical depth of the line at that frequency.

The radiative acceleration due to absorption in an optically thin ($\tau_l \ll 1$) line is proportional to the number of absorbing particles, and also to the term $(R_*/r)^2$, which means that the flux decreases with distance. It is independent of the velocity gradient.

The radiative acceleration due to absorption in an optically thick line ($\tau_l \gg 1$) does not depend on the number of absorbing ions, but rather on the velocity gradient in the wind.

To attain total radiative acceleration, we would have to sum over all possible absorption lines. The number of spectral lines to be summed over can be reduced, though, by the assumption that the wind desity is low enough to neglect collisional excitation. In that we only have to account for lines from the ground level, lower excitation levels, and metastable levels, which is about 10^5 lines [91].

Castor, Abbott and Klein have introduced a parameterized form of the total radiative acceleration, that simplifies the computation of mass-loss rates through stellar wind:

$$g_L = \frac{\sigma_e^{ref}}{4\pi c}\frac{L_*}{r^2}M(t)$$

Here σ_e^{ref} is a reference value for the electron scattering opacity. $M(t)$ is the 'force multiplier' that can be expressed simply by

$$M(t) = kt^{-\alpha}(10^{-11}n_e/W)^{\delta}$$

The quantities k, α and δ (introduced by Abbott [69]) are force multiplier parameters. They depend on T_{eff} and $\log g$. Further, $W(r)$ is the geometrical dilution factor, and n_e is the electron density; t is the dimensionless optical depth parameter, introduced so that the optical depth scale depends only on the structure of the wind:

$$t \equiv \sigma_e^{ref} v_{th}\rho\frac{dr}{dv}$$

where v_{th} is the thermal velocity of protons in the wind with $T = T_{eff}$.

These expressions show that the total radiative driving force depends on the mass-loss rate, $g_L \sim \dot{M}^{-\alpha}$. The terminal velocity of the wind v_∞ does not, though, because it is a free parameter.

2.1.5 Basics of a Stellar Atmosphere Model

What we call a Be disc very well may be essentially an extremely rotationally distorted extended stellar atmosphere (for example if the disc was formed from material ejected from the star mostly due to rapid rotation close to break-up velocity), and many authors do treat it that way.

To describe a stellar atmosphere means to describe the state of every particle in the atmosphere of chosen geometry (planar, spherical, other). A closed thermodynamical system may be found in total thermodynamical equilibrium. In such a case the description through distributions of velocities, excitation, and ionization states of particles is greatly simplified, and the system can be described by two variables (for example, the absolute temperature T, and particle density N).

A stellar atmosphere, however, is not an isolated thermodynamical system, because it emits photons (that enable us to see the star) , therefore it can only be in the state of local thermodynamical equilibrium (LTE) at most. In this case we can only use the relations of thermodynamics *locally*, with the locally specific temperature $T(\vec{r})$ and particle density $N(\vec{r})$, and apply this method throughout the whole stellar atmosphere. We also allow for the radiation (photons) to depart from its equilibrium (the Planckian distribution), while massive particles remain in equilibrium (the Maxwellian velocity distribution). Or, in other words, in position $\vec{r}$ all particle processes (excitation and ionization) must be in detailed balance microscopically, where the rate of every process is weighed out by a reverse one.

A non-LTE state is any state that departs from local thermodynamical equilibrium. It means that some level populations in some atoms are allow to depart from their values in local thermodynamical equilibrium, but the velocity distributions of all particles remains Maxwellian.

When modelling a stellar atmosphere, it is usually assumed to be in LTE. The assumption of LTE is generally valid for low temperatures, in the inner parts of the atmosphere (where close to no photons escape, thus radiation helps to establish thermodynamical equilibrium), and high particle densities (as particle collision tend to establish equilibrium). It is not valid for high temperatures and low densities. We see that in the hot atmospheres of Be stars, especially in the outermost parts that form the disc the departures from LTE will occur, but are rarely accounted for.

The state of LTE is described by three distributions [51]. These are, as follows, the Maxwellian distribution of particle velocities

$$f(v)dv = \frac{m}{2\pi kT}^{\frac{3}{2}} \exp\frac{-mv^2}{2kT}dv \tag{2.1}$$

with $T = T(\vec{r})$; the Boltzmann excitation equation, and the Saha ionization equation.

When describing a stellar atmosphere it is necessary to realize that the radiative part (mass-less, assumed to be non-interacting photons) and the material part are not separable. Radiation determines level populations, and atoms in turn absorb and emit radiation depending on their level populations. Such a self-consistent problem is difficult to solve, therefore we must choose a starting point (given radiation field, or given population levels) and continue from there, using the equations below.

- For a given radiation field, and the assumed geometry, the rate equations give level occupation numbers in atoms of the atmosphere. The Boltzmann and Saha equations represent the equilibrium of collisional, radiative, and their respective reverse processes in the atmosphere.

$$\frac{n_j}{n_i} = \frac{g_j}{g_i}\exp\left(-\frac{E_j - E_i}{kT}\right) \tag{2.2}$$

Here n_j and n_i are the populations of level j and i respectively; g_j, g_i are the statistical weights of the respective levels; E_j, E_i are the energies of the respective levels measured from the ground state.

$$\frac{N_I}{N_{I+1}} = n_e \frac{U_I}{U_{I+1}} \left(\frac{h^2}{2\pi mk} \right)^{\frac{3}{2}} T^{-\frac{3}{2}} \exp \frac{\chi_I}{kT} \tag{2.3}$$

The N_I, N_{I+1} denote the number density of the respective ionization stages. U is the partition function, defined as $U = \sum_1^\infty g_i \exp \frac{-E_i}{kT}$. χ_I is the ionization potential of ion I.

If there are any departures from LTE, this equation represents the equilibrium of rates of all processes in the atmosphere:

$$\frac{dn_i}{dt} = 0 \tag{2.4}$$

- The equation of radiative transfer, which for given level occupation numbers and the assumed geometry yields the radiation field.

$$(\frac{1}{c}\frac{\partial}{\partial t} + \vec{n} \cdot \nabla) I(\vec{r}, \vec{n}, \nu, t) = \eta(\vec{r}, \vec{n}, \nu, t) - \chi(\vec{r}, \vec{n}, \nu, t) I(\vec{r}, \vec{n}, \nu, t) \tag{2.5}$$

Here I is specific intensity of radiation at position $\vec{r}$, going in direction $\vec{n}$ in time t, with frequency ν; η is the absorption coefficient; and χ is the emission coefficient.

The equation of radiative transfer denotes that the change of the radiation field through interaction with matter is given by absorption and emission of radiation in the volume of matter.

- Equations of hydrodynamics used are the continuity equation, the momentum equation, and the equation of energy balance. In case of no outflow or inflow of matter the equation of hydrostatic equilibrium would be used, but that is not the case of the extended atmospheres of Be stars.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{2.6}$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v} = -\nabla P + \vec{f} \tag{2.7}$$

$$\frac{\partial}{\partial t}(\frac{1}{2}\rho v^2 + \rho\epsilon) + \nabla \cdot [(\frac{1}{2}\rho v^2 + \rho\epsilon + P)\vec{v}] = \vec{f} \cdot \vec{v} - \nabla \cdot (\vec{F_{rad}} + \vec{F_{con}}) \tag{2.8}$$

Here $\vec{v}$ means macroscopic velocity, ρ density, ϵ internal energy, P pressure, $\vec{f}$ external force (for example gravitational force), $\vec{F_{rad}}$ the radiation flux, and $\vec{F_{con}}$ the conducive flux.

2.2 Disc Models

If we assume from here on that Be discs are in general thin equatorial discs, which is the most widely accepted notion, it is possible to describe them in terms of the classical theory of thin accretion discs ([52]), or its slightly modified version ([53]) for the description of the disc's kinematics and dynamics.

A disc around a star may have developed either by accretion, which requires an outside source of matter, or by the reverse process, designated excretion, in which the matter that the disc consists of has come from the star itself. Thus the possibilities of a Be disc are following:

Protostellar accretion disc

There is a possibility that a Be star may have kept its protostellar disc, but it is very small. Since the main-sequence lifetime of a Be star is 10^7 to 10^8 years, it is reasonable to assume that only the youngest Be stars could have retained their protostellar accretion discs, if any at all. This assumption is supported by the fact that the Be phenomenon is spread throughout the subclasses $B0$ through $B9$, although the statistical distribution of Be stars in the B class does have a maximum at B2 [3]. In any case, such a star would be classified as a HAEBE object rather than a classical Be star.

Accretion discs in mass-transfer binaries

This is another possibility for a Be star to gain an accretion disc. Since a mass transfer occurs, when one of the stars has evolved off the main-sequence and filled its Roche lobe, and classical Be stars are by definition main-sequence stars, the mass-gainer of the binary would start developing a Keplerian accretion disc, and become a Be star. The transfer of angular momentum from the infalling material would in effect cause the Be star to spin up, which could explain the typical faster-than-average rotation of Be stars ([48]).

The scenario that Be stars are mass-transfering binaries with undetected companions has been proposed by Harmanec and Kříž ([24]) to explain the Be phenomenon. Later this theory was found rather less probable, as for example according to Pols et al. ([48]) at most 40% of Be stars can come from mass-transfering binaries, while according to Van Bever and Vanbeveren ([50]) it is only 20%. Moreover, Be stars in post-mass transfer systems have been observed as well, which excludes binary mass-transfer as the universal explanation.

A distinctive example of interacting systems are the Be/X-ray binaries, consisting of a Be star and a compact companion (as such neutron stars have been observationally confirmed). The X-ray bursts (temporary enhancements of X-ray emission by a factor of 10 and more) occur when the companion passes through the extended envelope of the Be star, or when its close passage possibly produces the warping of the disc by changing the boundary conditions (Roche lobes).

An argument in the favour of a binarity-related Be mechanism could be the fact that up until now no Be-Be binaries have been detected ([49]).

Accretion discs are the kind of Be discs which can be described using the classical accretion disc theory ([52]; [54]).

Reaccretion of excreted material

It also may be possible for a star (not retaining the protostellar disc) to build an accretion disc from material previously cast away from the star during an outburst. While outbursts, periodic and aperiodic, have been observed in Be stars, this mechanism is usually at best considered as an additional one.

Excretion discs

In single Be stars (as we have already stated the improbability of all single Be stars retaining their proto-discs) their discs must be excretion discs containing material cast away from the star by some mechanism, and expanding outward. They will be described differently. Concerning their origins, the excretion process is largely unknown, although a few suggestions have been made, which will be described later.

2.2.1 The Viscous Excretion Disc

This is a very simple and effective theoretical model of an excretion disc around a single Be star by Lee, Saio&Osaki ([53]), a modification of the classical accretion disc as described by Shakura&Sunyaev ([52]) or Pringle ([54]) in the sense that the drift of matter is reversed. The authors do not discuss the mechanism that supplies angular momentum to the matter of the disc at its inner boundary, but simply assume it exists.

The assumption is of a viscous, axisymmetric, geometrically thin disc has its inner boundary defined on the equatorial surface of the star. Because it is postulated as geometrically thin, its radial structure will be dominated strongly by centrifugal force over pressure gradient, and the authors may treat the radial and vertical structure of such a disc separately. Matter in the disc can be presumed to rotate at Keplerian speed (because of the prevailing centrifugal force) and drift outward on a viscous timescale due to transport of angular momentum. While angular momentum is conserved in the disc as a whole, viscosity is the agent that enables redistribution of angular momentum among particles. The authors use the α prescription, where viscous stress is proportional to gas pressure.

In examining the radial structure of the disc, following basic equations in cylindrical coordinates (r,ϕ,z) are used. They differ from the ones of a viscous accretion disc only in the boundaries imposed during their integration.

- Equation of mass conservation

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{2\pi r}\frac{\partial \dot{M}}{\partial r} = s(r,t),$$

where $s(r,t)$ is the mass source function; Σ is surface density given by

$$\Sigma = \int_{-\infty}^{\infty} \rho dz,$$

and $\dot{M}$ is the mass excretion rate, given by

$$\dot{M} = 2\pi r \Sigma v_r$$

.

- Equation of angular momentum conservation

$$4\pi\frac{\partial r^2 W}{\partial r} + \dot{M}\sqrt{\frac{GM}{r}} = 0,$$

where M is the mass of the star, and the term $\sqrt{\frac{GM}{r}}$ is obviously Keplerian velocity, here consistent with v_θ.

In this equation W is defined by

$$W \equiv -\int_{-\infty}^{\infty} w_{r\phi} dz = \int_{-\infty}^{\infty} \alpha p dz$$

where $w_{r\phi}$ is the $r-\phi$ component of the viscous stress tensor, p is pressure, and α is the viscous parameter.

- Equation of Energy Conservation

$$\sum \left(T\frac{\partial S}{\partial t}\right)_{vert.structure} = Q^{+}_{vis} - Q^{-} \tag{2.9}$$

Here S is the specific entropy, $Q^- \equiv 2F$ is the cooling rate with F as the radiative flux from the surface of the disc, and Q^+_{vis} is the integrated viscous heat generation rate defined as

$$Q^+_{vis} \equiv \int_{-\infty}^{\infty} \frac{3}{2}\alpha p \Omega dz = \frac{3}{2} W \Omega$$

According to the authors this theoretical model may be able to reproduce the observed B to Be transition, were the mechanism of mass-excretion time-dependent, that is, it would supply matter to the disc in episodes lasting years, for example. In such a case, after the mass excretion stops, the radial velocities (meaning velocities in the radial direction in the disc), however small, would be sufficient to dissipate the disc altogether. They conclude that the value of the viscous parameter $\alpha \rightarrow 1$ is needed for larger v_r, so as to fit the timescale of the disc dissipation and re-formation into a span of years to decades suggested by observations.

One of the disadvantages of the model is that it automatically produces a disc with a fixed - Keplerian - rotational law, in which the mass-loss rate is given by an unknown parameter α. Also, unless there is an additional force responsible for the ejection of material, this model would require a star rotating at break-up velocity to produce a disc at all. Which, in turn, would imply that all Be stars rotate at break-up velocity, contrary to values derived from observations [21].

2.2.2 The Empirical Model for γ Cas

The model of the envelope of γ Cas formulated by Poeckert and Marlborough in 1978 ([55]) until this day remains one of the most detailed quantitatively orientated models. In comparison with the earlier empirical models available, e.g. by Marlborough, 1969 ([56]); Cassinelli and Haisch, 1974 [59], it was based on a much larger amount of the authors' original data (H_α and H_β emission line profiles, and continuum and H_α and H_β polarization measurements), and attempted to emulate observational features beside the typical double-peaked emission profiles. As opposed to the viscous excretion disc model, the γ Cas model treats the problem more distinctly as a case of a star's extended, purely hydrogen atmosphere, and the processes inside the atmosphere irradiated by the star.

The envelope is assumed to be symmetrical in regards to the rotational axis and the equatorial plane, and described in cylindrical coordinates. The underlying star is spherically symmetrical and rotating at its critical velocity (as the authors presumed, but did not postulate, rotationally ejected mass to be the source of material for the disc). The envelope considered is steady-state, radiatively, and expanding along streamlines that converge at a certain distance from the rotational axis in the equatorial plane. The radial (expansion) velocity is given beforehand. The envelope is also assumed to rotate in the same direction as the star below, and the rotational law is given as

$$V_{rot} = V_*[\beta r^{-2} + (1-\beta)r^{-1}]^{\frac{1}{2}}$$

which in the case of $\beta = 0$ would take viscous effects into consideration, and in the case of $\beta = 1$ would mean conservation of angular momentum.

The principle of this model lies in assuming a certain density distribution for the envelope, and, taking the suppositions stated above into account, determining the degree of ionization and populations of the ground and the five higher levels of hydrogen at points of a grid created throughout the envelope. Then the radiation transfer equation along different lines of sight is solved by taking into account thermal line broadening, line polarization, continuum opacity and changing emission and absorption coefficients, while optical depth is assumed to be a linear function of position along the line of sight. The model was then fitted with stellar parameters - mass, radius, effective temperature and inclination - of γ Cas. In this way, Balmer line profiles (H_α, H_β, H_γ), line polarization, continuum polarization and overall energy distribution were obtained.

The obtained Balmer line profiles were compared with observations by Poeckert ([58]) and Poeckert and Marlborough ([57]), giving a good agreement especially in the case of H_α, although a different inclination (45 instead of the original, literature based 60) had to be implemented in the end for better results. The model with the best H_α fit provided the expansion velocity used. On the other hand, the predicted energy distribution was not consistent with observation, polarization predicted by the model was too large, and the H_β and H_γ profiles were too broad. Especially the energy distribution discrepancy may have been caused by the fact that the observations for comparison were two years old at the time, while γ Cas had undergone an increase in emission line intensity in the meantime. Further decrease of the star's rotational velocity to 75% of the critical value was also able to improve the H_β and H_γ fits.

2.3 Disc Origins

The question of what causes the accretion of a disc in a Be star still remains the most important and unresolved problem. The reason seems to be the number of different spectral and photometric characteristics of Be stars, and their different types of variability, which are impossible to explain using just one concrete physical process.

There is a limited number of global mass ejection processes (known to us and physically possible) found in stars [60]:

- centrifugal force that Struve's original explanation was based upon, but not sufficient to eject matter from the equator unless the star rotates at break-up velocity

- radiation pressure, which figures prominently in the Wind Compressed Disc model, but is applicable only in stars with massive, high velocity stellar winds

- gravitational pull in interacting binary systems

- thermal expansion, as seen in explosions of novae

- additional mass ejection mechanisms applicable only in combination with the forces above, like non-radial pulsations, magnetic fields, acoustic waves, and others

Options that have been researched and deemed applicable to Be stars will be presented below, but most of them account only for a few selected Be characteristics and even then on a mostly qualitative level. Furthermore, most existing models concerning the origin of the disc are 1D models, meaning they model the equatorial outflow only, and are thus only partial solutions to the puzzle of Be discs with limited use, as the system of the star and its envelope is decidedly not spherically symmetrical. They will be discussed later in this chapter. The only 2D model concerning Be discs is the Wind Compressed Disc (WCD) model. The reason is that in two dimensions even a steady-state flow of matter would have to be described in partial differential hydrodynamic equations, which are difficult to solve. Usually various simplifying assumptions have to be applied, if we attempt the solution.

2.3.1 The Wind Compressed Disc

This model by Bjorkman and Cassinelli [61] does not merely explore the mass ejection mechanism in single Be stars, but also attempts to build a disc with the desired properties, using merely the in Be stars generally prominent radiatively-driven stellar wind and the intrinsically fast rotation, without any additional forces applied.

We have already mentioned that aside from the relatively dense and slow-moving equatorial disc there is a second component to the Be envelope consisting of hot and high-velocity stellar wind, as evidenced by the presence of asymmetric absorption profiles in the far UV range. The mass-loss implied by the terminal wind velocities ranging from 450 kms^{-1} up to 1000 kms^{-1} is 10^{-11} - 3.10^{-9} solar masses per year [13].

The basic assumption of the WCD model is that there is a connection between the well-described radiatively-driven stellar wind component over the whole surface of the star, and the equatorial disc of unknown origin. According to [66] the properties of the superionized resonance line of C IV indicative of the presence of radiatively-driven stellar wind were noted to vary in correlation with Hα equivalent width and V/R ratio. Further comparison of UV and IR observations by Bjorkman and Snow [67] has shown that the C IV producing region of the envelope seems to be concentrated around the equatorial plane rather than the poles of the star. Since the process of superionization would require an effective temperature four times the T_{eff} appropriate for the B subclass [67], the authors of the WCD model have suggested the process responsible for the required temperatures confined to the equator was shock heating of the material.

In the Wind Compressed Disc (further on WCD) model the authors considered mass-loss through rotating line-driven stellar wind in two dimensions. Because the star rotates fast the ejected material tends to orbit the star while it is expanding outward. If the expansion outward is sufficiently slow compared to the rotation, the material from the poles of the star will tend to spin down to the equator on an orbit (the plane of which is generally inclined with respect to the equatorial plane) before expanding away from the star. The authors expect that streamlines from opposite poles are dense enough to form a pair of superheated shocks above and below the equator, when after 1/4 of the inclined orbit they cross the equator and meet their corresponding counterparts from the opposite hemisphere. The outflowing material will be compressed between these shocks and form a dense disc that cools radiatively (Figure 2.2). Rotation in this case acts as a means of latitudinal redistribution of material from the poles of the star, so that the polar regions are rid of outflowing material and the equatorial region is enhanced in terms of density.

Figure 2.2: A schematic depiction of the standing superheated shock developed around the equator due to streamlines crossing. Here $\Omega = 0.5$. (Bjorkman and Cassinelli, 1993 [61])

The model itself employs many simplifications. The authors describe the stellar wind from a rapidly rotating hot star in terms of hydrodynamics - the equation of continuity and the momentum equation - in spherical coordinates (r, θ, ϕ), but assume a steady-state flow symmetrical merely about the rotational axis, as the envelope they expected to obtain was obviously disc-like. They also replace the energy equation with the isothermal equation of state, assuming the wind to be isothermal and of the same temperature as the T_{eff} of the star. The mentioned basic equations are below:

- Continuity equation in differential form

$$\frac{1}{r^2}\frac{\partial(r^2\rho v_r)}{\partial r}+\frac{1}{r\sin\theta}\left(\frac{\partial\sin\theta\rho v_\theta}{\partial\theta}\right)=0 \tag{2.10}$$

- Components of the momentum equation

$$v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{{v_\theta}^2+{v_\phi}^2}{r}=-\frac{a^2}{\rho}\frac{\partial\rho}{\partial r}+{f^{ext}}_r, \tag{2.11}$$

$$v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}-\cot\theta\frac{{v_\theta}^2}{r}=-\frac{a^2}{r\rho}\frac{\partial\rho}{\partial\theta}, \tag{2.12}$$

$$\frac{1}{r\sin\theta}\left(v_r\frac{\partial}{\partial r}+\frac{v_\theta}{r}\frac{\partial}{\partial\theta}\right)(r\sin\theta v_\phi)=0, \tag{2.13}$$

where ${f^{ext}}_r$ is the sum of gravitational and radiative forces in the radial direction, given by

$${f^{ext}}_r=-\frac{GM}{r^2}+\frac{\sigma_e L}{4\pi c r^2}\left[1+fk\left(\frac{1}{\sigma_e\rho v_{\rm th}}\frac{dv_r}{dr}\right)^\alpha\right]. \tag{2.14}$$

The meaning of parameters and variables is as follows: for the spherical star, M is mass, R is radius, L is luminosity, σ_e is electron scattering continuum opacity; v$_r$, v$_\theta$ and v$_\phi$are the respective components of velocity in the direction spherical coordinates. The finite disc correction factor f and the so-called force multiplier constants, k and α are taken from Castor, Abbott and Klein [68], who worked out the original spherically symmetrical stellar wind model.

- Isothermal equation of state

$$P=a^2\rho, \tag{2.15}$$

where P is pressure, $a$is the isothermal speed of sound in the wind, and ρ is the density of the fluid.

For the streamlines from both poles to cross the equator and thus form the twin superheated shocks it is required that the orbital motion exceed 1/4 of an orbit. This means that there is a threshold rotational velocity at which a disc is produced by this mechanism. The authors have found that even if the rotation is less than the disc formation threshold, equatorial compression can still occur in places. They call this case the Wind Compressed Zone (WCZ) model [70].

The authors have also found that the produced density gradient with latitude corresponds to an ionization gradient in the stellar wind, which according to Bjorkman and Abbott [71] causes observable changes in the line profiles.

The WCD model, however, despite being the most complex attempt of a solution to the Be phenomenon to date, does have its weak points. For example, it does not explain Be stars with weak stellar winds that do not have sufficient wind-incited mass-loss rates to form a disc in this way. The model also automatically conserves angular momentum, which is contrary to many (the authors themselves included) who believe that Be envelopes are in fact thin Keplerian discs. The authors intone that it is also the reason why the resultant disc is too thin. To get a Keplerian (rotationally supported?) disc out of a WCD would require angular momentum to be fed into the disc by an additional mechanism. Later presented a modification of the WCD model has been presented with the addition of a magnetical field (MWCD).

Numerical Tests of the WCD model

The largely qualitatively formulated disc formation mechanism of the WCD model was, with certain adjustments, confirmed by Owocki, Cranmer and Blondin [63]. While the original WCD model in fact avoided complete solution of partial differential hydrodynamical equations in two dimensions by exploring merely the kinematics of the mechanism and the disc, Owocki, Cranmer and Blondin attempted to solve the WCD dynamically. Instead of computing a steady outflow in $2D$, they used a time-dependent hydrodynamical code in $2D$ that evolves from a model of a line-driven rapidly rotating stellar wind toward a steady flow in $2D$.

The adjustments to the original WCD model included a *dynamical* solution of the hydrodynamical equations with the addition of gas pressure to the radiative driving for a more realistic treatment of the formed equatorial shocks. Just like in the WCD model, merely a radial form of gravity (central gravitational force field)was assumed, by which the numerical simulation neglected any multipole components of the gravitational field that may have come from the rotationally distorted, oblate body of the star. However, the lower stellar wind boundary was set along an oblate surface.

The numerical simulation led to the formation of a disc as predicted by the WCD model, with some differences given by the aforementioned extensions.

- While the analytical WCD formed by a fixed outflow of matter will have its lower boundary detached from the surface of the star, the numerically simulated disc will extend to the surface, while its inner parts inflow and partially reaccrete back onto the star.

- The simulated maximum latitudinal wind velocities were a factor of two smaller than those predicted by the WCD model. This means a slower flow of matter toward the equator.

- The resulting equatorial disc is weaker than predicted analytically, with a wider opening angle, a lower disc/pole density ratio, and a smaller velocity jump at the edge of the shock.

- Discs of models rotating close to break-up velocity were found unstable, and exhibited variability.

Why It Doesn't Work...

In the WCD model the radiative force was considered to be so close to purely radial that non-radial effects could be neglected, as we mentioned earlier. However, later Owocki, Gayley and Cranmer [64] have shown that the non-radial components of the radiative force must be considered, and that, contrary to expectations, they do not help to achieve a denser disc. They are in fact significant enough to wholly stop the predicted formation of the disk, in the way of redirecting the mass flow towards the poles instead of the equator plane, even if the velocity threshold condition for the disc formation is fulfilled. In the end the radiative force will have a contrary effect on the disc formation, instead of being the cause of it.

The non-radial components of the radiative force come from several sources:

- Firstly, the fulfillment of the velocity threshold means that the star is rotating rapidly. Furthermore, we have already shown earlier, that most, if not all Be stars are intrinsically rapid rotators. Despite this all Be star models up until now have only considered *spherical* stars for the sake of simplicity. The rotating star itself in reality is oblate. The work of Owocki, Cranmer and Gayley [64]shows clearly that rotational distortion of the star can change the results of a simulation, and therefore must be taken into account.

- Also, the line force according to [68] depends on the velocity gradient, which, in turn, depends on direction. The authors [**?**, **?**, **?**] reason that the terminal velocity of stellar wind will be minimal in the equatorial plane (where there is minimal effective gravity), and maximal on the poles, therefore the velocity gradient is greater in the direction toward the equator than it is toward the poles. Such a gradient will produce lines with greater radiative force in the stellar wind around the equator, in effect pushing the mass flow away from the equatorial plane.

- Thirdly, the WCD model produces automatically an angular momentum conserving disc. To conserve angular momentum, the azimuthal velocity

v_ϕ has to decrease with radius, again producing a velocity gradient, which due to its direction will cause the CAK line force to slow down the mass flow, and hinder disc formation.

J. Bjorkman, one of the authors of the original WCD model, has discussed [62] various ways of reducing or obliterating the non-radial component of the radiative force. He deduces that since in optically thin absorption lines of the stellar wind the Sobolev line acceleration is radial unless the star rotates very rapidly, the non-radial component of the force is created mainly by the optically thick lines. Thus, if the stellar wind is driven by optically thin lines, the non-radial component will be small. Because the existence of stellar wind is conditioned by the presence of at least one optically thick line, the author rather advocates a case where stellar wind further away from the star shifts to being driven by thin lines only as a result of lower density.

Another proffered option is the presence of dense 'clumps' in the stellar wind. The radiative force on such clumps, assumed to be optically thick, would be radial even in the case of a rotationally oblate star, if the clumps are sufficiently large, leaving gravity as the only accelerating force. The clumps would therefore replace the streamlines considered in the original WCD model, and contribute to the disc formation in the same way.

The author also considered the ionization gradient close to the star, caused by the gradient in density. A rapid increase in ionization may reduce the number of UV lines responsible for the driving of the wind, which would reduce the non-radial component of the radiative force, as well.

2.3.2 Additional Mass Ejection and Variation Mechanisms

These are the mechanisms that are not capable of forming a Be disc by themselves, but can yield results in combination with fast rotation, and each other, or simply explain some specific parts of the Be problem (for example a type of variability).

Non-radial Pulsations

Non-radial pulsations (NRPs) are generally accepted as the explanation of the short-term periodic line profile variability (*lpv*) in Be stars. The line profile changes with period of $0.5 - 2.0$ days in the spectra of Be stars were first attributed to NRPs by Baade [29].

Line profile variations can be found in most B-class stars. Although there seem to be less later-type Be stars displaying *lpv*, the same short-term variations are present in later B stars without emission. Because *lpv* seems to be correlated with the presence of a circumstellar disc (since the Be/B ratio peaks at the B2 subtype), it has been inferred [86]that the cause of short-term *lpv* and the origin of the disc could be related.

It may be difficult to distinguish the mechanism behind *lpv*, since the period (ranging from hours to days) could correspond to non-radial pulsations, as well as the rotation period for example (Be stars are extremely rapid rotators), which is why the NRP model is currently competing with the model of rotational modulation (or, model of corrotating structures). We will look at the NRP model as an important mass ejection mechanism, rather than a definite explanation for *lpv*. Lately, the NRP model has been reliably confirmed in some singular cases (μ Cen [87] - multiperiodic, ω CMa [89], η Cen [86] - multiperiodic).

If we describe the star in spherical coordinates (r, θ, ϕ), non-radial pulsations are oscillations of a mass element in the radial direction and a direction transversal to it. The displacement of the element is a function of the azimuthal and polar coordinates. Non-radial pulsations involve much less pulsational energy, and produce less noticeable effects in the stellar spectra than radial pulsations. The solution to such small adiabatic oscillations are waves propagating in the radial, and the surface coordinates of the star.

Because the non-radial oscillations are small, the problem can be linearized, and the solution to the equation of the harmonic oscillator can be separated into a radial and spherical harmonics part.In such a case the sum of two solutions is a solution as well, which means that in a star they often combine in both coordinates to form multiperiodic pulsations.

In spherical coordinates, NRP modes can be described as a product of the radial displacement and spherical harmonics [90]:

$$\xi = \xi_{nl}(r)Y_l^m(\theta, \phi)e^{i\omega t}$$

Modes with $m < 0$ are prograde modes, while modes with $m > 0$ represent retrograde modes. The eigenfrequencies in the equations of motion can be grouped according to the dominant physical mechanism that propagates them. The *p*-modes, or pressure modes (pressure acts as the main restoring force for oscillations), are radial disturbances in pressure, or essentially acoustic waves. They are longitudinal. For $n = 0$ (where n is the order of the radial displacement) the *p*-mode becomes a radial pulsation. The *g*-modes are gravity waves (local gravity is the main oscillation restoring force). They are transverse, and characterized by small variations in pressure and density. The frequencies of *p*-modes are usually higher than those of *g*-modes. If the star is rotating, a new kind of modes becomes possible, in which the restoring force is the Coriolis force. They are called inertial oscillations.

Baade in [30] makes a point on why there could be a connection between mass loss and NRPs, even though the non-radial pulsations are capable of transfering much less momentum than radial pulsations: mass-loss in Be stars has been observed to be variable, as well as the amplitude of the NRPs (while their periods are not). In some cases there has been a decrease in the NRP amplitude after a Be episodic mass-loss (outburst). He deems the amount of kinetic energy needed for an outburst to be similar to the pulsational energy of some NRP modes, but because no evidence has been found for a direct dependence of the mass-loss rate on NRP amplitude, he suggests that long-term effects of NRPs are more important. It also seems that for that to take place rapid rotation is necessary, since low-order NRPs occur in slowly-rotating B-stars, but this does not lead to mass-loss.

Osaki [90] has shown that non-axisymmetric prograde waves transport positive angular momentum to the matter, while retrograde waves transport negative angular momentum. He suggested a scenario for an episode of mass-loss in a Be star, where prograde ($m < 0$) waves are excited by some mechanism inside the star, and then the angular momentum is deposited close to the surface through dissipation. This will increase the rotation of that concrete surface layer, possibly to above break-up velocity. In that case it would start mass-loss. Afterwards, the NRP waves would deposit their angular momentum in the extended envelope, and accelerate the mass-loss further. The NRP will be gradually damped due to increasingly leaky boundaries, and eventually stop fulfilling mass-loss conditions.

So far in at least one Be star (μ Cen) observations [87] suggest a correlation between retrograde multiperiod NRP modes and star-to-disc transfer. The observed periods agree with the non-radial gravity waves, in which the maximum height of the pulsation occurs when the lateral velocity is aligned with the direction of stellar rotation. Due to this Owocki in [88] proposes a scenario of Pulsationally Driven Orbital Mass Ejection (PDOME), where the NRPs are the mechanism responsible for launching material into Keplerian orbit of a rotating star. However, the orbital launch by NRPs requires for the difference between orbital and rotation speed $\Delta V = V_{crit} - V_{rot}$ to be comparable to the pulsation velocity amplitude ΔV_{NRP}, which can be about, or slightly more than the speed of sound $a \approx 25 km/s$. It means that the star would have to rotate at $0.95V_{crit}$, contrary to the commonly cited values of $0.7 - 0.8V_{crit}$. This mechanism has been hydrodynamically simulated in a star rotating at near break-up velocity, where the circumstellar density did increase, albeit prograde g-modes were required, which seems to contradict the observations of μ Cen.

Global Disc Oscillations

An effective model of a Be disc must be capable of reproducing the variations found in their spectra. In this sense the theory of global disc oscillations (GDOs) by Okazaki and Kato [18] offers a partial solution to the problem in that it explains the long-term (period ranging from months to decades) V/R variations.

This theory describes the long-term V/R variations dynamically. Earlier models like the elongated disc with apsidal motion [78], where the period of the V/R variations is the period of rotation of the main axis of the ellipsoidal disc, could sufficciently explain the V/R variations and the line profile shift that accompanies them. The model, however, did not explain how the elongated disc was formed, nor how its long-term, low-frequency apsidal motion was held up, and treated the problem merely geometrically.

The theory of GDOs is based on the fact found by Kato [73] that in near Keplerian discs there can exist low-order, global density oscillations. The source of the oscillations may be for example NRPs, or gravitational pull in close binary system with an eccentric orbit.

Further, if there are any global oscillations in the Keplerian disc, it can be proven that it have to be oscillations of order $m = 1$, where m is the number of azimuthal waves, and with their frequency ω much smaller than the angular frequency of disc rotation Ω. This is expressed by the condition $\omega \sim \Omega - \kappa \ll \Omega$, where κ is the frequency of oscillations in the disc due to the Coriolis force. The period of the global disc oscillation was estimated to be about 10 years when applied to Be discs, which agrees with the observed period of V/R variations.

The presence of one-armed low-frequency density waves in Be discs has been also observationally confirmed. For example, Telting et al. [75]have found evidence of a prograde $m = 1$ density wave by analysing the spectra of β Mon. Vakili et al. [76] have shown direct interferometric evidence also of a prograde desity wave with $m = 1$ in ζ Tau.

The authors conclude that since in Be stars there is very little flow of matter in the radial direction close to the star ($< 10R_*$), and the radial pressure gradient is much smaller than gravity (due to disc temperatures of 10^4K), the Be disc must be rotationally supported, and thus in general rotates at a velocity close to Keplerian. They find the GDO theory to be in most agreement with, and appliable to the viscous decretion disc by Lee, Saio, and Osaki [53].

Okazaki [79] later introduced a GDO theory that included rotational distortion of the star (adding a quadrupole term into the expression for gravitational potential) and radiative pressure exerted on the disc. This model, however, contains a higher number of free parameters, which enables it to closely match observations, but smears out the actual physical information, and makes it impossible to decide to what extent the model is correct. Recently, Fiřt and Harmanec [74] have managed to reduce the number of parameters in this modified theory, but remain rather skeptical in the sense that the GDOs are an elegant solution, but far from proven.

Magnetically Torqued Disc

After the WCD model has been found invalid due to the non-radial radiative force component overwhelming the formation of a disc, various possibilities to enhance the equator-ward flow have been researched. Among the most advocated to date is a dipolar magnetic field that would channel the stellar wind outflow.

The Magnetically Torqued Disc (MTD) [80] model explores analytically a near-Keplerian disc dominantly supported by centrifugal forces around a rapidly rotating Be star with a dipolar magnetic field aligned with the rotational axis. It uses the kinematically described azimuthal velocity $v_\phi(r)$ and a parametric model of the magnetic field with intensity $B(r)$ to derive an analytical description of disc properties. From there on the field strength on the stellar surface is deduced that would be required to transfer sufficient angular momentum to the line-driven mass outflow, and to direct it toward the equatorial plane to produce a disc with properties that have been observed in Be stars.

This model also attempts to give an explanation of why the Be-to-B ratio is highest for main sequence stars around the spectral class B2. Stars of early types have fast and dense stellar winds that would require magnetic fields too strong ($> 10^3 G$), while later type B stars have weaker outflow of matter, and thus produce weaker Be characteristics, even though they do have a circumstellar envelope. In comparison, according to the theory, a B2 V star would require a magnetic field intensity of about $300G$.

The analytical MTD disc can reproduce intrinsic linear polarization, flux in H_α, and disc radius that are comparable with typical observed and estimated values.

The effectiveness of a magnetic field in redirecting stellar wind outflow can be characterized by the ratio of magnetic energy density to wind energy density [77]:

$$\eta(r) \equiv \frac{B^2/8\pi}{\rho v^2/2} = \frac{B_*^2 R_*^2}{\dot{M} v_\infty} \frac{(r/R_*)^{2-2q}}{(1 - R_*/r^\beta)}$$

The term $\dot{M} v_\infty$ is the terminal momentum of the stellar wind; B_* denotes the intensity of the magnetic field at the equator, and R_* the stellar radius. Radial variation apparent in the two fractions on the right side is separated into the variation described by the magnetic power-law index q ($q = 3$ for a dipole), and the variation described by the velocity index β ($\beta \approx 1$ for line-driven stellar wind and a finite disc).

If we neglect the radial velocity variation, we can derive an Alfvén radius, $\eta(R_A) \equiv 1$, where the density of magnetic energy equals the density of wind kinetic energy:

$$R_A = \eta^{1/4} R_*$$

The Alfven radius provides an estimate of a maximal radius of closed magnetic loops in a star with stellar wind outflow of density ρ, velocity v, and terminal velocity v_∞.

Owocki and ud-Doula [77] have carried out magnetohydrodynamical simulations of line-driven stellar wind from a rotating hot star with a dipolar magnetic field aligned with the star's rotation axis to test the MTD model. They find that a magnetic field (moderate to strong) will cause the outflowing material to move along closed magnetic force lines that nearly corotate with the star, and thus in fact do provide additional angular momentum to the stellar wind. Contrary to the analytical MTD prediction, the material then does not form an equatorial disc, but tends to either fall back to the star (if it is below the Keplerian corotation radius, R_K, at which the equatorial centrifugal acceleration of a rigidly rotating body is equal to the gravitational acceleration in the same place), or break away completely (if it is above the corotation radius).

They conclude that the formation of a disc is still possible for very strong magnetic fields, albeit a magnetically rigid disc (MRD), in which the field maintains rigid-body rotation of the outflowing material. The typical Be emission, though, cannot be reproduced through a rigidly rotating circumstellar disc.

Acoustic Waves

Koninx and Hearn [81] have constructed a model of stellar wind combining radiative pressure in spectral lines (the theory of radiatively driven wind by Castor, Abbott and Klein [68] and the pressure of sound waves around the equator (theory of sound wave driven wind by Pijpers and Hearn [85]) to account for the dual nature of Be atmospheres: the fast and tenuous polar wind, and the equatorial slow and dense component.

The authors presume that both components are created by the stellar wind. In a first approximation the polar wind has the characteristics of a spherically symmetric purely radiatively driven outflow, while the equatorial wind is a spherically symmetric outflow driven by a combination of radiation pressure and acoustic waves. As a possible source of sound waves in Be stars non-radial pulsations are considered. The propagation of waves is described by linear theory.

The addition of a small flux of sound waves to the CAK-approximation radiatively driven wind results in increased mass-loss, and, surprisingly, lower wind velocity (presumably because both the radiative driving and the sound wave driving mechanisms depend on the same velocity gradient). If the sound wave flux is increased further and become the dominant driving mechanism, the mass-loss rate increases linearly with the mechanical flux. The solution places an upper limit on the mechanical flux input at the base of the wind.

Even with the maximum mechanical flux possible and the addition of rapid rotation, the over-all contribution of linear acoustic wave driving to Be star outflow turns out to be limited due to the stars' high gravity. Only waves with a very large amplitude can increase the mass-loss significantly, so there is still the possibility that non-linear waves could be more effective in increasing the mass-loss. The authors conclude that even when interaction between the radiation field and the acoustic waves is considered instead of treating them separately, it produces reverse rather than forward shocks needed to drive a Be stellar wind. Thus the linear acoustic wave theory is deemed applicable only in stars with very low effective gravity.

Bi-stability Mechanism in Stellar Winds

The so-called bi-stability mechanism was proposed for the stellar winds of P Cygni by Pauldrach and Puls [82], and later applied to Be and B[e] stars by Lamers and Pauldrach [83].

The basic principle is that if the stellar wind is optically thin, the mass-loss is mainly driven by strong lines in the Lyman continuum; if it is optically thick, the mass-loss is prevailingly driven by numerous weak lines in the Balmer continuum, which leads to higher lass-loss rates and lower terminal wind velocities.

In the case of P Cygni this duality can be seen as a discontinuity in mass-loss and terminal velocity depending on optical depth in the Lyman continuum. This bi-stability jump in the stellar wind is due to the change of opacity in the wind in the Lyman continuum from $\tau_L \lesssim 1$ to $\tau_L \gtrsim 3$. What happens is that when effective gravity decreases (for example due to the centrifugal force), the mass-loss increases and the terminal velocity in the wind is lower. At some point the mass-loss is so large and the outflow is so slow that the wind becomes optically thick (no longer see through for the photons) in the Lyman continuum ($\tau_{\lambda=912nm} \simeq 3$, while $\tau_{\lambda=600nm} \simeq 1$). Therefore the wind driving ions shift to lower ionization stages, as the photons at wavelegths below Lyman edge are blocked by the high hydrogen opacity, the flux in the Lyman decreases, and the flux in the Balmer continuum increases. So, the lower ionization lines in the Balmer continuum become dominant in the radiative driving of the wind, which causes the mass-loss rate to increase, the terminal velocity to decrease, and in effect raise the optical depth at the bi-stability jump about ten times, because wind density $\rho \sim \dot{M}/v$.

To apply the bistability mechanism to increase mass-loss in Be stars and B[e] supergiants the authors considered a B-star rotating at a considerable fraction of its break-up velocity. In such a star, because of the fast rotation, both mass-loss and the terminal velocity of the wind will depend on the latitude: mass-loss increasing at the equator due to lower effective gravity, and the terminal wind velocity decreasing because of the lower escape velocity around the equator. The brightness temperature will also decrease toward the equator due to the Von Zeipel theorem (so-called rotational darkening - in case of radiative transfer the flux of radiative energy on equipotential surfaces is proportional to effective gravity, therefore the star will appear darker on the equator).

Considering these effects, the dependence of optical depth τ_L on the stellar latitude β is as follows:

$$\tau_L(\beta) \sim (1 - \Omega^2 \cos\beta)^{-6.5}$$

Ω is the angular rotation of the star.

We see that the optical depth at the equator of a rotating B-star is much larger than at the poles. Depending on the pole optical depth, bi-stability jumps will occur at a certain latitude above and below the equator, and an outflowing disc with the according opening angle will form between them.

It has been shown that this mechanism does work for the B[e] supergiants [84]. But when applied to classical Be stars with a typical rotation velocity of less than $0.8 V_c rit$ and polar mass-loss rates of $\sim 10^{-9} M_\odot/year$ the bi-stability jump in opacity of the wind will not occur at any latitude. To be effective in Be stars bi-stability requires the presence of other mechanisms, such as a magnetic field, or non-radial pulsations.

When combined with another mechanism and functioning, according to [83] the bi-stability mechanism might also be able to explain the transition between Be, B, and B-shell phases if the mas-loss were variable. The opening angle of the disc would vary with the changing mass flux, and could also disappear completely, if the mass-loss becomes too small.

Chapter 3

Rotation Law in Be Discs

3.1 Rotation Law

The change of rotation velocity in the radial direction - the rotation law of the circumstellar envelope - is parameterized as

$$V(R) = V(R_*) \left(\frac{R}{R_*} \right)^{-j}, \tag{3.1}$$

$R > R_*$.

Here $V(R_*)$ is the rotation velocity on the surface of the star that we derive in the form of $V(R_*) \sin i$ from the broadening of the absorption lines in the spectra (we have already mentioned in Section 1.3.6 that obtaining the true rotational velocity of the star is a very complex problem). The exponent j is the rotational parameter that depending on its value determines the kind of rotation.

This equation can be expected to be a good approximation even in the presence of radial motions in the envelope, since they are small compared to the rotational velocity ($\sim 10^1$ km/s compared to $\sim 10^2$ km/s).

Rigid Body Rotation

This is the case with $j = -1 \Rightarrow V(R) = V(R_*)\frac{R}{R_*}$; the angular velocity everywhere in the envelope is the same as on the surface of the star.

In reality no rotating solid body is perfectly rigid. A non-rigid body, when rotating, will be distorted by the centrifugal force. If the rotation is slow enough, so that the distortion is small in comparison with the size of the body, the effect can be neglected, and the body approximated as rigid.

But extended atmospheres of Be stars are neither solid, as they essentially consist of radiation and gas in various stages of ionization, nor do they conform to the condition of moderately slow rotation. Therefore the case of rigid rotation does not occur for Be discs. Furthermore interferometric measurements of Be stars [6] have already confirmed that they are rotationally distorted.

Conservation of Angular Momentum

The case with $j = 1 \Rightarrow V(R) = V(R_*)R_*/R$. The angular momentum conserved is $L = RV(R) = R_*V(R_*)$, which means that the further from the star we get, the more does the angular velocity decrease. In effect the atmosphere will rotate in layers, each of them rotating at a different angular velocity.

For a Be star this would imply a scenario of a disc with continuous mass-loss, where new mass and angular momentum would have to supplied at the lower boundary (the surface of the star). The previously mentioned wind compressed disc is an example of a Be disc model that preserves angular momentum, as the disc is supplied by a steady flow of stellar wind.

Keplerian Rotation

This is the case with $j = 1/2 \Rightarrow V(R) = V(R_*)\sqrt{R_*/R}$ (and $V_* = \sqrt{GM/R_*}$).

Keplerian rotation is stable rotation with particles in circular orbits that is established in the case when centrifugal force just balances out the gravitational force. If we want a Keplerian disc to be formed, we have to allow for some small radial motion. This implies that it has to be formed by episodic mass-loss with times of quiescence in-between, so that sufficient time was provided for the orbits to stabilize and circularize.

Another possibility to obtain Keplerian rotation is accretion. If the pressure forces in the accretion disc are negligible when compared with gravity, an accretion disc can be approximated as thin, its rotation will be differential and Keplerian due to viscous transport of angular momentum outward.

3.1.1 Importance of the Rotation Law

Why is it important to find the rotation law in a Be circumstellar envelope? We have introduced the main branches in Be-disc modeling. All of those models have concentrated first and foremost on reproducing the distinguishing Be characteristics (Hα emission, linear polarization in the continuum, IR excess, variability), since comparison with observations is the only test for a model. Most of the models that were described here are indeed able to do that with various success.

An ideal model for the Be envelope would , of course, have to include and reproduce every distinguishing Be feature, and every kind of variability observed. Some of the presented models have had provided results reasonably close to the observation, but the proposed mechanism has turned out to be incorrect in the end. So, if we have two physically different models of the Be disc, capable of reproducing observed features to the same degree, how do we decide which one of them is acceptable? Here it is important to note that even when the WCD model was discarded on the grounds of hydrodynamical simulations, it was because the simulation did not produce the observationally inferred disc density.

If a model does not produce the same picture of a Be star as observation, we know it is not valid. However, if it does, it doesn't necessarily mean it is correct. A model can be painfully constructed to fit the observed values, and still need not be correct in the physical sense. Therefore the rotational law of the envelope should be considered as an additional measure of validity. This is because to determine the rotational law (or, more specifically, the rotational parameter) of a Be envelope means to place constraints on the models physically possible for that envelope.

3.2 Methods of Obtaining the Rotational Parameter

3.2.1 Line Profile Modeling

This is a method that Hummel and Vrancken ([92]) have used to determine the rotational parameter. They adapted the theory of Horne and Marsh ([94]) on shear broadening in accretion discs to Be envelopes, simulated H_α profiles at different rotation laws in the disc, and then fitted them to observed H_α profile to derive disc parameters.

Usually H_α emission profiles in Be stars are complex due to the combination of absorption and strong emission, and various broadening processes (rotational broadening, pressure broadening, electron scattering in the wings) that, although they are qualitatively understood, tend to reduce the information carried by the profile. The authors consider H_α lines of low equivalent width more suitable for modeling. They also suggest metallic lines (Fe II), but the strength of emission in low W_{eq} H_α lines is empirically correlated with emission in metallic lines.

The principle of shear broadening is that line photons can escape from an emitting, optically thick, and differentially moving layer along the direction of the largest Doppler gradient. It is an anisotropic scattering process that will deepen the central depression in the H_α emission profiles, arising from the fact that the layers of the envelope rotate differentialy.

The shear velocity is given by ([94])

$$V_{sh} = -\frac{H}{2R} V_K \sin i \tan i \sin \phi \cos \phi$$

where H is the height of the disc, i is inclination, ϕ is the azimuthal angle in the disc plane, and V_K is the local value of Keplerian velocity. $V_K \sin i$ is the kinematical broadening that determines the total width of the calculated lines most. Keplerian rotation is assumed.

The total line profile width is a combination of the thermal broadening V_{th} and shear broadening V_{sh}:

$$\Delta V = (\Delta V_{th}{}^2 + \Delta V_{sh}{}^2)^{1/2}$$

The authors neglect electron scattering, therefore their theory can be applied only to low-emission Be stars, in which the broadening in the wings of the H_α line can actually be neglected. However, they do take finite size of the stellar disc, shell absorption and obscuration of the star by the disc at certain inclinations into account.

The rotationally broadened theoretical lines were then compared with observed H_α emission lines. Because these had low W_{eq}, the underlying absorption profile was dominant, and had to be subtracted. In a first approximation the modeled lines were fit to the observed ones by varying circumstellar parameters like disc radius or number occupation density.

The shear velocity for a general parameter j instead of the assumed $j = 0.5$ is

$$V_{sh} = -\frac{jH}{R} V \sin i \tan i \sin\phi \cos\phi$$

For profiles of optically thin lines the velocity gradient merely re-scales the velocity along the line of sight, therefore some of the best fit parameters obtained in the Keplerian case could be used in the fitting for the case of conservation of angular momentum, $j = 1$. Only number occupation density, the density gradient and disc radius were varied. The $j = 1$ best fit corresponded with a smaller and thinner disc, than the values for $j = 1/2$. It was, however, possible to fit the optically thin lines for both values of j reasonably well. Although results were obtained, they were indistinguishable for angular momentum conserving and Keplerian rotation. Optically thin lines cannot be used to constrain the rotational parameter j, since their profile does not depend on the velocity gradient.

The profiles of optically thick lines are dependent on the shear velocity. For the case of $j = 1$ the shear velocity will be larger than for $j = 1/2$ if the radius $R < 2$, and lower if $R > 2$. Since we know that Be discs generally extend to up to 10 and more stellar radii, we can say that shear broadening is less effective for $j = 1$, so much so that the fitted line fails to form the central depression between peaks.

The authors also found that the peak separation in H_α emission lines increases with decreasing rotational parameter j.

They constrained the rotational parameter to $j = 1/2$ on the grounds of lines fitted to observational spectra of the Be-shell star HR 5440. Yet this result holds merely under the assumption that shell stars are not actually physically different from Be stars (the stellar obscuration, for, example was treated in a purely geometrical way), but are in fact Be stars seen at large inclinations.

A different approach from the previous method represents the work of Hanuschik ([95]), who derived the velocity structure of a Be envelope by calculating optically thin, rotationally broadened Fe II emission lines, instead of Balmer lines, from a cylindrically shaped disc. Fe II emission lines are much less disturbed by non-kinematic smearing like thermal motion or electron scattering, and have smaller line opacities. Because the intrinsic broadening of these lines is small in comparison with the rotational broadening (in Fe II $\lambda 5317$ $\Delta v_{th} = 1.7km/s$ compared to e.g. $400km/s$ [97]), they carry more local kinematic information about the velocity and density of the envelope than Balmer lines.

He finds that the Fe II profiles can be sufficiently fitted with optically thin, purely rotating models with disc-like geometry, where very little radial motion is present (the disc is close to Keplerian).

The author suggests that true Keplerian rotation is most probable in the innermost, densest parts of the envelope, where the transport of angular momentum becomes more effective through collisions. Since the Fe II lines are formed in the innermost regions, he adopts the value $j = 1/2$.

3.2.2 Half-Peak Separation in H_α

Half-peak separation of an emission line reflects the external rotational velocity of the circumstellar disc, seen at an inclination i.

Mennickent et al. [40] analyzed a sample of 84 stars and the half-peak separations in the H_α, H_β and H_γ lines using the following relations.

Half-peak separation $\Delta\lambda_n$ in the n-th line of the Balmer series is related to the rotational velocity given by the n-th Balmer line originating near the outer edge of the emitting disc through [98]

$$c\frac{\Delta\lambda_n}{\lambda} = V_n \sin i \tag{3.2}$$

When combined with the expression for the rotation law, we get an equation for the radius of the emitting disc:

$$\frac{R_n}{R_*} = \left(\frac{V_n}{V_*}\right)^{-\frac{1}{j}} \tag{3.3}$$

Here V_* is the critical velocity, $V_* = \sqrt{GM_*/R_*}$.

Each n-th line was then fitted by linear least squares

$$\left(\frac{V_n}{V_*}\right) = a\left(\frac{V_\beta}{V_*}\right) \tag{3.4}$$

The resulting coefficients a give a direct measure of rotation velocity normalized to the stellar rotation velocity of each of the Balmer line-emitting regions. In this way the authors obtained relations of the rotational velocity of the H_α line in the units of stellar rotation velocity of the disc seen in H_α against the H_β values; the rotaional velocity of the H_γ against the H_β values, and so on, for the lines H_α, H_β, H_γ, H_δ and H10. The rotational parameter j can be obtained by combining the results with net equivalent widths (equivalent width of the underlying absorption profile plus the equivalent width of the emission). The net equivalent width of an emission line in an optically thick disc is proportional to mean electron density and to the visible area of the disc:

$$W_n^{net} = f n_e^2 \left(\frac{R_n}{R_*}\right)^2 \tag{3.5}$$

where f is an inclination factor that is considered constant in the first approximation. Combined with equation (3.1) this gives the final formula used in obtaining j by this method [39]:

$$\log\left(V_n/V_*\right) = (j/2)\log\left(f n_e^2\right) - (j/2)\log\left(W_n^{net}\right) \tag{3.6}$$

This is a linear relation between the logarithms of V_n/V_* and W_n^{net} with a slope of $(j/2)$. According to the values employed by the authors, the least square fit of this relation gives a rotational parameter of 1.4 ± 0.2 in H_α for their whole sample of Be stars. The same result of $j = 1.4$ was reached by Mennickent [39] in H_β for a sample of seven southern Be stars.

A different way of deriving the rotational parameter from half-peak separation was adopted by Hanuschik in [96] from a theory by Huang [98] that approximates the Be envelope as an axisymmetric, cylindrical, and optically thin rotating disc. In such an approximation the half-peak separation (given in units of velocity) normalized by $V \sin i$ of the star is only a function of the outer radius of the emitting disc:

$$\frac{\Delta V_{peak}}{2V \sin i} = R_d^{-j} \tag{3.7}$$

Investigating the correlation between $V \sin i$ and half-peak separation in H_α and Fe II emission lines of a large sample of stars, he derived $j \approx 0.6 - 1$.

Due to the fact Fe II emitting radii are much smaller (Fe II lines are emitted deeper in the envelope) than the H_α radii, he states that rotational velocity must be monotonously decreasing in the outward direction.

These calculations support the existence of a non-Keplerian disc in Be stars.

3.2.3 Comparison of Interferometric and Spectroscopic Disc Radii

Another constraint for the rotation law in Be discs is a comparison between the values of radii derived directly by interferometric methods [6], and disc radii obtained from the half-peak separation in emission lines.

However, one must keep in mind that the inteferometric disc radii of [6] measured at half-width half-maximum (HWHM) of the lines are the lower limit when compared to the method of half-peak separation. This is because the measured radius of the star depends on the wavelength of the line - if we measure the radius in the center of the line, the star will appear larger, since the optical depth in the line varies from the centre to the wing, with it being largest in the centre of the line.

Emission strength in the spectra of Be stars depends on the size of the emitting region, therefore disc radii can be derived from the half-peak separation of emission lines modeled with an assumed velocity law and fitted to observed emission [93]. The value of disc radii derived in this way, though, is dependent on the V/V_crit ration, or more specifically $V \sin i$, because the half-peak separation is a function of both $V \sin i$ and equivalent width [96]. Here [92] the values 1.0 and 0.8 of said ratio were used.

The interferometric and spectroscopic values are then compared to find out at which value of the rotational parameter j the interferometric radius matches the spectroscopic one:

$$j = \frac{\log R_d/R_*}{2 \log a/a_*} \tag{3.8}$$

Because the interferometric radius was the lower estimate, the resultant parameter j is an upper estimate of the real value.

The values of the rotational parameter calculated by [92] using this method ranged from $j < 0.76$ for $V = V_{crit}$ to $j < 0.65$ for $V = 0.8V_{crit}$. We consider the second costraint placed on the rotational parameter to be more realistical, since the considered rotation velocity is closer to the usual observationally inferred value.

Interferometry offers a by far the most direct way of finding out the rotational law of the envelope - to measure the radius of the envelope in each sufficiently bright spectral line. In this way we would have an immediate relation of $V \sin i$ of that concrete line (derived from rotational broadening) and the emitting radius (depth in the envelope where the line was emitted.

3.3 Implications for Be Models

If we agree that the rotational parameter is an important way to constrain the physical possibility of Be envelope models, from this point of view we may have a look t the Be models available.

Many global (attempting to model a multitude of Be characteristics) and partial models (simulating one particular Be feature) tend to assume the rotational law to be Keplerian, without further delving into the issue. This is because Be envelopes are generally understood to be thin discs. The accretion theory by Shakura and Sunyaev [52], as well as the viscous excretion disc by Lee, Saio and Osaki [53] produce a thin Keplerian disc. On the other hand, the viscous excretion disc is able to reproduce the dissipation and reformation of a disc only if there is a time-dependent flow of matter present from the surface of the star. We know that there indeed is such a source - fast stellar wind, possibly in combination with other factors like outbursts. But the result in our opinion would not be a disc rotating in a Keplerian fashion.

Keplerian rotation is an approximation valid for large systems where most of the mass is concentrated at the center (in a volume very small in comparison with the dimension of the system), and mass of the rest is fairly negligible. In that case the velocity of anything orbiting the centre at a reasonably large distance will be inversely proportional to the square root of that distance from the center, and its orbit will be nearly circular. In Be stars the gaseous disc extends to 10 - 15 R_*, has non-negligible mass, and furthermore, roughly about every decade a massive event such as the disappearance and reformation of the disc takes place, which would involve considerable mass-outflow and inflow. Simply on physical grounds, in our opinion the Be disc diverges from purely Keplerian rotation, even if the viscous energy transfer that propagates stability and Keplerian rotation in the disc is very effective. Even calculations of the rotational parameter that supported Keplerian rotation [92], allowed for variations from $j = 1/2$ to up to 0.65.

More probable than a strictly Keplerian/angular momentum conserving rotation is a different scenario tentatively proposed by some (e.g. [95]). Here the inner, denser parts of the Be disc rotate close to Keplerian fashion, because the viscous transport of energy outward needed to stabilize the thin disc is more effective. The outer, more tenuous parts near the conservation of angular momentum, since there is outflow at the outer boundary. The assumption of outflow is only natural, since the disc will dissipate sometime during the life of the star. Furthermore, if the inner parts of the disc are fairly stabilized and their rotation is near Keplerian (e.g. $j \sim 0.65$, as according to [92]), the outflow at the outer edge will be continuous, even if the disc has been formed by episodic enhanced mass-loss.

Recently, [65] have shown that in the radiative force there is always an azimuthal component that causes a transfer of angular momentum from equatorial stellar wind, if the radial change of the azimuthal component is less than that of rigid rotation. In this way the wind driving radiation can actually spin-down the stellar wind. In effect the rotation velocity of the envelope would then decrease with radius faster than required for the envelope to conserve angular momentum. When applied to the WCD model this could mean that the resulting disc need not be strictly angular momentum conserving.

We may further look at the rotation law from the viewpoint of the conditions that must be met by a thin Keplerian accretion disc. The WCD model (later proven invalid, but for different reasons) had been criticized by many for the sole reason that it was an angular momentum conserving model, and thus was incompatible with the best explanation for long-term V/R variability, which is the model of one-armed oscillations (GDOs). The GDO model requires a Keplerian disc to work. It introduces a global azimuthal pressure wave in the circumstellar disc that it requires to be a thin Keplerian one. But the assumption of Keplerian rotation in thin discs is only valid if the pressure forces inside the disc are negligible compared to gravity. If that is not the case, the disc becomes thick, which is clearly the case of a globally oscillating disc.

The greatest problem of said Be models is physical inconsistency. To develop a more reliable way of obtaining the rotational parameter may provide us with a way to finally rule out those that are physically impossible for a Be star.

Chapter 4

Applied measurement of rotational index j

4.1 Applied method

In order to apply one of the methods of determining the rotational parameter j the method described by Mennickent et al. [40] based on half peak separation in double-peaked emission lines was chosen.

The method determines j by applying equation (3.6)

$$\log(V_\alpha/V_*) = -\frac{j}{2}\log W_\alpha^{net} + \frac{j}{2}\log(fn_e^2)$$

to a large sample of Be stars. The procedure is simplified by two important assumptions:

a. the inclination function f and the mean electron density n_e are considered constant in a first approximation, and

b. the rotational parameter j is more or less the same for all Be stars in the sample.

In that case, the equation above is reduced to a linear relation between logarithms of rotational velocity of the H_α line normalized by the rotation of the star, and the net equivalent width of H_α, with the slope of the relation being $-j/2$. According to the procedure applied by Mennickent et al., the rotational parameter j here is the mean rotational parameter for the overall Be group. Because the functions of f and n_e are considered constant, they will merely cause a shift of the plotted linear relation, and can in the end be estimated, albeit very roughly, from the shift.

For the purposes of this study we modified some of the procedures due to the amount of data available to us. Instead of examining a large sample of stars by analyzing one H_α line per star, we selected three Be stars - 4 Her, κ Dra, and ϕ Per, analyzing all spectra available for these stars respectively. We assumed that equation (3.6) must be valid for every single Be star in the sample, as well as for the sample as a whole, therefore we first analyzed each selected star separately, and then examined the sample of merged data for three stars, and compared the results.

4.2 Measuring Procedure

The spectra used were taken with the 2-meter telescope at the observatory in Ondrejov, Czech Republic, with the echelle spectrograph HEROS. Thereafter they were continuum-normalized and the Balmer lines were analyzed with the IRAF software.

The Balmer lines processed in each spectrum available for each of the stars were H_α, H_β and $_\gamma$. The results of J presented here are for H_α only, as the simple relation between equivalent width and the square of the normalized radius of the line-emitting region is valid only for H_α lines in optically thick envelopes. Half-peak separation and rotational velocities were calculated and are presented for all lines that have been analyzed.

The features measured were: emission maximum in the violet peak V_{max}, emission maximum in the red peak R_{max}, estimated centers of both peaks V_{cen} and R_{cen} in spectra where the peaks were strongly asymmetric, minimum in the center of the line $line_{min}$, equivalent width of the line W_{eq}, and center of the line produced in the measurement of equivalent width $line_{cen}$.

The values of V_{max}, R_{max}, V_{cen}, R_{cen}, line_{min}, and line_{cen} were used in different combinations to determine the rotational velocity of the line, and also to estimate the systematical error brought on by the person conducting the measurement. The combinations of these values that were calculated are

$$V_n sini = c\frac{R_{max} - V_{max}}{line_{min}}$$

$$V_n sini = c\frac{R_{cen} - V_{cen}}{line_{cen}}$$

$$V_n sini = c\frac{R_{max} - V_{max}}{line_{cen}}$$

4.3 Analysis of Results

4.3.1 4 Herculis (HD 142926)

4 Her is an often observed, long-term variable spectroscopic binary, with an orbital period of 46 days [99]. The primary is a B9pe star. The secondary is light and invisible. It is a non-interacting binary, meaning that no mass transfer between the components has been detected. Since 4 Her has been observed for over 75 years, the change from normal B star to Be star to shell

Balmer line	theoretical W_{eq}
H_α	59.0912
H_β	47.7545
H_γ	45.9066

Table 4.1: Theoretical equivalent widths of Balmer lines supplied by a non-LTE model for 4 Her

star has been documented in this star as in one of the very few. Aside from these, 4 Her does not seem to undergo significant, more rapid variability.

The values adopted from an LTE model of a Be atmosphere in [99] and applied to our data are $T_{eff} = 12500K$, $\log g = 4.0$ and $v \sin i = 300$ km s^{-1}. As the measured equivalent widths were of lines combined of absorption and emission, they had to be corrected by subtracting the theoretical W_{eq} of the underlying photospheric absorption, provided by a non-LTE model of J. Kubát and listed in 4.1.

The data table for the half-peak separation analysis of 4 Her are included at the end of the chapter. Here we will merely present the results of linear regression of the $\log(V_\alpha/V_*)$ versus $\log W_{eq}$ plot, where W_{eq} equals the directly measured equivalent width of the H_α line corrected by the theoretical equivalent width.

Figure 4.1: A $\log(V_\alpha/V_*)$ versus $\log W_{eq}$ graph for 4 Her

The linear regression in 4.1 is described by the equation $\log(V_\alpha/V_*) = -1.0248 \log W_{eq} + 1.4792$, where the slope of -1.0248 corresponds to $-j/2$.

The result of $j = 2.0496$ suggests neither Keplerian nor angular moment-conserving rotation. Of all the rotation index-finding methods that have been discussed in chapter 3, though, this result is closest to the one reported by Mennickent et al. [40].

4.3.2 κ Draconis (HD 109387)

κ Dra is a bright, highly variable, single-line spectroscopic binary [104], with an orbital period of $61^d.55$. Its primary, a B6IIIpe star, has been observed as a Be star since 1895 [100]. The variability of κ Dra has been a questionable issue for a long time, but recently the variations of emission in H_β and the more rapid line profile variations have been shown to be periodic. The long-term variations of emission intensity in the Balmer series are presumed to

Balmer line	theoretical W_{eq}
H_α	21.1081
H_β	13.272
H_γ	10.7743
H_δ	10.127

Table 4.2: Theoretical equivalent widths of Balmer lines supplied by a non-LTE model for κ Dra

be connected the Be phenomenon, more precisely to an unknown mechanism supplying material to the disc of the Be primary with a period of 23 years.

We adopted the results of a non-LTE model fit by [100] to use in our calculations here. They are $T_{eff} = 14000K$, $\log g = 3.5$ and $v \sin i = 170$ km s^{-1}. The non-LTE theoretical values of W_{eq} of the underlying absorption seen in 4.2 were again supplied by J. Kubát.

We present the result of linear regression in 4.2.

Figure 4.2: A $\log(V_\alpha/V_*)$ versus $\log W_{eq}$ graph for κ Dra

Again, the values of $\log(V_\alpha/V_*)$ against $\log W_{eq}$, and their linear regression were plotted. The resulting linear regression is described by the equation $\log(V_\alpha/V_*) = -1.059 \log W_{eq} + 1.1021$. The result of $j = 2.118$ is almost identical to the calculated rotation parameter of 4 Her above, and, again does denote neither Keplerian, nor angular momentum-conserving rotation, at least at the outer edge of the envelope, if we keep in mind the restriction given by our use of the relation between half-peak separation and rotational velocity of the respective emission line in equation (3.2).

4.3.3 ϕ Persei (HD 10516)

ϕ Per is a highly variable spectroscopic binary consisting of a B0.5IVe primary and what is believed [102] to be a hot (up to 53 000 K [101]) O-type subdwarf secondary, possibly a remnant of an extensive mass transfer onto the now-Be primary, with an orbital period of $126^d.7$. Compared to the previous two observed non-interacting binaries, in ϕ Per the contribution of the secondary to the overall radiation of the binary is much more significant, therefore a difference in results when compared to 4 Her and κ Dra can be expected.

For our purposes we adopted following values of the Be primary from [103]: $T_{eff} = 29300K$, $\log g = 3.7$ and $v \sin i = 450$ km s^{-1}. We can notice that the

Balmer line	theoretical W_{eq}
H_α	97.6596
H_β	83.117
H_γ	73.6012

Table 4.3: Theoretical equivalent widths of Balmer lines supplied by a non-LTE model for ϕ Per

rotational velocity of the primary is quite close to break-up rotation of a star its size (550 km s^{-1} [101]), which can be viewed as an argument in favor of the primary spin-up by mass-transfer from the secondary.

The non-LTE model theoretical equivalent widths of the absorption spectrum applied are included in table 4.3.

In the $\log(V_\alpha/V_*)$ against $\log W_{eq}$ plot in 4.3 we immediately notice that the data are very scattered.

Figure 4.3: A $\log(V_\alpha/V_*)$ versus $\log W_{eq}$ graph for ϕ Per

The slope of linear regression $\log(V_\alpha/V_*) = 21.236 \log W_{eq} - 46.576$ is negative even, which does not correspond to any physical case of rotation, as it implies (from equation (3.3), a different form of the rotation law) that the line-emitting radius (or, the radius at which the emission in question is formed) is below the surface of the star. We find that it is not possible to draw any conclusion whatsoever from the result for $j = -42.472$ given by this method when applied to ϕ Per, except that it is not reliable in this case.

4.3.4 Distribution of Three Stars

Finally, we merged the processed data of the three stars regardless of the results the half-peak separation method dealt in individual cases, and applied linear regression. This is the procedure originally used by the author of the method [40], except that his distribution contained a sample of 51 stars, each represented by one pair of values only.

The resulting plot can be seen below.

Figure 4.4: A $\log(V_\alpha/V_*)$ versus $\log W_{eq}$ graph for a distribution of three Be stars

The resulting linear regression of $\log(V_\alpha/V_*) = -0.3104 \log W_{eq} + 0.025$ gives a result of $j = 0.6208$. This is, from our point of view, a very sensible result. It does not quite correspond to Keplerian rotation, but rather to what we would expect to measure if we were limited to the outer edge of a circumstellar disc that is very close to Keplerian at its inner edge, because the radial mass in-flow/out-flow at the edge will be greater, thus corrupting the Keplerian stability.

This is a result that can be more easily understood and explained in physical terms than the previous discrepancies, even though it does not agree with the result of Mennickent et al.

However, the question offered in this case is can we accept such a result to be a correct estimate of the rotational parameter, even though we used such a small sample, and processed it differently? How reliable is such a seemingly reasonable result, if we have included the interacting, and therefore inapplicable, binary of ϕ Per?

We therefore excluded the data of ϕ Per while taking upon us the great risk and inaccuracy of linearly regressing a sample of only two stars, to see what would happen when the unusable data are removed. A valid method should even in these conditions give an at least physically understandable result.

The 'corrected' distribution is shown in 4.5.

Figure 4.5: A $\log(V_\alpha/V_*)$ versus $\log W_{eq}$ graph for a distribution of the merged data of κ Dra and 4 Her

The result of $j = -2.2142$ is, again, insensible in the physical sense, after correctly removing inapplicable data from the sample. This in fact confirms that the previous result of the three-star sample is invalid and not to be taken into account.

4.4 Discussion

We have seen that the reliability of the rotational parameters gained by this method is to be doubted in the very least. In the following discussion we will try to explain what parts of the method of Mennickent et al. that we applied here may have caused this. We will also discuss the appropriateness of the procedures used in our application of the method, and the overall reliability of the method.

First of all the basis of the method itself, which is the measuring of half-peak separation itself possibly brings in a strongly subjective systematical error. The term of 'half-peak separation' in this case is not properly defined:

it may be considered the separation of the peaks themselves, or - which would in our opinion be more accurate - the separation of the peaks' centers, as in the processed spectra, which were taken over years in some cases, the peaks often vary from completely symmetrical to asymmetrical to a single-peaked emission and back. However, in IRAF profile fitting of individual peaks is not possible, therefore we had to rely on estimates of the asymmetrical peaks' centers.

We attempted to minimize this subjective error by calculating various combinations of half-peak separation and line center in deriving $v \sin i_{line}$, but the results for other combinations not shown here have proven that this systematical error is after all quite small and does not change the overall character of the resulting $\log(V_\alpha/V_*)$ vs. $\log W_{eq}$ plot (the slope of the linear relation varies only by approximately ± 0.05).

Further let us note that we have assumed that the approximated linear relation between the terms $\log(V_\alpha/V_{star})$ and $\log W_{eq}$ is valid for individual stars as well as for the distribution of these stars, which would be true if the stars in the distribution have approximately the same rotational law as stated in [40]. However, the results differed greatly when compared.

Here it may be argued that our sample is much too small for any conclusions to be drawn, moreover after one of our stars turned out to be unusable for this purpose. We wish to point out that were the rotational parameters merely quite different this may have been a valid argument, but we have in fact two individual results that are unusual but can potentially be explained and a rotational parameter for the merged data that cannot be taken into consideration in a physical sense at all.

It may be argued as well that the spectra used by the author of the method originally were taken over a shorter time, thus our sample is more prone to errors occurring due to variability of equivalent width and half-peak separation. For this purpose we have plotted the time-dependencies of equivalent width and rotational velocity of the line-emitting envelope for each of the examined stars (see Appendix) to estimate the influence of mid-term variability on j. It became obvious that in 4 Her and κ Dra $V \sin i_\alpha$ remain close to constant over time, regardless of which points in the emission line we have measured. The equivalent widths in 4 Her vary by approximately 2 continuum intensity units, and in κ Dra by approximately 6 units. This, however, in the logarithmic scale cannot make a difference large enough to render a method useless. In ϕ Per we find what appears to be a spin-down of the calculated rotation of the H_α-emitting envelope that we have no explanation for, but one that can be certainly considered to be one of the reasons for the peculiar results in this star.

Therefore we must conclude that the largest error is caused by the restrictions placed by the theoretical equations that this method is based on. If we neglect the fact that the method is limited to the outer edge of the disc and the H_α line due to the use of equations (3.2) and (3.5), it is important to note

Figure 4.6: Time-dependency of rotation in the H_α-emitting region for 4 Her, κ Dra, and ϕ Per

Figure 4.7: Time-dependency of equivalent width of H_α for 4 Her, κ Dra, and ϕ Per

Figure 4.8: Source data and calculations for H_α in 4 Her

Figure 4.9: Source data and calculations for H_β in 4 Her

Figure 4.10: Source data and calculations for H_γ in 4 Her

Figure 4.11: Source data and calculations for H_α in κ Dra

Figure 4.12: Source data and calculations for H_β in κ Dra

Figure 4.13: Source data and calculations for H_γ in κ Dra

Figure 4.14: Source data and calculations for H_α in ϕ Per

Figure 4.15: Source data and calculations for H_β in ϕ Per

Figure 4.16: Source data and calculations for H_γ in ϕ Per

that the inclination function f and mean electron density n_e are assumed to be constant throughout the disc. In reality, for an individual star its inclination can be easily considered to remain constant over time, but reasonably not for a sample of stars. On the other hand, mean electron density will not remain constant throughout any disc, if only for the fact that the disc is finite.

The results shown here - especially the notable difference between the results of the same method when applied to individual stars and then applied to a group of the same stars - in our opinion prove that a first approximation is by far not accurate enough to base estimates of the rotational parameter on. In other words, the linear relation of $\log(V_\alpha/V_*)$ and $\log W_{eq}$ is so simplified a case that it is rendered inapplicable. The plots for individual stars and also for their distribution do close to no trend, linear or otherwise, at a mere sight, and the following linear regression is riddled with a large error margin.

For these reasons we must consider even the very reasonable-seeming rotational parameter resulting for the distribution of 4 Her, κ Dra and ϕ Per erroneous.

Chapter 5

Conclusion

Be stars are hot, massive, main-sequence stars of the spectral type B displaying Balmer emission that presumably originates in an equatorial disc.

While defined by the presence of Balmer emission, there are many other characteristics, the distinct combination of which sets the Be stars apart from the rest of the B class. Especially the fact that Be stars rotate so rapidly that they form a different velocity distribution than other B stars makes them suitable objects to study the effects of extremely fast rotation on massive hot stars. Rapid rotation is also considered to be one of the keys of the so-called Be phenomenon - that is the question of how the equatorial disc forms around a star of such specific characteristics.

The Be phenomenon can be viewed in two ways : as an universal unknown mechanism capable of explaining the circumstellar disc around each and every Be star; or as a mechanism constructed specifically to explain the origin and structure of a disc around one Be star.

An important point to note is that every Be star is unique. Except the defining factors the Be stars as a group can be described only in relative terms. For example, they rotate rapidly in general, but there are also intrinsically slow rotators among them, as found by [40] and others. Yet, the differing projected rotation of Be stars may also be understood as representing a variety of alignments of the rotational axis. In general they are variables, but there are Be stars in which variability has not been observed, presumably because they have been under observation for to short a time, if we take into account the possible period of some long-term variations occuring in Be stars, but that must not necessarily be true. Also, the type of variability seems to depend on spectral subtype and $V \sin i$, therefore some Be stars are more likely to display more rapid variations like *lpv*, and others slower variations (B - Be - Be-shell transitions). Of course, they differ in strength of emission, polarization rate, IR excess, and other obsevable characteristics, as well.

So the question remains, does it make sense to look for one specific Be mechanism or combination of mechanisms?

An universal solution to the Be problem would have to account for all observed Be features, all types of variability, and also be capable of believably explaining the differences among concrete Be stars. This is what models such as the viscous excretion disc, the viscous excretion disc in combination with one-armed oscillations, and the wind comressed disc attempt. On the other hand, it is entirely possible that there are , in fact, *different* mechanisms acting to form and sustain a disc among different Be stars, or at least, observationaly differing subgroups of Be stars (rapidly and slowly rotating, rapidly or slowly variable, and so on, and combinations thereof). These Be mechanisms would be appliable to a certain star according to its features.

The physical difference between single Be stars and binaries certainly must be considered. The spectra of these two groups are virtually indistinguishable from each other, except for Be/X-ray binaries that display X-ray bursts at times of the companion's passage through the plane of the disc. Yet we know that there are different physical restrictions placed on the disc model. An equatorial disc around a single B star, whatever its origin, was clearly formed in very different conditions from those for a disc around a star in a mass-transfering binary. The origin of a single Be star would have been determined by its rotation, luminosity, mass, while in a Be containing binary it would be first and foremost by the initial mass ratio that establish the mass-loss or mass-gain, the eventual spin-down of one star and spin-up of the other, and their evolution. Why do the spectra of single versus binary Be stars not reflect this fact?

Furthermore, in mass-transfering Be binaries we have a rather good idea of the way a disc is built up. It is entirely conceivable that even if that cannot be the case in all Be stars, around those in close binaries the discs do form through accretion of material onto the mass-gainer, or even that the effective gravity much lowered due to the presence of a more massive object would enhance natural, rotationally induced mass-loss, and thus help the decretion of a disc around the mass-loser. The first case mentioned are accretion discs that we are able to describe satisfactorily [52], in the second case the description could be perhaps analogous to the viscous excretion disc model [53], with a known source of enhanced mass-loss.

The Be stars in binaries on the whole seem to be a more transparent problem, although not solved by far. Binary Be systems can be found that are simply too wide (as is the case of 4 Her and κ Dra, for example), and the origin of the Be disc cannot be explained away by mass-loss or significant gravitational interaction. That binarity must have some sort of significance for the Be phenomenon is highlighted by the fact that as of yet there are *no* Be-Be binaries known.

Single Be stars represent the another part of the puzzle of Be discs, since we neither know the source of the disc, nor the cause of such high rotational velocities. These are the main unknowns, as the other observable Be peculiarities like emission, IR excess or resonance UV lines can be explained either by the presence of a disc (not addressing its origin) or that of a strong radiatively-driven stellar wind (which is a fairly typical feature in high-luminosity stars).

The search for the Be mechanism should be conducted in a parallel way on single, interacting and non-interacting Be stars to map in detail the similarities and the differences between them. We propose that all models should be tested on on binaries and single Be stars, as well, since both groups present a test that a successfull model should be able to pass. This, of course, applies only in the case when by searching for a 'Be mechanism' we truly mean a specific one mechanism that would be capable of explaining the Be phenomenon throughout spectral subclasses, the range of rotational velocities, and the different physical restrictions of singularity/binarity. We would like to stress that due to Occam's razor, it is most advantageous not to look for a completely new, previously unheard of physical process, but to explore the possibilities and combinations of the processes we know. Since we know that rotation and stellar wind are the key elements in the formation of a disc around a single Be star, the forces taken into account in particular are gravity, centrifugal force and radiative force. An example of an elegant solution using only these three basic global forces present in most Be stars is the Wind Compressed Disc [61], even though the model itself is not functional.

The WCD model actually clues us in on where the solution of the Be pehnomenon may lie. It may be in the although known, but comparatively small, and thus neglected factors, and processes not accounted for in sufficient detail. These are especially the extreme rotation and its effects on the underlying star, which is important, as in the case of single Be stars we presume that they themselves are the sources for the disc.

First of all, rotational distortion of the star's body must be taken into account. Be stars are not spherically symmetric. The oblate shape provides a lower boundary for the disc, causes latitudinal dependence of the radiative force, and enhances its non-radial components, which strongly influences the line-driven wind part of a Be envelope, and the resultant mass-loss.

Secondly, if the star is the source of the circumstellar disc, it will lose a part of its angular momentum in favour of the equatorial disc through mass-loss. This could cause a Be star to slow its rotation during its main-sequence life, so that Be stars of later luminosity subclasses would rotate slower and have a lower ratio among normal B stars, which could explain te observed distribution. This could presumably happen because rotation is one of the two known keys to the Be phenomenon, so one can imagine that when the rotation velocity falls lower than some threshold and approaches the rotation rates of normal B stars, the Be phenomenon would become less frequent.

Thirdly, rapid rotation may have an effect on the composition of the star [21]. The radiative force from a rotating star depends on latitude due to the Von Zeipel effect, which means that radiative flux is larger around the poles than on the equator. Due to this its driving of heavy elements (metals) will also be latitudinally dependent, drifting towards the equatorial plane and enhancing the mass-loss there. This could further alter the evolution of the star slightly, since the more metallic equatorial regions would tend to have larger convection zone. This is only a tentative proposition, but offers an example of what kind of effect it could have to account for the internal structure of the underlying star.

A rotation law obtained from observations should be the first step in a model of a Be star, not an assumed function. This would enable to cut down on the number of possible combinations of known processes running the Be stars. Interferometric maps in a large number of lines offer probably the most direct way of acquiring the rotation law.

Our application of the half-peak separation method has shown that finding the rotational parameter may be an ambiguous task. While the method used seems simple and effective, the uncertanties of this procedure raise more questions than answers. To decide, if the results obtained for 4 Her and κ Dra individually at least can be in taken into consideration would require an individual test of more Be stars. On the other hand, we consider the rotational parameter obtained for their distribution invalid, as well as the assumption of uniform Be rotation that justifies the method's use on a sample of stars. A possibly more reliable test of the method may be to apply it to a model atmosphere with a known rotation law, to ascertain if it is applicable at all, or if the linearization is too much of a simplification.

For a moment let us assume that the results in the individual cases of 4 Her and κ Dra can be taken as orientational values. This would mean that the rotation velocity in the disc drops off even faster than in an angular momentum-conserving case, which could be caused by the presence of a magnetic field or a strong non-radial component in the radiative force (like it happens in the wind-compressed disc). It is also interesting to note, that it would be the case in *both* of the Be binaries with a small, light, non-interacting companion that we have examined.

Lastly, it may very well be possible that there is no universal Be mechanism applicable to all sorts of Be stars. According to our opinion since the Be stars vary so greatly among each other in spectral, photometric and physical characteristics it seems more probable that the Be problem requires individual treatment. Meaning to use combinations of known global processes typical for all Be stars tailored to simulate the observations of a concrete Be star and then look for common features. It may be necessary to develop qualitatively different models for subgroups of Be stars differing among themselves in some basic feature like rotation, for example.

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