

**Charles University in Prague**

Faculty of Social Sciences  
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RIGOROUS THESIS

# **Reduced-form Approach to LGD Modeling**

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## **Declaration of Authorship**

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Signature

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## Abstract

The rigorous thesis deals with the advanced methods for estimating credit risk parameters from market prices: probability of default ( $PD$ ) and loss given default ( $LGD$ ). Precise evaluation of these parameters is important not only for banks to calculate their regulatory capital but also for investors to price risky bonds and credit derivatives.

We develop two forward looking reduced-form analytical methods for the calculation of  $PD$  and  $LGD$  of the corporate defaultable bonds based on their quoted market prices, prices of equivalent risk-free bonds and quoted senior and subordinated credit default swap spreads of the issuer of these bonds. This is reversed to the most of the studies on credit risk modeling, as the aim is not to price instruments on the basis of the estimated credit risk parameters, but to calculate these parameters based on the available market quotes. Furthermore, compared to other studies, the  $LGD$  parameter is assumed to be endogenous and we provide the method for its simultaneous calculation with the probability of default. Finally, using the developed methods, we estimate implied  $PD$  and  $LGD$  for nine European banks assuming that the risk is priced correctly by other investors and the markets are efficient.

**JEL Classification** C02, C63, G13, G33

**Keywords** credit risk, loss given default, probability of default, credit default swap

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## Abstrakt

Táto rigorózná práca sa zaoberá pokročilými metódami odhadu parametrov kreditného rizika na základe tržných cien. Týmito parametrami sú pravdepodobnosť zlyhania ( $PD$  - probability of default) a strata v prípade zlyhania ( $LGD$  - loss given default). Ich presné ohodnotenie je dôležité nielen pre bankové inštitúcie pri výpočte regulačného kapitálu, ale aj pre investorov pri oceňovaní rizikových dlhopisov a kreditných derivátov.

Prezentujeme analytickú metódu výpočtu  $PD$  a  $LGD$  rizikových dlhopisov použitím tržných cien týchto dlhopisov, cien ekvivalentných bezrizikových dlhopisov a kótovaných rizikových prémie príslušných credit default swap derivátov. V porovnaní s väčšinou štúdií v oblasti kreditného rizika je náš proces výpočtu obrátený, keďže cieľom nie je oceniť rizikové inštrumenty na základe odhadnutých rizikových parametrov, ale počítať tieto rizikové parametre z dostupných tržných cien. Navyše, použitím tejto metódy je možné vypočítať  $LGD$ , a to simultánne s pravdepodobnosťou zlyhania. Na záver, za predpokladu, že ostatní investori ocenili tržné riziko správne a trhy sú efektívne, aplikujeme túto metódu na tržné dáta 9 Európskych bánk.

**JEL Klasifikácia**

C02, C63, G13, G33

**Kľúčové slová**

kreditné riziko, strata v prípade zlyhania, pravdepodobnosť zlyhania, credit default swap

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# Chapter 1

## Introduction

Nowadays, measuring of credit risk is considered as an important issue for financial institutions as well as for non-financial companies. Due to Basel regulation, banks are allowed to calculate their own estimates of the credit risk parameters under the IRB approach and therefore to more precisely align their regulatory capital with the underlying risk in a credit portfolio. Another possibility how to cope with the credit risk is to hedge or to trade the risk. Financial markets with credit derivatives significantly raised in the last decade and are more and more used for speculation rather than hedging, for which they were primarily designed. Both, the regulatory reason and the speculation on derivatives market reason gave rise to new methods for the credit risk estimation.

Main components of the credit risk are the probability of default ( $PD$ ) and the loss given default ( $LGD$ ). These are included in the credit spread, which is the difference in market prices between defaultable and default-free bonds. While much attention was paid to modeling of the probability of default, the loss given default was often assumed to be constant and exogenously given. Lack of studies on the  $LGD$  modeling is mainly due to the fact that the probability of default and the loss given default are difficult to separate based on the price of single financial instrument.

Objective of this master thesis is to endogenously estimate loss given default. The focus was put on the explanation of different theories concerning modeling of the credit risk parameters and possibilities of their interconnection in order to gain more information and calculate the probability of default and the loss given default parameters simultaneously. This approach is different to other studies, as the aim is not to price risky instruments based on the econometric estimation of credit risk parameters, but to extract these parameters

from the available market prices.

In Chapter 2 we explain the basic motivation of the loss given default modeling, we provide three approaches of its measuring and discuss main characteristics of *LGD*. From Chapter 3 onwards, we use the implied market approach, which is the forward-looking method of measurement of the credit risk parameters from prices of non-defaulted financial instruments. The goal of third chapter is to explain main ideas behind two implied market approaches: structural and reduced-form approach. Using the later, under the assumption of constant exogenous *LGD*, we show how the time-varying probability of default estimates can be extracted from the market prices of risky and risk-free bonds. Moreover, we describe relationship between *LGD* of bonds with different seniorities.

In Chapter 4, we describe reduced-form approach for pricing credit default swaps, which include investor's estimates on *PD* and *LGD* of the reference entity. Market prices of CDS provide the additional information that is necessary in order to calculate the loss given default and the probability of default simultaneously. Based on the reduced-form models for pricing credit default swaps and defaultable bonds, we introduce a method for the calculation of both credit risk parameters. Secondly, modify the adjusted relative spread method to show how *LGD* can be extracted from the market prices of subordinated and senior CDS of the same reference firm.

Finally, in the last chapter we illustrate the empirical use of these methods. Compared to the Master Thesis, for the first model we extend the analysis of the method and explain the modification that provides us with better applicability to market data. For the relative spread model, we slightly generalize a methodology and enlarge the sample to nine European banks. We conclude with comparison of these two methods and their findings.

# Chapter 2

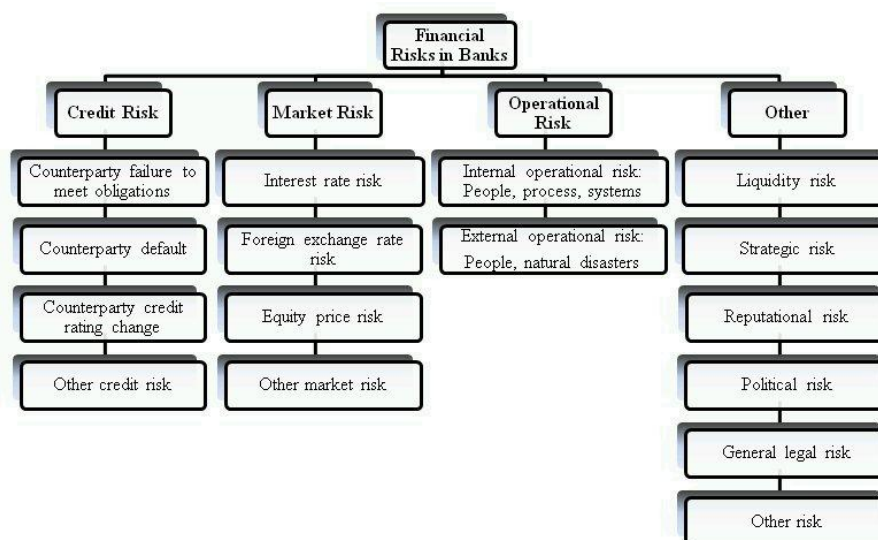
## Loss Given Default in Credit Risk

### 2.1 Credit Risk Management

Banks and other financial institutions have been always facing various financial risks. Many financial crisis, recent or experienced in the past, have shown how important is to recognize and estimate risks correctly. In order to maintain sustainability of business activities, banks need to manage risks and capture potential losses. Banking risks can be divided into several categories. According to the Bank for International Settlements (BIS) banking risks can be classified as shown in Figure 2.1 into credit risk, market risk, operational risk and other risks. Risk classification varies in the literature, the credit and the operational risk can be thought as a part of the market risks. The classification in Figure 2.1 reveals the BIS opinion that special attention should be paid to the credit and the operational risks.

In this work, we will first examine the *counterparty credit risk* in bond agreements and then the *reference credit risk* in derivative agreements. Terms are adopted from Bielecki & Rutkowski (2002), who distinguishes these two types of risk, where the former refers to the credit risk of the second player involved in the agreement, while the latter refers to the third party credit risk that is not directly involved in the agreement. The name “reference” is based on the fact that the third party in the credit derivative agreements is called the reference entity. According to Giesecke (2004), the counterparty credit risk can be defined as a “distribution of financial losses due to unexpected changes in the credit quality of a counterparty in a financial agreement”. It is important to notice that the counterparty credit risk includes counterparty’s risk of insolvency, risk of decrease in creditworthiness and risk of delay in payment. The

Figure 2.1: Financial Risk Classification



Source: Chernobai *et al.* (2007)

reference credit risk is defined in each derivative contract. In the standardized contracts it includes risk of failure to pay and risk of bankruptcy. Even though the counterparty credit risk is to some extent present also in derivative agreements, we would assume it to be significantly smaller compared to the reference credit risk.

In recent years, banks' attitude to the credit risk has changed. First of all, the main instruments to mitigate credit risk used in the past were collateral and covenants, while nowadays the development of credit derivatives markets and raise in securitization bring more opportunities for the banks' credit risk management. Secondly, due to the Basel Capital Accord published in 2004 (Basel II), banks were given more flexibility concerning the credit risk estimation. While under Basel I banks had to use the standardized approach for calculation of economic capital<sup>1</sup>, Basel II allows banks to employ their own credit risk models, which helps to better differentiate risks and to include the effect from diversification of bank's portfolio.

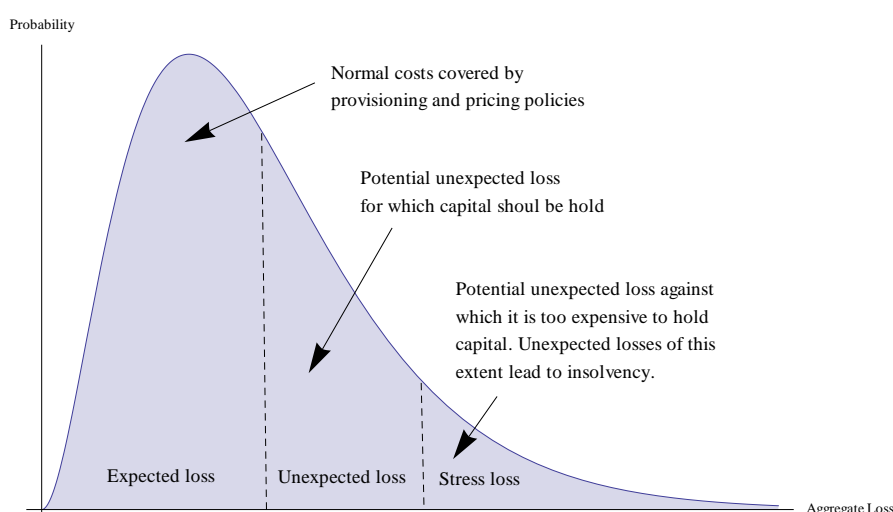
Key parameters for the credit risk management recognized by BIS are the

<sup>1</sup>Basel I imposed minimal capital requirements for banks, which is calculated as ratio of regulatory capital to total risk-weighted assets. Also it defined risk-weights for specific asset classes. For example, loans collateralized by mortgages on residential property were risk-weighted at 35%, whereas other exposures to individuals in the retail portfolio were weighted at 75%. Furthermore, to estimate credit risk banks had to use credit ratings provided by external rating agencies such as Standard & Poor's, Moody's, Fitch Ratings.



*probability of default*  $PD$ , expressing the probability of the counterparty to default within certain time period; the *exposure at default*  $EAD$ , representing the amount of outstanding obligations at the time of default; and the *loss given default*  $LGD$ , expressing the percentage loss incurred relative to exposure at default. Besides the standardized approach, banks can use the internal rating based approach (IRB) to estimate credit risk, either the foundation or the advanced IRB. The former allows banks to estimate internally the probabilities of default, while the latter allows banks to employ their own models for  $LGD$  and  $EAD$  estimation. However, these models must be first approved by national regulator. See Roy (2005) or BCBS (2006) for more information about the internal rating based approach, its requirements, methodology and implication for banks.

Figure 2.2: Distribution of credit losses



Source: RBNZ (2005)

Under the IRB approach, banks need to estimate expected and unexpected credit losses. Distribution of these losses is shown in Figure 2.2. According to BCBS (2006), § 212, risk-weighted functions produce the capital requirements only for the unexpected losses ( $UL$ ) portion, while the expected losses ( $EL$ ) are considered to stand for ex-ante estimated average losses, therefore being already incorporated into the price of the risky instrument. Generally,  $EL$  can be calculated as the product of  $PD$ ,  $LGD$  and  $EAD$ . Aim of this work will be to investigate and estimate these credit risk parameters separately. While lot of studies paid attention to  $PD$  modeling,  $LGD$  was often assumed to be constant, as it is not straightforward to estimate. Also it was due to unavailability of

historical data concerning *LGD*, as banks do not disclose them publicly. Only lately *LGD* received a bit more attention, as it was realized it is important for precise pricing of financial instruments. Furthermore, as mentioned above, the accurate estimation of *LGD* can also help banks to effectively allocate regulatory and economic capital. Following section explains different concepts of the loss given default measurement.

## 2.2 LGD Measurement

Previous part briefly explained motivation to estimate *LGD* parameter. Before discussing the *loss given default* measurement, the precise definition of *default* is needed. Unfortunately, there is no consensus about the standard definition. According to BIS, the reference definition is: “A default is considered to have occurred with regard to a particular obligor when either one or both of the following events have taken place:

- The bank considers that the obligor is unlikely to pay its credit obligations to the banking group in full, without recourse by the bank to actions such as realizing security (if held).
- The obligor is past due more than 90 days on any material credit obligation to the banking group. Overdrafts will be considered as being past due once the customer has breached an advised limit or been advised of a limit smaller than current outstandings.”<sup>2</sup>

Loss given default is usually defined as the percentage loss rate on the exposure if the counterparty defaults. It is important to distinguish between *LGD* and actual loss incurred, which can be computed as  $LGD \times EAD$ . Given the default of a counterparty, according to Seidler & Jakubik (2009), the total loss consists of:

- The loss of principal
- The carrying costs of non-performing loans
- The workout expenses

However, the carrying costs and other expenses are very small relatively to the principal loss, therefore it is reasonable to assume they will not significantly

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<sup>2</sup>See BCBS (2006), p. 100, §452

influence the loss given default rate. Following this assumption, recovery rate, the percentage rate of exposure that lender receive after the obligor defaults can be defined as a complement to  $LGD$  as

$$R = 1 - LGD$$

$LGD$  will eventually depend on the definition of default. The most controversial are events of fully recovered exposure. In such situations, under BIS definition, default occurs but there is no loss incurred. For example, the firm that is more than 90 days delayed with its payments can possibly repay all its obligations. Under BIS definition this event would be considered as a default and  $LGD$  would be zero. However, many banks do not consider such events as credit events and therefore full recoveries would not be included in their loss data. This leads to underestimation of recoveries or in other words, pessimistic view on overall bank's loss given default.

Methods of loss given default measurement can be divided into the *ex-post* default measurement and the *ex-ante* default estimation. According to Schuermann (2004), there are three broadly recognized concepts of measuring the loss given default:

- Market  $LGD$  - based on the market prices of defaulted bonds or loans,
- Workout  $LGD$  - based on the estimated cash flows resulting from the workout process,
- Implied market  $LGD$  - derived from the market prices of non-defaulted bonds or loans.

### 2.2.1 Market LGD

This methodology of measuring  $LGD$  ex-post is based on the idea that market prices of defaulted bonds or marketable loans reflect the actual investors' expectations about the recovery. The main advantage of this method compared to other ex-post methods is that data can be observed immediately after the default. Also, as it is a market price, it reflects the total expected present value of the recovery, including recovered principal, missed interest payments and costs associated with the restructuring process, all already properly discounted. Actual prices on defaulted bond markets are based on par, thus can be easily transformed into percentage of the recovery. As a result, the most

rating agencies use the market  $LGD$  for the recovery estimation. On the other hand, the main disadvantage is that market  $LGD$  is not observable for some instruments, when there is either illiquid or no market for them. Traditionally, defaulted bank loans are not further traded, thus application of the market  $LGD$  is limited.

Another possible market approach is to estimate the recovery rate based on the market value of newly issued bonds. This is based on the idea that firms issue the emergence bonds after they reorganize and restructure the initial debt. These bonds are valued by investors showing their expectation about the firm's value. As new bonds are not issued immediately after the default, price of new bonds must be appropriately discounted to compute the recovery of defaulted bonds. This market approach is called *emergence LGD*.

### 2.2.2 Workout LGD

Another ex-post methodology is based on the process of recovery workout. It considers bank as an investor who invests into the defaulted asset. It takes into account all cash flows from distressed asset related to the recovery. The workout  $LGD$  at default of a single debt instrument would be computed as follows

$$LGD(\tau) = \frac{EAD(\tau) - PV \left[ \sum_{t=\tau}^T R(t) \right] + PV \left[ \sum_{t=\tau}^T C(t) \right]}{EAD(\tau)}$$

where  $\tau$  is the default time,  $T$  is the time when workout process is finished,  $PV[C(t)]$  and  $PV[R(t)]$  denote present value of costs and recoveries throughout recovery workout process. Even though this formula is mathematically simple, compared to directly observed market  $LGD$ , it is actually much more difficult to calculate. Firstly, because it is not unambiguous, how these cash flows should be discounted. Not only the timing of cash flows but also the discount rate are subject to discussion. Banks usually discount at hurdle rate<sup>3</sup>, but the risk-free (Treasury) rate is not exceptional as well. Secondly, recoveries are often not in form of cash, but in form of securities that might have illiquid or no secondary market, therefore their price is not clear. For banks this would imply that they cannot compute precise workout  $LGD$  until all recovered claims are sold

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<sup>3</sup>Hurdle rate can be generally defined as the minimum return on investment that is required to cover all associated costs. It depends on the investor's specific structure of the cost of capital.

which could take a long time. Instead of waiting, banks can use their expected value of recovered securities. Then calculated loss given default will be also in the form of its expected value.

Despite these difficulties, workout loss given default measure is considered to precisely reflect the bank's losses. For example, it incorporates specific cost of bank during the workout process and compared to market *LGD*, it does not include risk premium for unexpected losses. As the market *LGD* is observed immediately after the default, not only after the workout process finishes, when adjusted for mentioned differences, it can serve as good estimation of workout *LGD*. Other way around, banks often use long time-taking workout *LGD* approach for illiquid loans, when the market *LGD* is not observable at all.

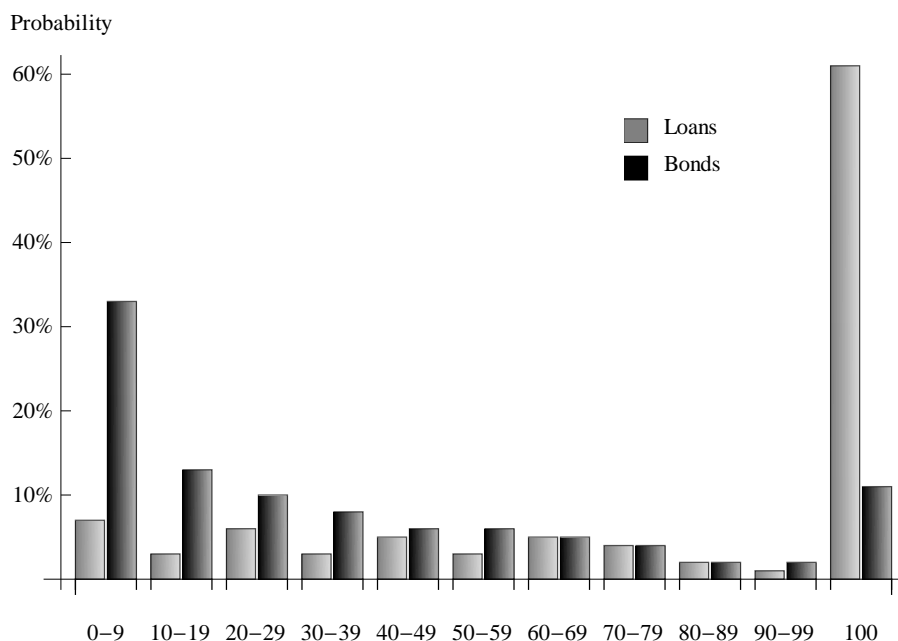
### 2.2.3 Implied Market LGD

Different approach to the *LGD* estimation is ex-ante implied market approach. Similarly to market *LGD*, this methodology is also based on the assumption that the market prices reflect the precise valuation of the security. The implied market *LGD* estimation, however, does not use the data from defaulted bonds or loans, rather it examines the credit spreads of non-defaulted risky bonds over the risk-free (government) bonds. This spread is equal to the risk premium investors demand for buying risky bond instead of risk-free bond. The spread is believed to express the investors' expectation about the possible expected loss. In order to estimate *LGD*, the expected loss needs to be broken into *PD*, *EAD* and *LGD* component. However, as claimed by Jarrow (2001), the spread can beside the expected credit loss reflect also the liquidity premium and other risks.

This approach is not yet widely used in banks, but it provides an important tools for pricing fixed-income securities and credit derivatives. One of the limitations is the risk-neutral measurement used in the implied market models which is not fully consistent with the physical measure. The implied market models estimating the credit risk parameters can be further divided into structural models and reduced-form models. We will examine these models, their advantages and disadvantages in more detail in next chapters.

In the following section, we provide the summary of the loss given default characteristics, which are common, regardless the approach for its measurement.

Figure 2.3: Probability Distribution of Recoveries, 1987-2006



Source: Moody's (2007)

## 2.3 Main characteristics of LGD

To estimate *LGD* properly, it is important to understand what drives the differences in *LGD* among different default events. Generally, characteristics such as seniority of the debt, industry of the issuer, stage of the business cycle or collateral are believed to influence the recovery and consequently loss given default rate [Schuermann (2004)]. We will have a closer look on each of these characteristic. The basic understanding on how it influences *LGD* will be expanded by review of empirical results.

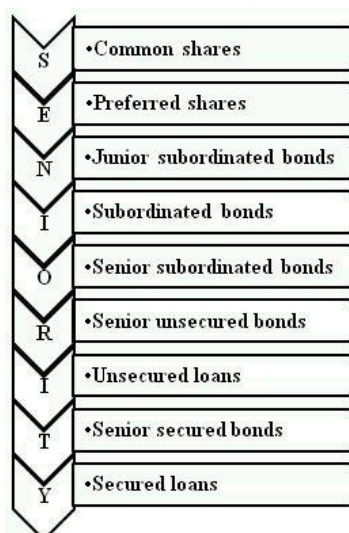
Most significantly, empirical results about distribution of recoveries<sup>4</sup> show that recovery is either quite low or quite high. This is shown in Figure 2.3. High recovery peak is much higher for loans whereas distribution of bond recoveries is more significantly bimodal with low recoveries more probable than high ones. This bimodality is believed to be mainly influenced by the collateral associated with it, whether it is secured or unsecured debt.

<sup>4</sup>Distribution of recoveries can be easily transformed into distribution of *LGD*, as it is complement to each other.

### 2.3.1 Seniority of Debt

Most persistent result in literature over years is that seniority of the debt has the most significant impact on debt recovery. According to *absolute priority rule*, in case of bankruptcy senior creditors must be fully satisfied before capital is distributed to junior creditors and those should be fully satisfied before shareholders. The basic scheme of different types of the debt according to its seniority is shown in Figure 2.4. However, this rule is often violated, either due to higher bargaining power of specific debtholders, regardless seniority of their debt, or simply because senior creditors are willing to give up part of the claim in order to resolve bankruptcy process faster.

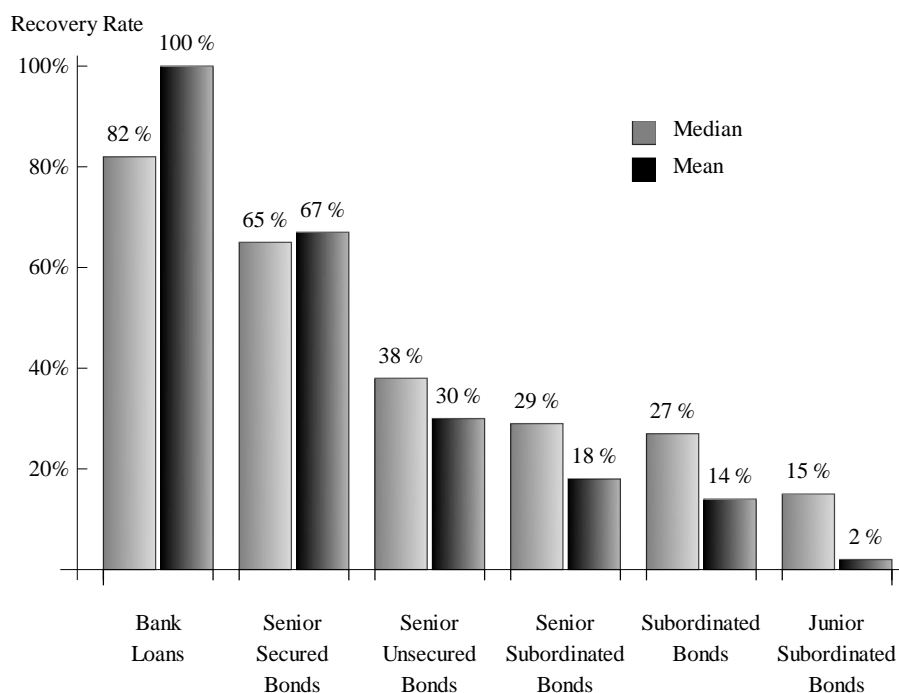
Figure 2.4: Capital structure of a firm



*Source:* Adopted and changed from Schuermann (2004)

Using data published in Moody's (2007), we can see in Figure 2.5 significant relationship between the seniority of the debt and the mean recovery rates observed. Data include information on 3500 loans and bonds from over 720 U.S. non-financial corporate default events in period between 1987 and 2007. Bank loans recovered on average at 82% at resolution on a discounted basis. In contrast, senior secured bonds recovered on average at 65% and average recovery rates on unsecured bonds vary from 38% for senior unsecured bonds down to 15% for junior subordinated bonds.

Figure 2.5: Recovery rates by seniority of debt instruments, 1987-2006



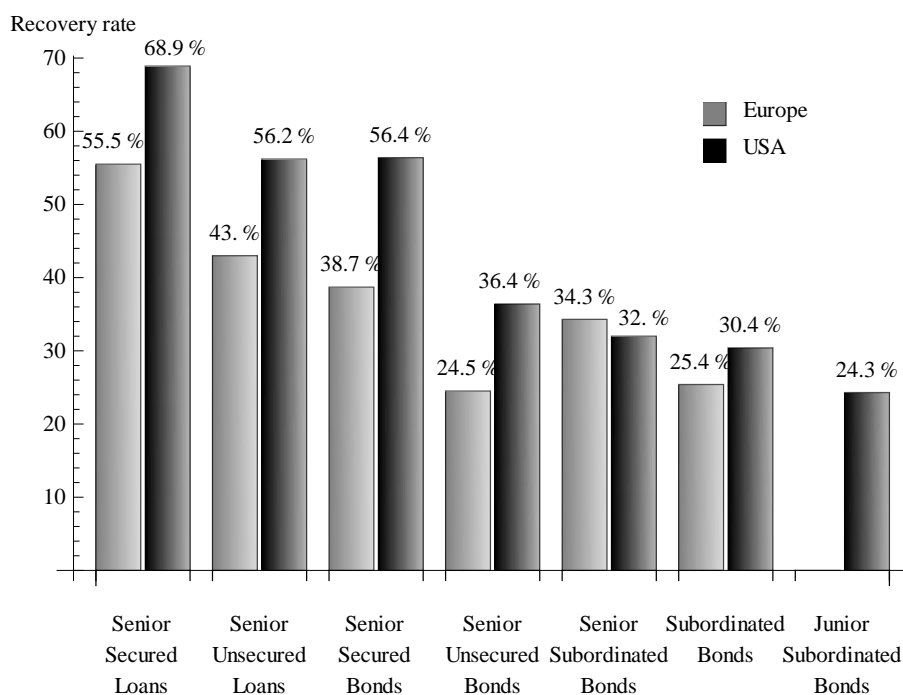
Source: Moody's (2007)

Furthermore, higher median recovery values than mean recovery for secured loans and senior secured bonds in Figure 2.5 supports already mentioned bimodality of recovery distribution with significant high recovery peak for secured debt. On the other hand, for unsecured debt, low recovery peak is more significant which is consistent with lower median than mean values.

This higher seniority - higher recovery relationship was supported by many other studies on the European as well as the U.S. debt recoveries in the past. Even though, as can be seen in Figure 2.6 based on Moody's (2010), it is not that straightforward for the European corporate recoveries. However, this might be due to unavailability of data for the European recoveries, as the market approach was used for gathering data and market with defaulted bonds is not yet sufficiently developed in Europe.



Figure 2.6: Recovery rates by seniority of European and U.S. debt instruments, 1985-2009

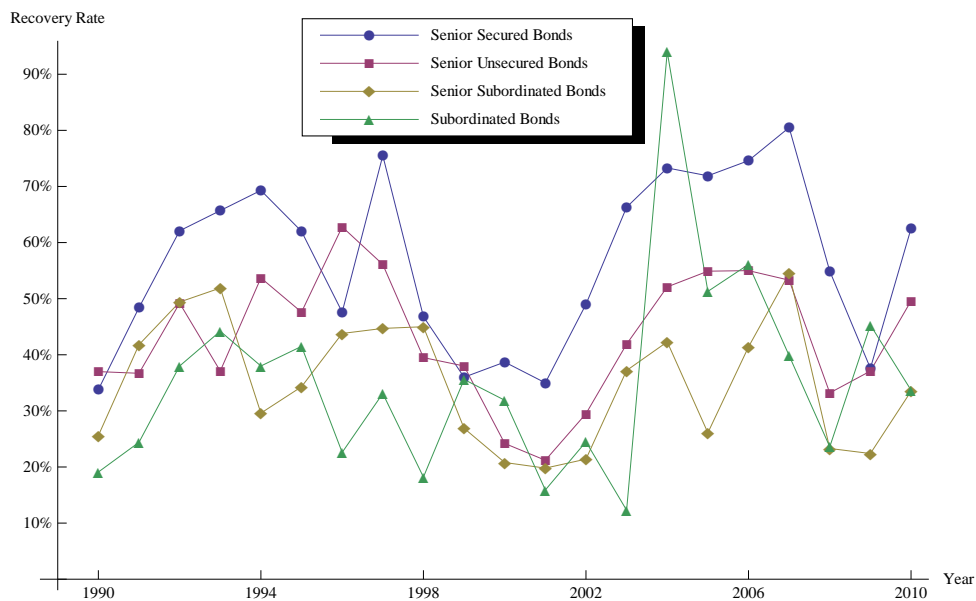


Source: Computed from Moody's (2010)

### 2.3.2 Business Cycle Impact on Recoveries

Apart from seniority, there is strong evidence that the business cycle has the impact on recoveries. It can be observed from Figure 2.7 that average recovery rates are significantly changing in time. Moreover, for different seniority they follow cyclical variation, which is based also on macroeconomic conditions. Also, it is interesting to notice that the U.S. mean recovery rates for all senior bonds and loans are higher for period 1987-2006 (Figure 2.5) than for years 1985-2009 (Figure 2.6), where recent years of global financial crisis are included.

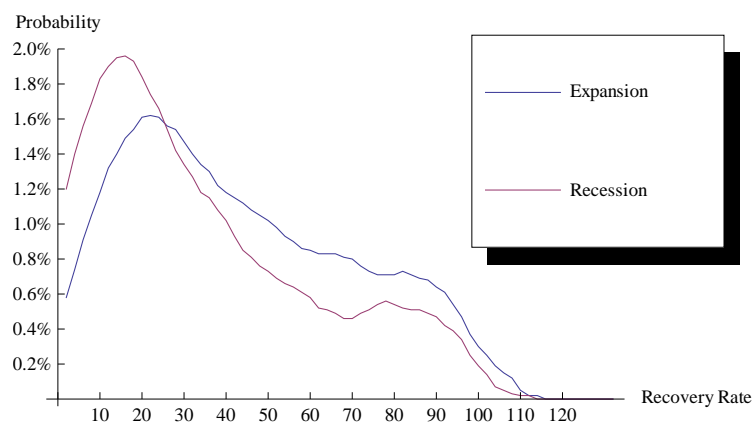
Figure 2.7: Recovery rates during 1990-2010



Source: Computed from Moody's (2011)

According to Moody's data (1970-2003) is average recovery 32% in recessions and 41% in expansions. Furthermore, as can be seen in Figure 2.8, left peak is much higher during recessions, thus situations with very low recoveries are more probable at that part of cycle. This can be intuitively a result of all markets being less liquid during recession, therefore firm might have difficulties when selling its assets during the liquidation process. During expansions, recovery values are more equally distributed.

Figure 2.8: Probability Densities of Recoveries across the Business Cycle, 1970-2003

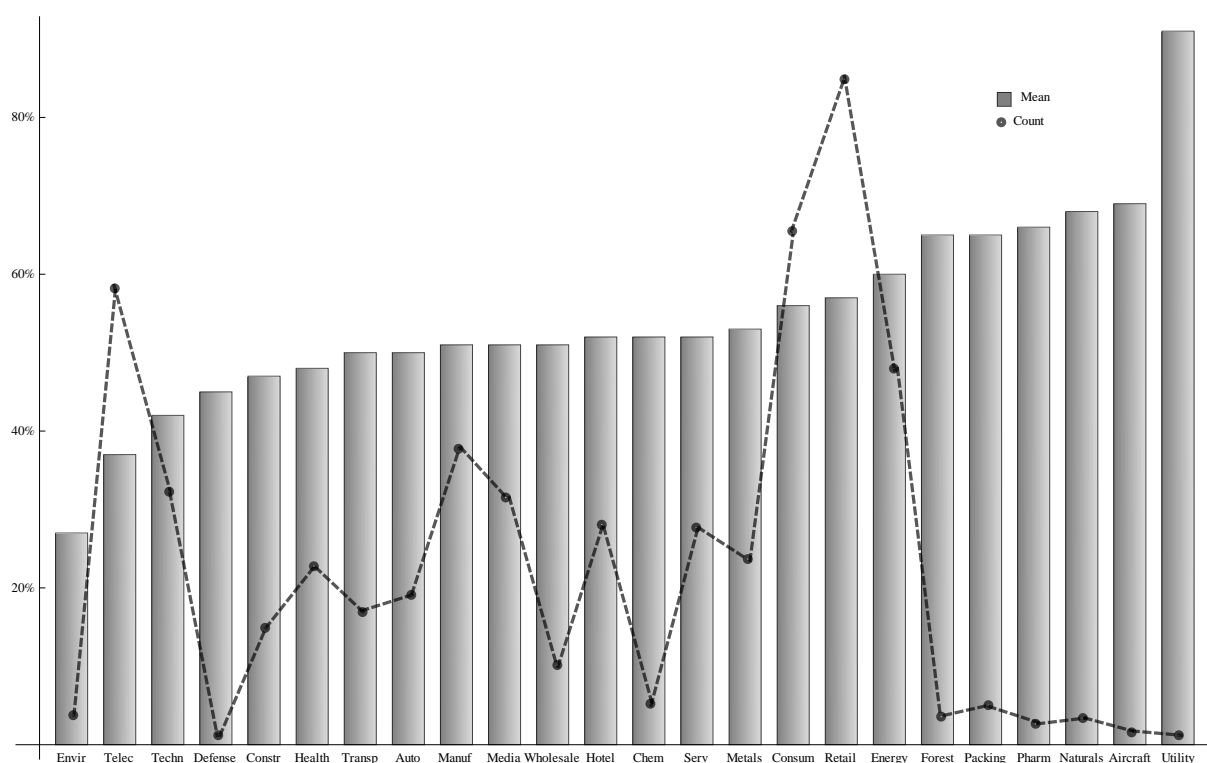


Source: Moody's (2004)

### 2.3.3 Industry Impact on Recoveries

Impact of industry of the issuer on recovery rates is not that straightforward as the impact of seniority and business cycle. Generally, after the default debtholders receive money according to the value of firm's assets. However not all types of assets can be sold easily at reasonable time and price. Each industry has specific assets that comprise most of the firm's values. Therefore industries with mostly liquid and easy to sold assets should be performing with higher recoveries. Intuitively, the capital structure of the firm has also an impact on *LGD*, as in lower leveraged firm there is proportionally less debtholders to share firm's assets.

Figure 2.9: Recovery Rates by Industry, 1987-2006



Source: Moody's (2007)

According to research by Moody's (2007), certain industries may have features that are correlated with higher or lower than average recovery rates. For example, firms in quickly growing and highly competitive industries may experience higher than average recovery rates because assets can be easily sold at the liquid market. Furthermore, lower average recovery rate is probable for firm that defaults in a concentrated industry — with fewer potential buyers for the defaulted firm's assets. However, they did not find any statistically sig-

nificant relationship. Most of industries have mean recovery rate between 40% and 60%, see Figure 2.9. Those that are over 60% have very low number of observations, therefore we cannot predict if recovery rate will be high in future default cases within these industries.

Altman & Kishore (1996) claimed that some industries, especially utilities, have significantly higher average recovery returns, which is consistent with Moody's (2007). However, this might be due to fact that utility industry is still in many countries a regulated market. Summary of data from their research on industry impact can be found in Table 2.1. Because definition of certain industries is not clear, it is difficult to compare results between different studies. It would need deeper analysis to decide to what extent the industry has an impact on recovery or loss given default rates.

Table 2.1: Industry Impact on Recoveries, 1971-1995

<b>Industry</b>	<b>Avg. Recovery</b>	<b>Industry</b>	<b>Avg. Recovery</b>
Utilities	70%	Communication	37%
Services	46%	Financial Institutions	36%
Food	45%	Real Estate	35%
Trade	44%	General Stores	33%
Manufacturing	42%	Textil	32%
Building	39%	Paper	30%
Transportation	38%	Hospitals	26%

*Source:* Altman & Kishore (1996)

In this chapter we provided a brief overview of the loss given default problematics, its general characteristics, methods of calculation and motivation behind them. Next we will continue with the implied market approach of *LGD* measurement, focusing on reduced-form modeling.

## Chapter 3

# Implied Market Modeling Based on Bonds

Credit risk measurement became lately more important for banks as well as for corporate firms. New methods were developed in the academic research as well as among banks. The importance of *LGD* modeling (*LGD* is complement to recovery rate) became recognized only in the last decade. There are two main approaches how to measure the implied market default probability: structural and reduced-form models. These models differ in basic assumption whether default time is predictable or not. Structural models assume predictability of the default event which is based on the timing when firm's asset value falls below a certain barrier. This barrier can be represented by a firm's value of debt, which would mean that default occurs when firm's equity is negative, therefore firm is not able to pay all its obligation. These models are often called Merton's type models based on Merton (1974).

In contrast, reduced-form models treat the default as an unexpected event, assuming only that default can occur. Default time is based on the default-intensity process, which can be parametrized e.g. using macroeconomic information. Some intensity processes have been modeled based on credit ratings, where frequency of default for companies with the same rating is expected to be the same. However, according to Duffie & Singleton (2003) this might be a bit simplifying because ratings show other parameters than default probability (*EAD*, *LGD*). Moreover, as claimed by Ederington & Yawitz (1987), credit ratings remain more stable than default probabilities during different parts of business cycle.

Another point of view on credit risk models based on implied market in-

formation was by Jarrow & Protter (2004) who claimed that structural form and reduced-form models are basically the same, but only based on different assumptions about available information. Structural models can be seen from managers point of view with full information (e.g. about asset value), whereas reduced-form models are from point of view of investors with only partial information. This idea of common principle was further developed by Guo *et al.* (2009) into extended reduced-form models which use information about firm's asset value under two different information sets: full and partial, thus combining structural and reduced-form approach in one model.

Goal of this chapter is to introduce the credit risk pricing models with emphasis on different possibilities of implied market measurement of probability of default ( $PD$ ) and loss given default ( $LGD$ ). Chronologically, we begin with explanation of former Merton's model and further improvements made on the field of structural models, continuing with basic reduced-form models developed mainly in last two decades. In order to understand reduced-form models that focus on loss given default parameter, first we present analytical tools which enable ex-ante measuring of default probability, assuming constant expected  $LGD$ . This is thereafter followed by more sophisticated reduced-form model for measuring  $LGD$ . In this chapter, all models are based on information implied from market data of risky but not yet defaulted bonds.

### 3.1 Structural Models

The theory of option pricing introduced by Black & Scholes (1973) and their suggestion that the technique could be used for pricing of corporate debts gave rise to structural models for measuring the credit risk. This framework was firstly developed by Merton (1974) with main focus on measuring probability of default at maturity of a firm's debt. Generally, in structural models, firm defaults when is not able to meet its obligations. According to Merton (1974), this happens when value of firm's asset is lower than value of its liabilities when these mature<sup>1</sup>. Hence, it is obvious that term "structural" comes from the fact that credit risk parameters are dependent on structural characteristics of the firm: asset volatility (business risk), and leverage structure (financial risk).

Merton's model, even though employing many simplifying assumptions, introduced a new approach for credit risk pricing of corporate bonds, which is

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<sup>1</sup>By asset value it is referred to its market value rather than accounting value.

now widely used in theory as well as in practice. Business models for credit risk measurement and estimation of  $PD$  and  $LGD$  developed and used by Moody's (KMV model) and J.P. Morgan (CreditMetrics TM) are both based on Merton (1974).

Next, basic ideas of the model will be explained, followed by brief overview how some of its assumptions have been relaxed. Figure 3.1 shows an example of asset value evolution in the past and graphically explains how it can evolve in the future so that there is no default. Probability of default in the figure is then the probability that asset value will fall below "debt value", which reflect the face value that must be repaid at maturity  $T$ . Below see the list of assumptions for Merton's model. They are mostly based on traditional Black-Scholes option pricing theory. Merton (1974) assumed:

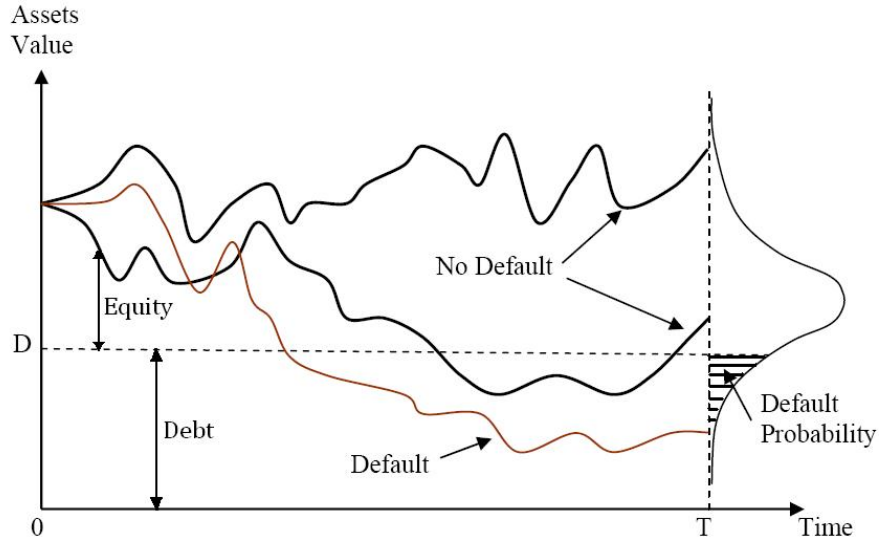
- continuous-time trading of perfectly divisible assets with sufficient number of investors who have comparable level of wealth,
- no friction, no transaction cost, no taxes, no bid-ask spreads in markets,
- short-selling without any restrictions is allowed,
- term structure of risk-free interest rate is flat and known,
- value of a firm's assets  $V$  is financed by equity  $E$  and one zero-coupon bond with market price  $D$ , maturity at time  $T$  and face value  $F$ ,
- capital structure does not influence value of a firm (Modigliani-Miller theorem holds),
- no dividend payout, no new issues of equity or debt,
- default can only occur at maturity of the debt,
- no reorganization or bankruptcy costs in case of default and absolute priority rule holds.

In structural models, it is necessary to estimate evolution of firm's value of assets and liabilities (or other boundary process or value instead of debt's value) in order to be able to predict default probability. According to Merton (1974) the asset value  $A_t$  follows diffusion type stochastic process with standard Gauss-Wiener process<sup>2</sup> and can be described through following stochastic differential

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<sup>2</sup>See the definition of Gauss-Wiener process in Appendix A.

Figure 3.1: Probability of default in Merton model



Source: Adapted from Crouhy *et al.* (2000).

equation

$$\frac{dA_t}{A_t} = (\mu_A - \gamma_A)dt + \sigma_A dW_t^A, \quad (3.1)$$

where  $\mu_A$  denotes expected rate of return on firm's assets per unit time,  $\sigma_A$  denotes volatility of return on firm's assets per unit time,  $\gamma_A$  is total cash outflow in means of dividends or coupons per unit time and  $W_t^A$  is the standard Gauss-Wiener process. Under Merton's assumptions, default can only occur at maturity of the bond  $T$  and if value of assets  $A_T$  is not sufficient to repay face value of the bond  $F$ . Therefore the probability of default under the stochastic evolution of firm's value is given by

$$PD = P[A_T \leq F] \quad (3.2)$$

At maturity, debtholder will receive either face value  $F$  in case of no default ( $A_T > F$ ) or whole value of the firm  $A_T$  in case default occurs ( $A_T \leq F$ ). This is based on assumption of absolute priority rule, so that bondholders have to be fully paid before shareholders and therefore shareholders receive nothing in case of default. Market value of firm's debt is similarly to  $A_t$  assumed to follow the diffusion type stochastic process with constant drift  $\mu_D$  and standard Gauss-Wiener process  $W_t^D$

$$\frac{dD_t}{D_t} = (\mu_D - \gamma_D)dt + \sigma_D dW_t^D \quad (3.3)$$



Merton (1974) expressed value of equity, as well as value of debt, as a functions of the firm's asset value  $A_t$  and the time to maturity  $\bar{t} = T - t$ , for which holds

$$A_t = D(A_t, \bar{t}) + E(A_t, \bar{t}) \quad (3.4)$$

Then by using Itô's lemma<sup>3</sup> and Equation 3.4 it is possible to deduce fundamental differential equation for equity value

$$\frac{\partial E_t}{\partial t} + rA_t \frac{\partial E_t}{\partial A_t} + \frac{1}{2} \sigma_A^2 A_t^2 \frac{\partial^2 E_t}{\partial A_t^2} - rE_t = 0 \quad (3.5)$$

To solve Equation 3.5 it is necessary to examine initial and bounding conditions. The initial conditions are based on the time to maturity  $\bar{t}$  equal to zero. As stated before, at the maturity of the debt, shareholders receive nothing in cause of default, otherwise face value of bond is paid out and therefore initial condition is  $E(A_T, 0) = \max(0, A_T - F)$ . First bounding condition that applies at any time is that if value of assets is zero, debt and equity must be zero as well:  $D(0, \bar{t}) = 0$ ,  $E(0, \bar{t}) = 0$ . Secondly, debt value cannot be higher than asset value:  $D(A, \bar{t}) \leq A$  for any  $\bar{t}$ . Equation 3.5 with defined initial and bounding conditions is identical to Black & Scholes (1973) option pricing formula for European call option. Therefore solution to Equation 3.5 is

$$E(A, \bar{t}) = A\Phi(d_1) - Fe^{-r\bar{t}}\Phi(d_2) \quad (3.6)$$

where  $\Phi(\cdot)$  stands for cumulative standard normal distribution function and

$$d_1 = \frac{\ln \frac{A}{F} + r\bar{t} + \frac{1}{2}\bar{t}\sigma_A^2}{\sigma_A\sqrt{\bar{t}}} \quad (3.7)$$

$$d_2 = d_1 - \sigma_A\sqrt{\bar{t}} = \frac{\ln \frac{A}{F} + r\bar{t} - \frac{1}{2}\bar{t}\sigma_A^2}{\sigma_A\sqrt{\bar{t}}}$$

According to Black & Scholes (1973),  $\Phi(d_2)$  stands for the probability that option will be exercised, which would in this analogy of the model imply that equity holders repaid the debt at its maturity and receive a positive value of  $A_T - F$ . Probability of default might than be computed as  $1 - \Phi(d_2)$ , which is equal to  $\Phi(-d_2)$  due to standard normal distribution characteristics. It must be noted that this derivation of default probability was done under risk-neutral measure.

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<sup>3</sup>See the Appendix A for definition.

Compared to risk-neutral default probability, according to Crouhy *et al.* (2000), physical probability of default can be calculated according to Equation 3.2 assuming that physical asset value  $A_t$  has log-normal distribution, so that expected value at time  $t$  is  $E[A_t] = A_0 e^{\mu_A t}$ . Their empirical study based on real data supported assumption of log-normality of assets. Thanks to characteristics of normal logarithm, we get the distribution of  $\ln A_t$  from Equation 3.1, which is

$$\ln A_T \sim \Phi(\ln A_0 + \mu_A T - \frac{1}{2}\sigma_A^2 T, \sigma_A^2 T) \quad (3.8)$$

Physical probability of default is then computed as

$$PD^* = [\ln A_T \leq \ln F] = \Phi\left(-\frac{\ln \frac{A}{F} + \mu_A T - \frac{1}{2}\sigma_A^2 T}{\sigma_A \sqrt{T}}\right) = \Phi(-d_2^*) \quad (3.9)$$

Risk-neutral and physical probabilities of default measurements differ only in expectations about return on assets. In real world, investors demand higher return on asset ( $\mu_A$ ) than the risk-free rate  $r$ , which imply  $d_2^* > d_2$ , and from properties of normal distribution  $\Phi(-d_2^*) < \Phi(-d_2)$ . Risk-neutral probability of default is thus higher than actual physical probability. This must be taken into account when using risk-neutrality in structural models.

Few of Merton's assumptions were considered as limitation to empirical usage and have been relaxed in subsequent works by other authors. Black & Cox (1976) introduced more complex capital structure; Geske (1977) introduced interest paying debt; Vasicek (1984) brought an idea of distinction between short term and long term debt. Moreover, all of the above mentioned extensions of Merton's model assumed that default can occur before maturity and relaxed the condition of flat term structure of risk-free interest rate. The economic interpretation of default that occurs before maturity lies in the default barrier that represents a debt covenant and default time is the first time of its violation. In next sections we will examine a reduced-form approach to modeling credit risk.

## 3.2 Standard Reduced-Form Models

Reduced-form credit risk models has been firstly introduced by Jarrow & Turnbull (1995) as a reaction to structural form approach, especially trying to decrease informational difficulty when modeling credit risk. In order to achieve this, assumption of predictability of default time, which is present in struc-

tural models, was relaxed. On contrary, in reduced-form models default is not conditioned by particular economic parameters of a company, but is simply expected to occur at any time with some intensity. This intensity is modeled by a exogenous default process. Reduced-form approach belongs also to category of market implied approaches, therefore market prices of defaultable instruments are believed to disclose market expectation of credit parameters under no-arbitrage. In this section, intensity with which default occurs will be considered firstly as a constant, then as a deterministic time-varying and thereafter a stochastic variable, while recovery rate will be assumed to be constant or even zero in order to better understand ideas behind reduced-form intensity based modeling. Further simplification lies in using zero-coupon bonds in these models. Even though, this assumption is not always realistic, any coupon paying bond can be easily stripped into coupons and face value payment at maturity, each representing a zero-coupon bond with different maturity. Mechanism that can be applied to prices of coupon bearing bonds to calculate theoretical market value of zero-coupon bonds will be explained in empirical part of this work. Until that time, we will automatically assume zero-coupon for both, risky and risk-free, bonds.

### 3.2.1 Basic Model

As was already mentioned, default event in reduced-form models is not predictable. Only publicly available market information is used for its measurement. In order to achieve this low information approach, few assumptions are needed. Most important is the assumption about what leads the difference between prices of defaultable and default-free bonds, which are otherwise equal<sup>4</sup>. In basic reduced-form models, e.g. Hull & White (2000), this spread is assumed to be equal to expected loss in the case of default<sup>5</sup>. Therefore basic equation of the model says that the difference between today's market value of default-free bond with maturity at time  $T$  (we denote as  $g^T$ ) and today's market value of defaultable bond with same maturity (we denote as  $b^T$ ) is equal to expected

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<sup>4</sup>Defaultable and default-free bonds are considered to be equal if they have same maturity, same face value and same coupon structure

<sup>5</sup>In some more complex reduced-form models, e.g. Jarrow (2001) the spread is believed to disclose expected credit risk premium together with liquidity risk premium.

value of loss incurred given default<sup>6</sup>

$$g^T - b^T = PV[PD \times LGD \times EAD] \quad (3.10)$$

From market value of credit spread, given the assumption about *LGD* and exposure, it is straightforward to calculate default probability of the risky bond. This can be shown on simple example: assuming zero-coupon risky bond with maturity 3 years from now, face value 100 and yield to maturity 4% and risk-free bond with same maturity and face value yielding 3.5% to maturity. Using continuous compounding, the price of risky bond is  $100 \times e^{-0.04 \times 3} = 88.69$  and price of risk-free bond is  $100 \times e^{-0.035 \times 3} = 90.03$ , implying credit spread to be 1.34, which is equal to present value of  $PD \times LGD \times EAD$ . Taking face value of risky debt as exposure at default and zero recovery ( $LGD = 100\%$ ), probability of default satisfies  $1.34 = e^{-0.035 \times 3} \times PD \times 1.00 \times 100$ <sup>7</sup>. Therefore  $PD = 1.49\%$ .

Figure 3.2 shows the implied default probability sensitivity to *LGD*. For the given credit spread, which is calculated from risky and risk-free yields to maturity, the higher assumed *LGD* implies lower probability of default and vice versa, so that overall expected loss until maturity remains the same. Furthermore, intuitively we expect that greater difference between yields to maturity is due to higher overall expected loss. This is also depicted in Figure 3.2, as *PD* and *LGD* simultaneously increase as the yield to maturity of risky bond increases. However, assumptions used in this example would be too simplifying for the following reasons:

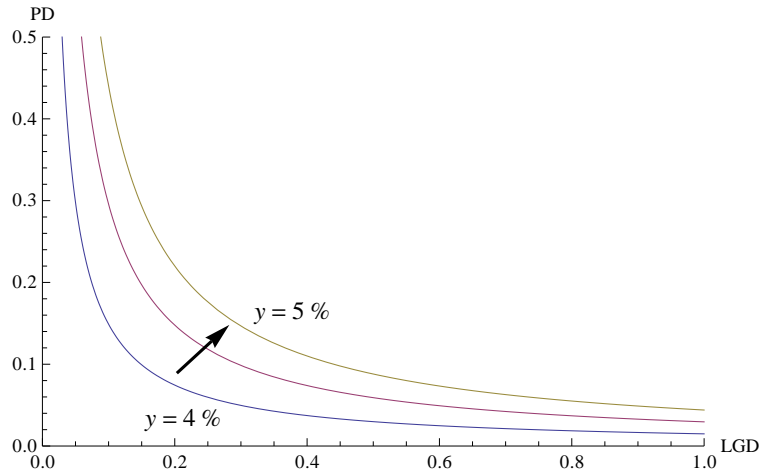
- probability of default of the bond can vary until maturity
- face value is only one of the possibilities for estimating exposure at default in reduced-form models
- recovery rate does not need to be zero and even more, it can vary in time

All these limitations are discussed further in this work. Firstly, we start with definition of probability of default parameter which can vary in time. In reduced-form approach, often called intensity-based, default time is not conditioned on financial situation of the company, rather it is set at the time equal

<sup>6</sup>Expected value of loss incurred given default is equal to the loss rate *LGD* times the exposure at default *EAD* times the probability that default will occur *PD*.

<sup>7</sup>The first term stands for discounting of payoff at maturity into present value, *PD* denotes the overall probability of default until maturity. These are multiplied by 1.00, that stands for 100%*LGD* and by face value of 100.

Figure 3.2: Sensitivity of default probability to  $LGD$  and yield  $y$  of defaultable bond



Source: Author's computation. Risk-free rate is assumed to be 3.5%.

to first jump of discrete time homogeneous Poisson process  $N$  with intensity  $\lambda$  [Jarrow & Turnbull (1995)], where

$$N_t = \sum 1_{T_i \leq t}$$

is count of Poisson process event arrivals  $T_i$  in the  $[0, t]$  interval<sup>8</sup>. Let's denote  $\tau$  as the default time, which is equal to  $T_1$  of Poisson process  $(T_i)$  with intensity  $\lambda$ . Probability of no default during interval  $[0, t]$  is the probability of  $N_t$  being equal to zero, which is identical to  $\tau$  being later time than  $t$ . We will denote such probability as  $p(0, t)$ , the survival probability until time  $t$ :

$$p(0, t) = P(N_t = 0) = P[\tau > t] = 1 - F(t) = e^{-\lambda t} \quad (3.11)$$

In order to be able to determine continuous  $PD$  function, let's denote the survival probability until time  $t + \Delta$ , given that no default occurred until time  $t$  as  $p(t, t + \Delta)$ . Probability of no default during interval  $[t, t + \Delta]$  can be then calculated as probability of  $N_t - N_{t+\Delta}$  being equal to zero. In analogy to forward interest rates,  $p(t, t + \Delta)$  is the forward survival probability, as it is conditional on information at time  $t$ .

$$p(t, t + \Delta) = P(N_t - N_{t+\Delta} = 0) = e^{-\lambda \Delta} \quad (3.12)$$

<sup>8</sup>See Appendix A for definitions of discrete and continuous time homogeneous Poisson processes.

Note that survival probabilities can be easily transformed into probabilities of default as complements to 1. Definition of  $PD(0, t)$  is then straightforward. It is important to notice that if we would define  $PD(t, t + \Delta)$  as  $1 - p(t, t + \Delta)$ , this would refer to forward probability of default in the interval  $[t, t + \Delta]$ , but it would not be clear if there was any default before time  $t$ . Therefore it is more convenient to define  $PD(t, t + \Delta)$  as probability of no default before time  $t$  and default occurring in the specified interval.

$$PD(t, t + \Delta) = p(0, t) \times (1 - p(t, t + \Delta)) = e^{-\lambda t}(1 - e^{-\lambda \Delta}) \quad (3.13)$$

To simulate continuous default probability, it is sufficient to decrease  $\Delta$  in Equation 3.13 limitely to zero:

$$\begin{aligned} PD(t) &= \lim_{\Delta \rightarrow 0} PD(t, t + \Delta) = p(0, t) \times \lim_{\Delta \rightarrow 0} 1 - p(t, t + \Delta) \\ &= e^{-\lambda t} \lim_{\Delta \rightarrow 0} 1 - e^{-\lambda \times \Delta} \end{aligned} \quad (3.14)$$

This was often used in literature to parametrize probability of default and calculate market price of risky bonds. Duffie (1998) provided the pricing model under affine settings, so that risk free rate and intensity rate followed affine state process. Then, he used Monte Carlo simulation to find solutions. We would not go into detail on different parametrization models or econometric estimation of parameters, as we will focus more on analytical solutions, rather than statistical estimates.

Hull & White (2000) proposed simple analytical model for iterative calculating of time-varying probabilities of default. They did not employ continuous  $PD(t)$  function, rather they assumed  $PD$  is constant in the certain intervals, which simplifies the mathematical tractability. Assuming that a firm has  $N$  bonds with maturities  $t_1 \leq t_2 \leq t_3 \dots \leq t_N$ , they set constant probability of default for  $t \in (t_{i-1}, t_i)$ . So that  $PD(t_i)$  denotes the probability of default occurring any time during the interval. These bonds, as issued by the same reference entity, are assumed to have the same probability of default in each defined time interval. Let's denote  $PD(t_i)$  as the probability that default occurs during  $(t_{i-1}, t_i)$ . We can easily adapt our basic Equation 3.10 for time-varying probability of default and we get

$$g^T - b^T = PV \left[ \sum_{i=1}^N PD(t_i) \times LGD \times EAD(t_i) \right] \quad (3.15)$$

In order to calculate the default probability in different intervals Hull & White (2000) used following data:

- today's prices of firm's bonds, denoted by  $b^{t_j}$  for a bond with maturity  $t_j$ , for  $j = 1, 2, \dots, N$  so that  $t_1 \leq t_2 \leq \dots \leq t_N$
- today's prices of equivalent government bonds, denoted by  $g^{t_j}$  for a bond with maturity  $t_j$  and the same face value as  $j$ 'th risky bond, for  $j = 1, 2, \dots, N$
- constant recovery rate  $\hat{R}$  (according to historical data provided by rating agencies)
- constant risk-free interest rate  $r$  used for calculation of the discount factor  $e^{-rt}$
- forward price  $F_j(t)$  of risk-free bond maturing at time  $t$  with the same face value as  $j$ 'th firm's bond
- claim  $C_j(t)$  made by bondholder in case that  $j$ 'th bond defaults at time  $t$

Based on these data, they defined present value of the loss from a default of  $j$ 'th bond at time interval  $(t_{i-1}, t_i)$  as

$$\beta_j^{t_i} = \int_{t_{i-1}}^{t_i} e^{-rt} [F_j(t) - \hat{R}C_j(t)] dt \quad (3.16)$$

which stands for difference the investor would get if he invested money into risk-free bond maturing at time  $t \in (t_{i-1}, t_i)$  instead of investing money into risky  $j$ 'th bond which happens to default at  $t \in (t_{i-1}, t_i)$ .  $\beta_j^{t_i}$  is already discounted value of the possible loss. The basic reduced-form expression of the market credit risk spread (Equation 3.15) can be then reformulated according to definition of loss value  $\beta_j^{t_i}$  for  $j$ 'th bond

$$g^{t_j} - b^{t_j} = \sum_{i=1}^j PD(t_i) \times \beta_j^{t_i} \quad (3.17)$$

Probabilities of default on time intervals determined by firm's bonds maturity dates as defined above, can be then calculated inductively (starting with  $j = 1$ , then  $j = 2$ , etc.).

$$PD(t_j) = \frac{g^{t_j} - b^{t_j} - \sum_{i=1}^{j-1} PD(t_i) \times \beta_j^{t_i}}{\beta_j^{t_j}} \quad (3.18)$$

Even though few simplifying assumptions were used in previous calculation, this inductive formula for probabilities of default was successfully applied on real data by Hull & White (2000). We intentionally skipped the definition of claim  $C_j(t)$  that bondholder will make in case  $j$ 'th bond defaults at time  $t$ . In literature there is still lots of discussion concerning what price describes the claims made by bondholders in the best possible way. Terms *bond's claim* and *exposure at default* can be used interchangeably. Also it can be referred to as *recovery assumptions*. However this have nothing to do with the recovery rate directly. Rather it is the assumption about the exposure at default from which some percentage is recovered. We will stay with "exposure at default" terminology as we find it more appropriate. The following part introduces three main "*EAD* assumptions", their implications, tractability and shortcomings.

### 3.2.2 Assumptions about Exposure at Default

In this part we will present assumptions about exposure at default used in different reduced-form models. We will follow previous notation as well as assumption of constant recovery rate to be  $\hat{R}$ , face values of all bonds to be 1 and default-free interest rate  $r$  to be constant within all *EAD* approaches.

In general, the price of defaultable zero-coupon bonds with the constant recovery rate can be calculated as the expected promised payment at maturity (face value of the bond) in case of no default plus expected proportion of exposure recovered in case of default. These possible payments must be properly discounted. We will use  $e^{-rT}$  as the discount factor for present value of payment at time  $T$ . Therefore the present value of default-free bond with face value 1 is  $g_0^T = e^{-rT}$ . The Equation 3.19 shows general formula for defaultable bond price, which will be further rearranged using different assumptions about *EAD*,

$$b_0^T = PV[P[\tau > T] \times 1 + (1 - P[\tau > T]) \times \hat{R} \times EAD] \quad (3.19)$$

where  $\tau$  stands for default time,  $T$  is maturity of the bond,  $P[\cdot]$  denotes probability measure under martingale  $\mathbb{Q}$  and  $PV[\cdot]$  denotes present value of future cash flows.

Firstly, as mentioned in the basic reduced-form example, exposure at default can be equal to face value, therefore recovered value is a fraction of face value in event of default. According to Giesecke (2004) this convention is called *recovery of face value*. Under this convention it is further assumed that recovery is paid



out at maturity  $T$ , thus both possible payments should be discounted with  $e^{-rT}$ . According to Equation 3.19 and definition of survival probability in Equation 3.11 the value of the bond at time zero is

$$\begin{aligned} b_0^T &= PV[P[\tau > T] \times 1 + \hat{R}P[\tau \leq T] \times 1] \\ &= e^{-rT}(e^{-\lambda T} \times 1 + \hat{R}(1 - e^{-\lambda T}) \times 1) \\ &= g_0^T - e^{-rT} \times (1 - \hat{R}) \times P[\tau \leq T] \end{aligned} \quad (3.20)$$

This shows that the credit spread between value of defaultable and default-free zero bonds is the risk-neutral value of expected loss from default, which is consistent with already explained basic reduced-form formula. Moreover, if we assume zero recovery rate, the equation can be simplified into  $b_0^T = e^{-(r+\lambda)T}$ . As stated by Jarrow (2001), defaultable bonds can be valued in the same way as default-free, but using risk-adjusted interest rate  $r + \lambda$  for discounting. As shown by Giesecke (2004), this holds for other than zero coupon bonds assuming zero recovery. The recovery of the face value convention was first empirically used in the study of Duffie (1998). He proposed the model with recovery as a fraction of face value as he believed that the bondholders should receive fixed payment proportional to the face value of their holdings, regardless time to maturity and coupon rate of these bonds. He realized that such assumption might not be very realistic, especially in case of reorganization at default, some bondholders have higher bargaining power than others and recovery is not proportional to face value. The main advantage of using recovery of face value convention in reduced-form models lies in its mathematical tractability. Hull & White (2000) presented model with extended recovery of face value convention for coupon bonds. To calculate the exposure they added accrued interest since last coupon payment to the face value of bond.

Secondly, exposure at default can be estimated as a value of otherwise equivalent default-free bond with same maturity and coupon payments, from which arises the name of this convention [Giesecke (2004)]. Under the *equivalent recovery* convention, recovery is calculated as a fraction of  $g_\tau^T$  and it is paid out at default time  $\tau$ . Contrary to face value convention, the remaining maturity affects the recovered amount. Value of bond is thus a sum of expected discounted value of the bond if no default occur until maturity and discounted

value of recovery at default time  $\tau$ :

$$\begin{aligned}
b_0^T &= PV[P[\tau > T] \times 1] + PV[\hat{R} \times P[\tau \leq T] \times g_\tau^T] \\
&= e^{-rT} e^{-\lambda T} \times 1 + e^{-r\tau} \hat{R} \times (1 - e^{-\lambda T}) e^{-r(T-\tau)} \\
&= e^{-(r+\lambda)T} - \hat{R} \times e^{-(r+\lambda)T} + \hat{R} \times e^{-rT} \\
&= (1 - \hat{R}) e^{-(r+\lambda)T} + \hat{R} e^{-r\tau} g_\tau^T \\
&= (1 - \hat{R}) e^{-rT} P[\tau > T] + \hat{R} e^{-r\tau} g_\tau^T
\end{aligned} \tag{3.21}$$

Assuming equivalent recovery and its payment at default time,  $\hat{R} e^{-r\tau} g_\tau^T$  is the certain part, received even in case of default, while the  $(1 - \hat{R})$  fraction of default-free bond (see Equation 3.21) is additionally received only if no default occurs until maturity. Based on Equation 3.21 it is obvious that using this convention, pricing of bond cannot be easily transformed into the basic model in Equation 3.15. Therefore difference between default-free and defaultable bond prices can not be expressed as simply as with recovery of face value convention.

Thirdly, if bondholders could have sold the bond at the market price just before the default then pre-default market value can be considered as their exposure. Under so called *fractional recovery* convention, recovery is a fraction of pre-default market value of the bond  $b_{\tau-}^T$  and is paid at default time  $\tau$ . Value of this bond is again sum of expected discounted value of the bond if no default occurs until maturity and discounted value of recovery at default time  $\tau$ .

$$\begin{aligned}
b_0^T &= E[e^{-rT} P[\tau > T] \times 1] + E[\hat{R} b_{\tau-}^T P[\tau \leq T]] \\
&= e^{-rT} e^{-\lambda T} + e^{-r\tau} \hat{R} b_{\tau-}^T (1 - e^{-\lambda T}) \\
&= e^{-(r+(1-\hat{R})\lambda)T}
\end{aligned} \tag{3.22}$$

This imply that defaultable bond under fractional recovery convention can be valued as if it was default-free, but using intensity and recovery-adjusted interest rate  $r + (1 - R)\lambda$  for discounting, which is similar to result under face value recovery convention. As Giesecke (2004) explained, there is simple intuition behind Equation 3.22. Supposing that bond defaults with intensity  $\lambda$  and in case of this default,  $b_{\tau-}^T$  is paid out with probability  $\hat{R}$  and nothing is paid out with probability  $1 - \hat{R}$  (this results into expected recovery value of  $\hat{R} b_{\tau-}^T$ ). Thus default with no loss can occur with intensity  $\lambda \hat{R}$  and default with zero recovery can occur with intensity  $\lambda(1 - \hat{R})$ . However using this convention for modeling loss given default of not yet defaulted bonds is possible only in terms

of expected  $LGD$ , because the market pre-default time value is not available and can only be estimated.

Reduced-form models can be divided according to the manner in which the exposure at default is estimated. There is no consensus about the best exposure at default estimation in literature. Jarrow & Turnbull (1995) assumed that bondholders receive fraction of market value of equivalent default-free bond, while Duffie & Singleton (1999) preferred fractional recovery convention which allows for closed-form solutions using risk adjusted rate for discounting cash flows. As mentioned above, Duffie (1998) assumed recovery of face value convention due to mathematical tractability. It would be correct to use this convention if the absolute priority rule is strictly obeyed. Further on, we will follow the recovery of face value convention, assuming that absolute priority rule holds in a sense that bondholders of same seniority have equal rights regardless coupon structure and time to maturity. However, as will be discussed in next section, even under recovery of face value convention, it is possible to allow violation of the absolute priority rule, so that junior bondholders can recover some part of their exposure before more senior bondholders receive whole exposures.

Given assumption about recovery rate and estimation of risk-free interest rate, the default intensity of a bond can be calculated from its market price. In most studies it is done other way around: estimating default intensity in order to calculate the price of the bond. However, as noted by Giesecke (2004), using constant intensity variable  $\lambda$  causes term structure of credit spreads to be flat. For more realistic modeling of spreads, time-varying and stochastic intensities are used. For a brief explanation and definitions of default probability under stochastic intensity  $\lambda$  and definition of Cox process in reduced-form models see the Appendix A. In the following part, we will conclude this section with suggestions on extension of described reduced-form models under assumption of constant recovery rate.

### 3.2.3 Possible Extensions of Standard Reduced Form Models

In standard reduced form models it is assumed that default automatically imply insolvency, bankruptcy and liquidation. All these terms are usually merged and default is used as a final state of a firm and recovery is paid at default time. However, Jarrow & Purnanandam (2004) and Guo *et al.* (2009) brought up a more realistic approach which distinguishes default from insolvency. Also

liquidation is not the only possibility after bankruptcy. In the event of default<sup>9</sup>, the maturity of debt is postponed to some random time, called *resolution time*, and firm has two possible evolutions:

- firm's asset value stays above some insolvency barrier until resolution time and debt is paid back at full or at significantly high fraction and thus stays *solvent* and continues to operate normally, or
- firm's asset value falls below mentioned insolvency barrier before the resolution and it becomes *insolvent* and starts bankruptcy process. Then a lower fraction of debt value is recovered<sup>10</sup> and paid to investor. This could be further specified to liquidation and restructuring processes.

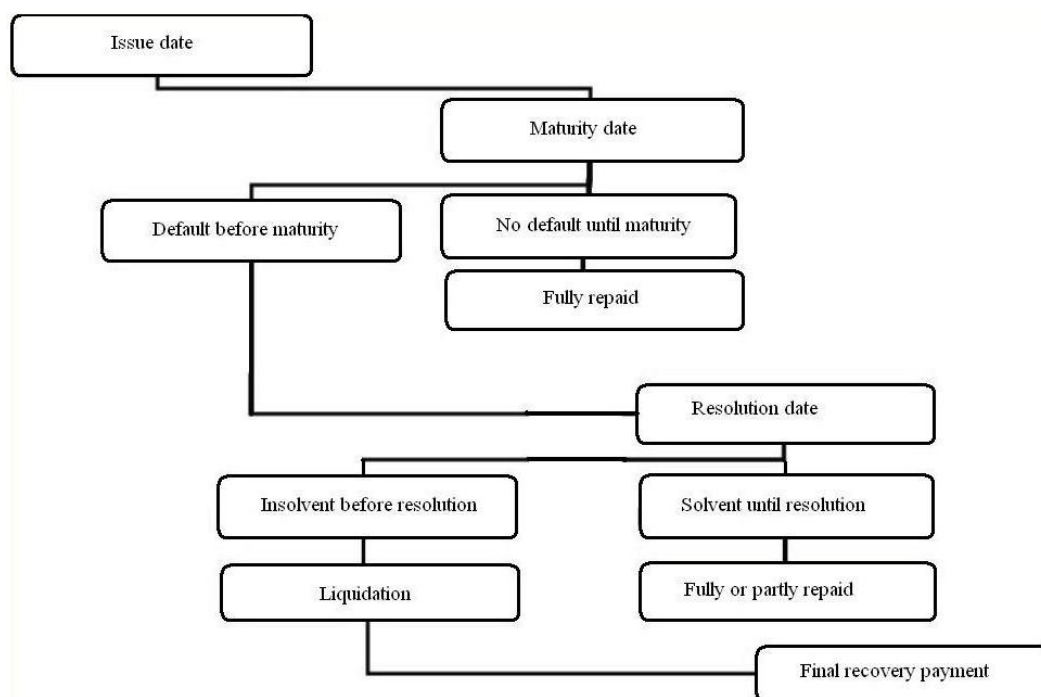
Therefore, whole process consists of three parts: default intensity process, bankruptcy intensity process conditional on default, and finally, recovery process conditional on previous processes. See the structure of the process in Figure 3.3. Firm defaults on particular obligation if default time  $\tau$  is lower than maturity  $T$ . Conditional on this, maturity of debt changes to resolution time  $\bar{T}$ . Firm becomes insolvent and thus bankrupts if bankruptcy time  $\tilde{\tau}$  is after default  $\tau$  but before resolution at  $\bar{T}$ .

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<sup>9</sup>It is important to remember that default includes situations such as late payment on any debt or violation of debt's covenants.

<sup>10</sup>Proof of recovery rate under bankruptcy being lower than recovery rate at resolution time can be found in Appendix C of Guo *et al.* (2009).

Figure 3.3: Time Scheme of Default, Bankruptcy and Recovery Process



Source: Author's construction, based on Guo *et al.* (2009)

Secondly, Guo *et al.* (2009) suggested to implement a basic idea from structural models but still retain within reduced-form information simplification. Thus they took into account firm's asset and liabilities processes and used this in modeling default as well as bankruptcy time. They used regime switching model and jump diffusion model to describe firm's asset value process. Both are based on continuous-time Markov chain process and models of asset value dependent on drift and volatility variables. In the later one, asset value is further dependent on random jump amplitude value.

Thirdly, Guo *et al.* (2009) quantified all processes under two different sets of information:

- *complete information*: management of the firm has precise information about actual asset value at any time,
- *partial information*: investors have delayed information about asset value. Information are revealed to investors in repeated times  $t_i$ , for example in form of quarterly reports, and in random times  $T_n$ , when state of a firm is changed, for example in form of newspaper articles.

Such different sets of information cause different valuation of debt by management and investors. This valuations are only same at time zero (when debt is issued). Also when time is converging to any revealing time  $t_i$  or  $T_n$ , the prices of debt converge to same values.

Next section pays attention to modeling loss given default parameter. As discussed in Duffie & Singleton (2003), the product of  $PD$  and  $LGD$  in the bond's credit spread is difficult to separate using the standard reduced-form modeling approach. Therefore we will focus on the theory of *adjusted relative spread* introduced by Unal *et al.* (2003), in which market prices of senior and junior debts are used to estimate senior and junior  $LGD$  parameter. This theory will be explained and modified so that it can be used for time-varying  $LGD$  calculation.

### 3.3 Extracting LGD from Bonds with Different Seniority

Most credit risk models use the constant expected  $LGD$ , as it was discussed previously. We already reviewed further possibilities within reduced-form modeling concerning probability of default and exposure at default. We described a model, based on Hull & White (2000), to calculate time-varying probability of default given an assumption on  $EAD$  and  $LGD$ . Thereafter, we discussed three main conventions for exposure at default and decided to use recovery of face value.

The last component of the credit spread, using the basic reduced-form relationship in Equation 3.15, not yet investigated is the loss given default rate<sup>11</sup>. The importance of modeling time-varying recovery rate was lately emphasized in few studies. See Bakshi *et al.* (2006) for the framework which shows importance of stochastic recovery rate process. They furthermore parametrized  $PD$  and  $LGD$  in a way that correlation between parameters can be modeled. In this section, we will present the approach for pricing the risk of recovery in default using market prices of bonds with different seniority based on Unal *et al.* (2003).

Unal *et al.* (2003) claimed that recovery of bonds issued by the same company differ only if these bonds have different seniority. Furthermore, they sup-

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<sup>11</sup>We still assume that recovery rate is complement to loss given default rate, so that  $PD(t) + LGD(t) = 1$  at any time.

posed all bonds of one issuer to be exposed to the same arrival risk of default. Similar characteristics were assumed by Jarrow (2001) who used market prices of bonds and equity to parametrize default probability and constant recovery rates for bonds and for equity and successfully overcame the problem of *PD* and *LGD* separation.

It was already stated that different types of debt and equity can be put into order based on its seniority (see Figure 2.5). Under *absolute priority rule*, in case of default (bankruptcy or liquidation), more senior debt should be repaid in full before any junior debt or shares are being repaid even partly. Therefore, when default occurs, market value of firm's assets would be evenly divided between all senior bondholders, thus all senior bonds would be subject to the same recovery rate, which is also referred to as *pari-passu* characteristic, which can be usually found in bonds prospectus. If these are fully paid off, then remaining value of firm's assets would be evenly divided between junior bondholders, so that same recovery rate would be applied among them. Same would follow for shareholders if all senior and junior bondholders are fully paid off. However APR rule is often violated in reality.

We would first describe a new statistic, the *adjusted relative spread*, introduced by Unal *et al.* (2003), which is positively related to recovery rates and is free of default timing consideration. This will be followed by discussion of reasons of absolute priority rule violation and extension of the model for extracting time-varying *LGD* of junior bonds relative to senior bonds of a firm.

### 3.3.1 Adjusted Relative Spread

According to Unal *et al.* (2003), senior and junior bonds of the same issuer face the same probability of default and their relative prices are therefore highly important in order to extract loss given default. In their study, the face value convention was assumed, therefore the price of defaultable bond can be expressed as

$$b_i^T = e^{-rT} (e^{-\lambda T} \times 1 + E[R_i](1 - e^{-\lambda T}) \times 1) \quad (3.23)$$

where  $E[R_i]$  denotes expected recovery rate for junior ( $i = J$ ) or senior bonds ( $i = S$ ).

Unal *et al.* (2003) defined a new statistics, the relative spread *RS*, which is equal to the ratio of difference between senior and junior debt prices over difference between default-free and junior debt prices. They denoted the ratio of sum of nominal of all senior bonds  $S$  to total issued nominal for all bonds

$S + J$  by  $p_S$ . Then adjusted relative spread  $ARS$  can be defined as

$$ARS = p_S \times RS = p_S \times \frac{b_S^T - b_J^T}{g^T - b_J^T}, \quad (3.24)$$

where  $b_S^T$  denotes the market price of senior bond with maturity at  $T$  and  $b_J^T$  denotes market price of junior bond with the same maturity.  $ARS$  can easily be calculated from market available data. The next aim is to express the adjusted relative spread in terms of recovery rates and without impact of the probability of default. This is done by substituting the formula for pricing defaultable bonds (Equation 3.23) with the same maturities and with different expected recovery rates for senior and junior bond. Therefore for  $ARS$  holds following relation

$$ARS = p_S \times \frac{E[R_S] - E[R_J]}{1 - E[R_J]} \quad (3.25)$$

Because  $ARS$  is not dependent on probability of default, it is referred to as pure recovery model. Consider a firm that issued senior bonds with the sum of total nominal  $S$  and expected recovery rate  $E[R_S]$  and junior bonds with sum of total nominal  $J$  and expected recovery rate  $E[R_J]$ . Expected aggregate recovery rate  $E[R]$  to all outstanding bonds is then calculated as

$$E[R] = \frac{S}{S+J}E[R_S] + \frac{J}{S+J}E[R_J] = p_S E[R_S] + (1 - p_S)E[R_J] \quad (3.26)$$

Then,  $ARS$  can be expressed in terms of junior and aggregate expected loss given default as

$$ARS = p_S \times \frac{E[R_S] - E[R_J]}{1 - E[R_J]} = \frac{E[R] - E[R_J]}{1 - E[R_J]} \quad (3.27)$$

$$ARS = 1 - \frac{E[LGD]}{E[LGD_J]}$$

According to Unal *et al.* (2003), it is necessary to obtain risk-neutral recovery density for  $R_J$  for better analysis of  $ARS$  dynamics. Then, based on the estimated relationship between  $R$  and  $R_J$ , and on  $ARS$  calculated from market prices spreads, it would be straightforward calculation to obtain market implied recovery rates. They expected  $R_J$  to be function of aggregate recovery  $R$  denoted as  $R_J = J(R)$ . If we have a density function of possible aggregate



recoveries  $f(R)$ , then expected recovery of junior bonds can be calculated as

$$E[R_J] = \int_0^1 J(R)f(R)dR \quad (3.28)$$

Unal *et al.* (2003) claimed that the density  $f(R)$  can be assumed as logit transformation of normally distributed variable  $x$ . Thus  $R = \frac{e^x}{1+e^x}$  satisfies that aggregate recovery rate should be in  $(0, 1)$  interval as it is describing percentage value. Another advantage against normally distributed  $R$  is that mean and variance are not related, while if assuming normal distribution, the variance is approaching 0 for both, zero recovery and complete recovery. Assuming  $x \sim N(\mu, \sigma^2)$ , the conditional aggregate recovery density is:

$$f(R) = \frac{1}{\sigma\sqrt{2\pi}R(1-R)} \times e^{-\frac{1}{2\sigma^2}(\ln(\frac{R}{1-R})-\mu)^2} \quad (3.29)$$

for  $R \in (0, 1)$ . Given the density function, it is possible to express mean and variance of the aggregate recovery in terms of  $\mu$  and  $\sigma$

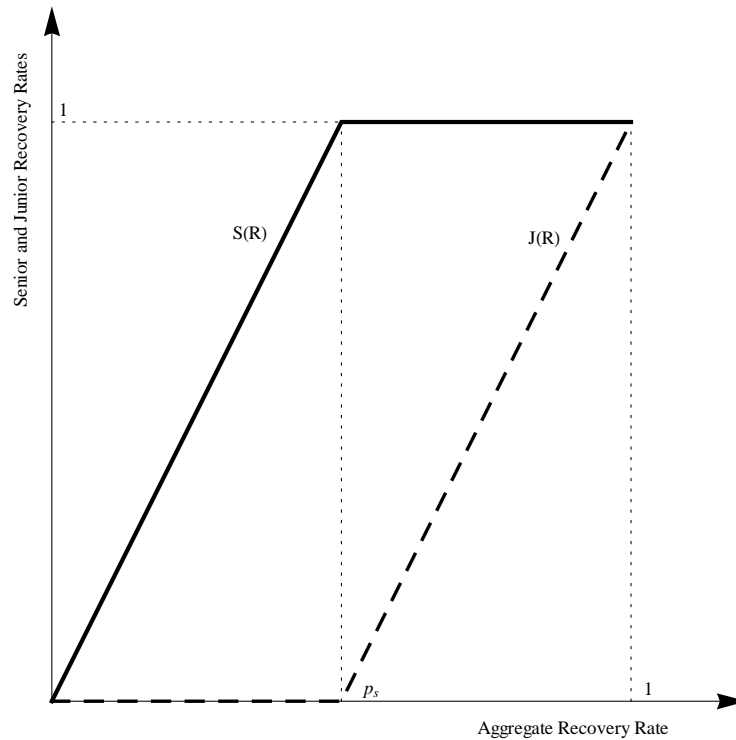
$$E[R] = 1 - \int_0^1 N\left(\frac{\ln(\frac{R}{1-R}) - \mu}{\sigma}\right) dR \quad (3.30)$$

$$Var[R] = \int_0^1 2(1-R) \times N\left(\frac{\ln(\frac{R}{1-R}) - \mu}{\sigma}\right) dR - \left(\int_0^1 N\left(\frac{\ln(\frac{R}{1-R}) - \mu}{\sigma}\right) dR\right)^2$$

where  $N$  denotes probability distribution function of standard normal distribution. See the Appendix in Unal *et al.* (2003) for derivation of the mean and variance.

To define payoff function  $J(R)$ , assumptions about absolute priority rule must be made. Under strict APR, junior bondholders only receive payment after all senior bondholders are fully paid off. As  $p_S$  is the ratio of senior bonds to all bonds issued by the company, aggregate recovery must be greater than  $p_S$  in order for junior recovery not to be zero. This payoff structure is the same as for long position on a call option with strike price  $p_S$ . See Figure 3.4 for graphical understanding of the relationship between aggregate recovery rate and junior and senior recovery rates. Number of corresponding units of call option is equal to the slope of payoff function  $(\frac{1}{1-p_S})$ .

Figure 3.4: Senior and Junior Recovery Rate Structure



Source: Adopted and changed from Unal *et al.* (2003)

In analogy to  $E[R_J]$ , we can define the expected senior recovery rate  $E[R_S]$  as

$$E[R_S] = \int_0^1 S(R) f(R) dR \quad (3.31)$$

where  $f(R)$  is the density function of aggregate recovery and  $S(R)$  is a payoff function for senior bonds in relation to the aggregate recovery. Assuming APR,  $S(R)$  is the same as short position on a put option with strike price  $p_S$ . This can be graphically seen from Figure 3.4. Senior bondholders sell  $\frac{1}{p_S}$  units of the put option.

However, assumption that absolute priority rule strictly holds is quite over-optimistic. Therefore we will next discuss how to estimate the extent of APR violation and then include this parameter into payoff functions  $J(R)$  and  $S(R)$ .

### 3.3.2 Absolute Priority Rule Violation

According to loan agreements and bond contracts, in case that borrower fails to repay, lenders have a right to receive back their full or partial investment. They can force borrower to bankruptcy and retrieve payments from liquidation

of assets or they can take possession of these assets. Absolute priority rule is the legal rule that should help to decide how much are bondholders entitled to receive from value of the assets. It states that more senior debtholders have priority over junior ones and over shareholders. Even though APR seems simple, it is not that easily implemented. Longhofer & Carlstrom (1995) provides overview of empirical studies on frequency of APR violation. Most studies concluded that absolute priority rule was violated to some extent in more than 70% of cases. Reasons behind this high number might lie in the increasing efficiency of bankruptcy process in cases when APR is not strictly followed. Some senior bondholders agree to decrease their recovery in favour of junior bondholders or shareholders if it fasten the reimbursement process.

It is important to notice that bankruptcy process in US and countries in Europe is quite different. In most countries there are two bankruptcy processes: liquidation of assets and reorganization of debts. According to Brouwer (2006), bankruptcy followed by reorganization is more common in USA (around 5% of cases) then it is in Europe (around 0.4% in Germany). She claimed it is a consequence of legal origins. While in USA, UK and Ireland<sup>12</sup> prevails *Common law*, under which judges have more flexibility in their decisions, in European countries legislation-driven *Civil law* is more widespread. In Europe it can be further divided between Scandinavian law countries, German civil law countries and France civil law countries. In general, in civil law countries, state and legislation is putted above the courts and judges. Most significant findings based on Brouwer (2006) are:

- in Germany, secured debtholder are always paid first
- in France, the protection of creditors is very low, employees are highly protected by legislation
- in Common law countries, shareholders receive more protection from courts than debtholders
- in Civil law countries, it is more difficult to decide about absolute priority rule violation in courts as this would be against legislation

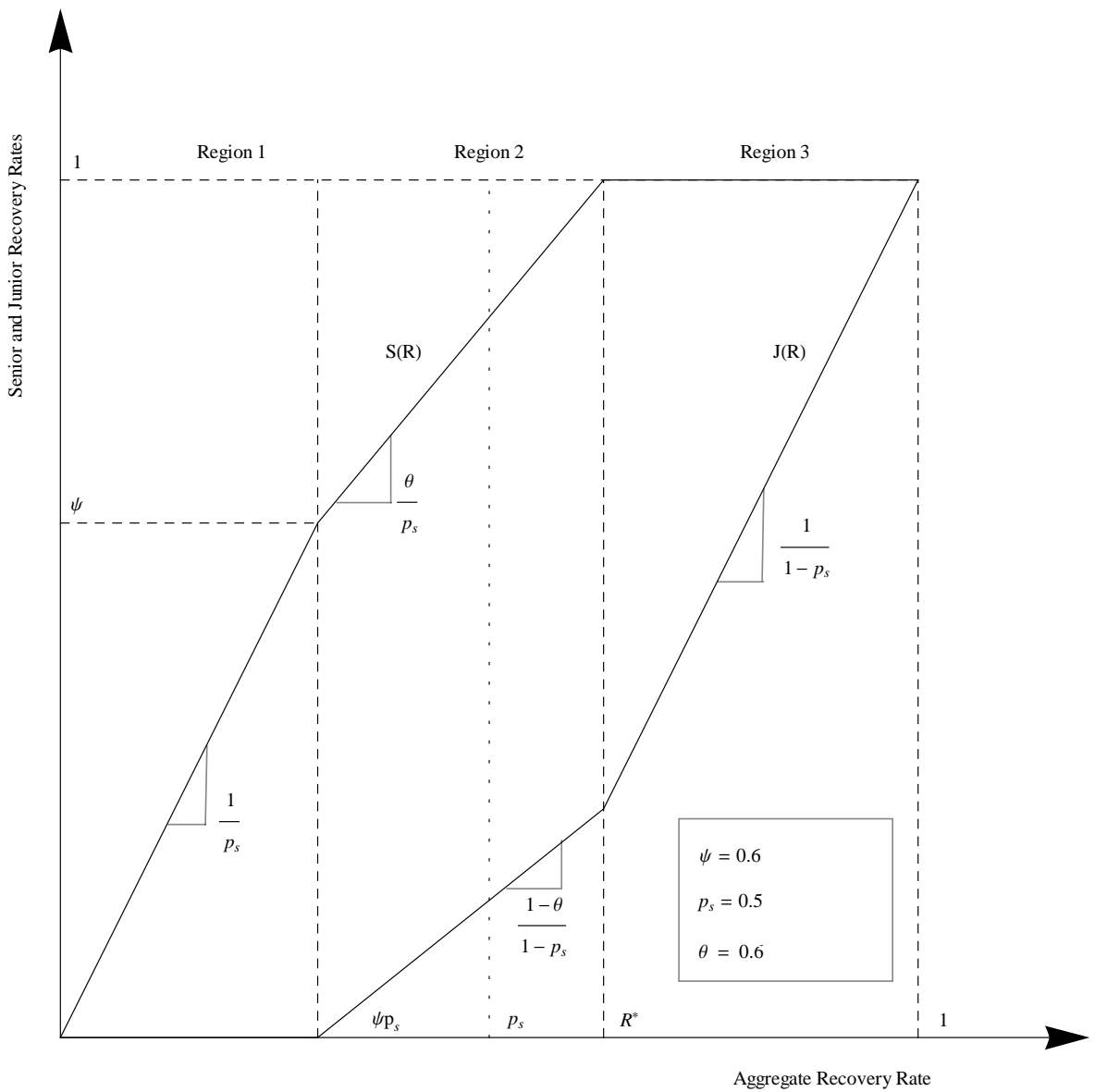
When modeling violation of absolute priority rule, it is necessary to define type and extent of the violation. Following approach of Unal *et al.* (2003),

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<sup>12</sup>From legal origin point of view, UK and Ireland are not considered as a European countries, rather they are similar to USA and other Anglo-Saxon countries.

violation can be represented by situation when senior bondholders are equally paid up to  $\psi\%$  of their claims, which represents the aggregate recovery level at which violation of APR occurs. Afterwards, if possible, both senior and junior bondholders are being repaid. This is distributed in the ratio of  $\theta : (1 - \theta)$  for senior to junior bondholders. Schema in Figure 3.5 demonstrates the relationship of senior and junior recovery rates to aggregate recovery rate.

Figure 3.5: Senior and Junior Recovery Rate Structure with Violation of APR



Source: Adopted and changed from Unal *et al.* (2003)

The first region represents aggregate recovery up to level  $p_s\psi$  when only

senior bondholders receive payments and  $J(R) = 0$ .  $S(R)$  is equal to  $\frac{R}{p_S}$ . In the second region all bondholders are sharing what is additionally recovered. The increase of senior recovery rate function is reduced due to multiplication by  $\theta \leq 1$ . Therefore starting at  $S(R) = \psi$  and increasing at rate  $\frac{\theta}{p_S}$  up to full recovery means that function for senior recovery rate in second region is

$$S(R) = \psi + \frac{\theta}{p_S}(R - \psi p_S) \quad (3.32)$$

Junior bondholders are better off compared to APR as their payoff function is equal to the long position on call option with lower strike price ( $\psi p_S$ ). On the other hand, senior bondholders are worse off, which can be seen from higher aggregate recovery necessary for their full recovery ( $R^* > p_S$ ). Aggregate rate  $R^*$  at which senior bondholders are fully paid off can be extracted from Equation 3.32 as

$$R^* = \psi p_S + \frac{(1 - \psi)p_S}{\theta} \quad (3.33)$$

Variable  $R^*$  stands for a rate in percentage, thus must be within interval  $\langle 0, 1 \rangle$ . Therefore, the bounding condition on  $\theta$ , using Equation 3.33 is following<sup>13</sup>

$$\begin{aligned} \psi p_S + \frac{(1 - \psi)p_S}{\theta} &\leq 1 \\ \frac{p_S - \psi p_S}{1 - \psi p_S} &\leq \theta \end{aligned} \quad (3.34)$$

Junior recovery rate is increasing at rate  $\frac{1-\theta}{1-p_S}$  in the second region, therefore at the moment senior bondholders reached full recovery (aggregate recovery is  $R^*$ ) junior recovery rate is

$$J(R^*) = \frac{(1 - \theta)(1 - \psi)p_S}{(1 - p_S)\theta} \quad (3.35)$$

In the third region, if aggregate recovery is over  $R^*$ , all senior bondholders are fully repaid and junior recovery is increasing from  $J(R^*)$  up to full recovery. To summarize, in terms of  $\psi$  - the recovery level of senior bonds at which APR is violated,  $\theta$  - rate of reduction of senior recovery due to APR violation,  $p_S$  - ratio of senior bond, and  $R$  - aggregate recovery rate, payments recovered by junior and senior bondholders can be expressed as

<sup>13</sup>In order to satisfy the bounding condition arising from  $R^* \geq 0$ , it is enough to assume that  $\theta \geq 0$ .

Table 3.1: Junior and Senior Recovery Rates under APR Violation

$J(R)$	$S(R)$	for $R$
0	$\frac{R}{p_S}$	$R \leq \psi p_S$
$\frac{(1-\theta)(R-\psi p_S)}{1-p_S}$	$\psi + \frac{\theta}{p_S}(R - \psi p_S)$	$\psi p_S \leq R \leq R^*$
$\frac{R-p_S}{1-p_S}$	1	$R^* \leq R \leq 1$

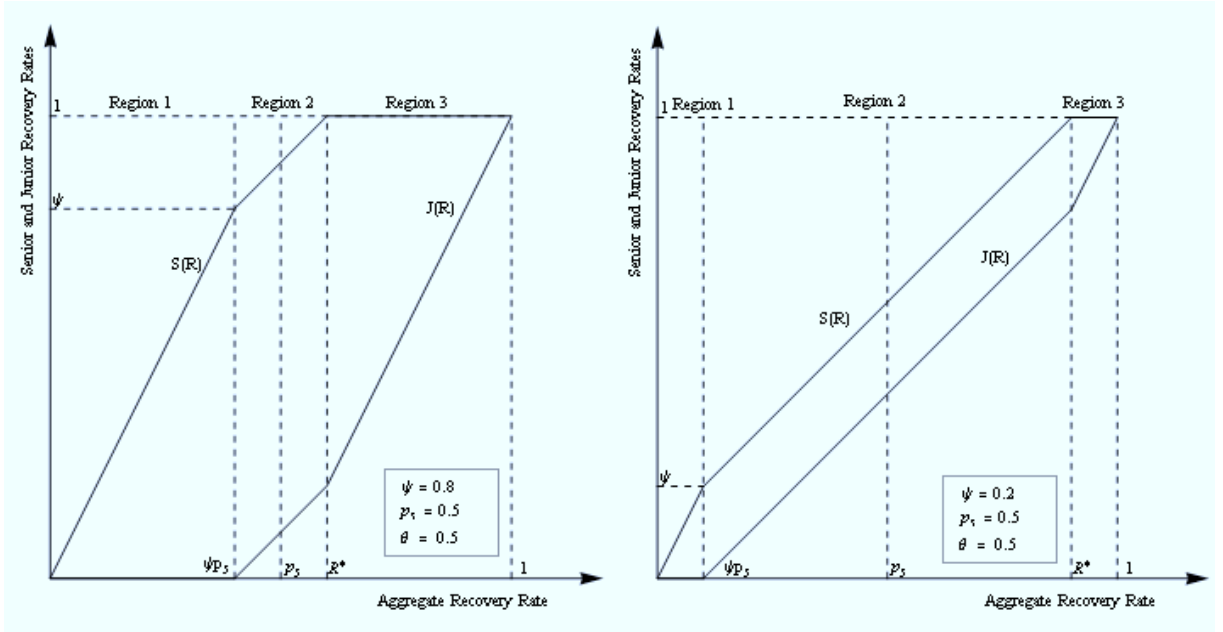
We would now discuss how are junior and senior recovery rate structures sensitive to  $\psi$ ,  $\theta$  and  $p_S$ . This analysis is done in three scenarios. In each one we fix two of these parameters at some non-extreme rate and choose one very low and one very high rate for the last parameter. When choosing these rates it is necessary to count for bounding conditions as these parameters all express a percentage. Another bounding condition in Equation 3.34 is due to the fact that senior bondholders must reach full recovery at least when aggregate recovery rate is 1.

Firstly, if  $\psi$  is low, senior bondholders are exclusively paid only small fraction of their investment before firm's asset value is distributed between senior and junior bondholders. This imply that senior recovery rate is significantly lower for  $R \in (\psi p_S, R^*)$ , see schema on the right in Figure 3.6. Also the minimum aggregate recovery rate when senior bondholders are fully repaid is much higher for low  $\psi$ .

Secondly, structure of senior and junior recovery rates is most effected by  $\theta$ . It stands for the ratio in which proceeds are divided between senior and junior holders when these should be repaid simultaneously. When  $\theta$  is approaching 1, structure looks very similar to strict absolute priority rule, regardless value of  $\psi$  and  $p_S$ . This can be seen in schema on the right in Figure 3.7. Similarly to  $\psi$  sensitivity, the minimum aggregate recovery rate after which senior bondholders are fully repaid is higher for low  $\theta$ .

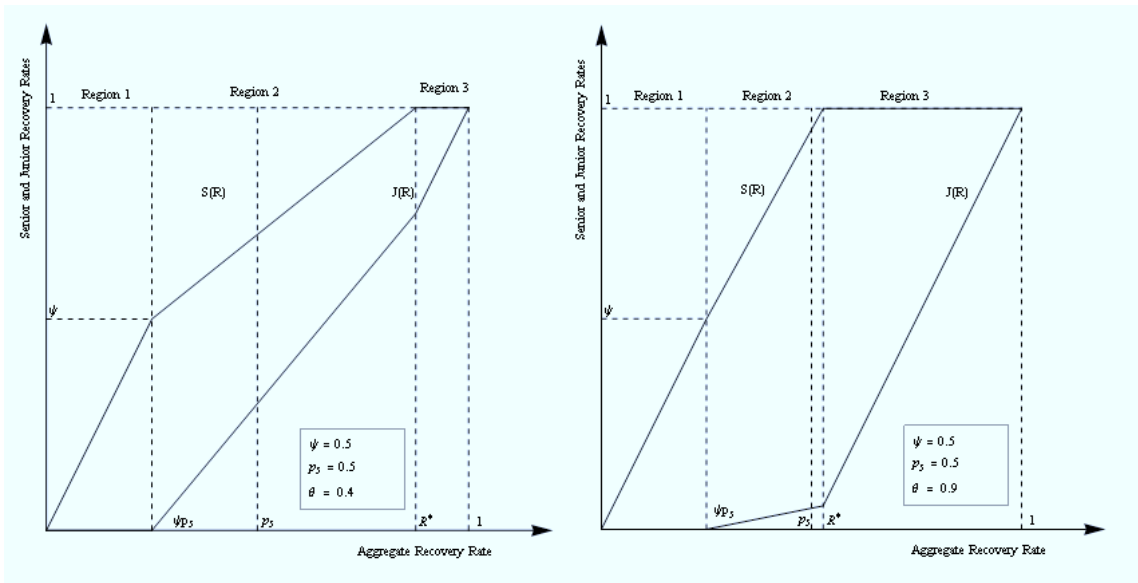
Finally, the structure of senior and junior recovery rates is also effected by ratio of total nominal of senior bonds to all bonds. If this ratio is high, so that total value all junior bondholders might recover is relatively small. From

Figure 3.6: Sensitivity of Senior and Junior Recovery Rate Structure to Recovery Level of Senior Bonds at which APR is Violated



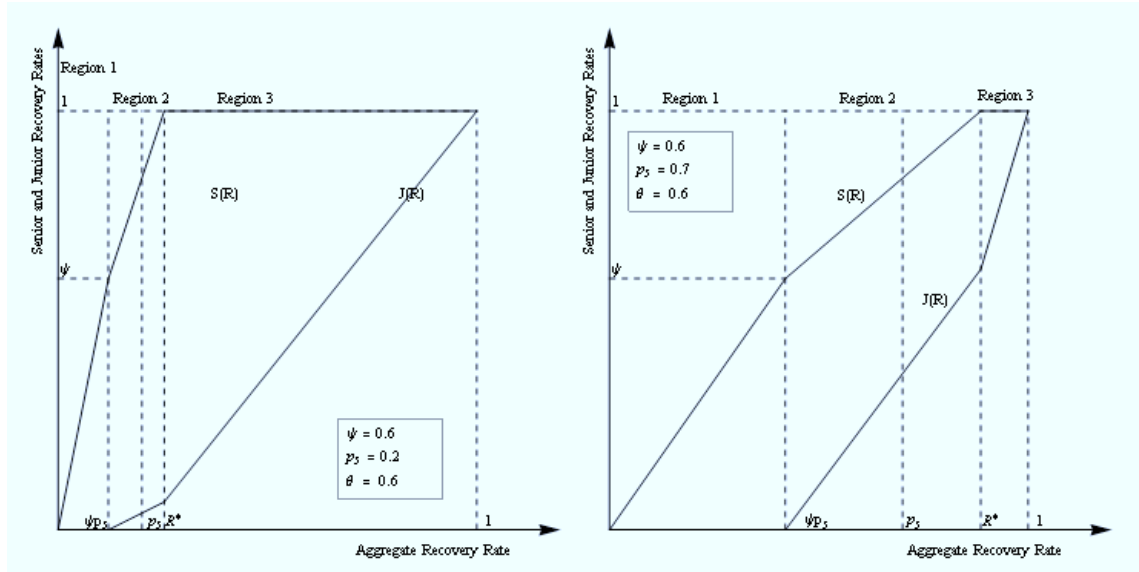
Source: Computed from Table 3.1

Figure 3.7: Sensitivity of Senior and Junior Recovery Rate Structure to Rate at which APR is Violated



Source: Computed from Table 3.1

Figure 3.8: Sensitivity of Senior and Junior Recovery Rate Structure to Ratio of Issued Senior Bonds



Source: Computed from Table 3.1

extreme point of view, even if junior bondholders are fully repaid before senior ones, this would not have significant impact on value possibly recovered by senior bondholders. Other way around, as can be seen in Figure 3.8, lower aggregate recovery is necessary for the same senior recovery rate if  $p_S$  is low.

As Unal *et al.* (2003) noticed, this relationship between aggregate and junior recovery rate can be transformed into sum of two call options, which can also be intuitively seen at Figure 3.5. First call option is in money if aggregate recovery rate is greater than  $\psi p_S$  and slope of the payoff function in the second region correspond to number of these call options. Second call option is in money only if aggregate recovery rate is greater than  $R^*$ . In these cases, first call option would always be in money as well, because it has lower strike price. Therefore, to simulate payment received by junior bondholders if  $R \geq R^*$  we must add payoffs from both call options. Thus the amount of second call option that correspond together with first option to payoff to junior bondholders is the difference between slope of payoff function in third and second region. Thus junior recovery rate, written as sum to two call options, is equal to

$$J(R) = \frac{1 - \theta}{1 - p_S} \text{Max}[R - \psi p_S, 0] + \frac{\theta}{1 - p_S} \text{Max}[R - R^*, 0] \quad (3.36)$$

Similarly, senior payoff function correspond to sum of two put options in



short position. Generally, writer of a put option with strike price  $k$  receive premium payment from the buyer and expects that price will no fall below strike price, when buyer would realize the put option and therefore payoff will decrease. Writer of the second put option receives premium of 1 unless  $R \leq R^*$ , then the payoff falls at rate  $\frac{\theta}{p_S}$ , which correspond to number of options. Due to the first put option the payoff is decreased if  $R \leq \psi p_S$  at rate  $\frac{1-\theta}{p_S}$ . The rate stands for difference in slopes in second and first region. Thus senior recovery rate, written as sum to two call options, is equal to

$$S(R) = 1 - \frac{1-\theta}{p_S} \text{Max}[\psi p_S - R, 0] - \frac{\theta}{p_S} \text{Max}[R^* - R, 0] \quad (3.37)$$

Then using derived density of aggregate recovery, the calculation of the expected junior and senior recovery rates, based on Equation 3.28 and Equation 3.31, is straightforward. Parameters of recovery density,  $\mu$  and  $\sigma$  can be estimated following the *ARS* reduced-form model. Then  $E[R_J]$  and  $E[R_S]$  can be calculated based estimated parameters of APR violation,  $\theta$  and  $\psi$ . However, as will be explained later together with other shortcomings of empirical application of this model, junior or subordinated bonds are not used by all firms, rather it is an attribute of financial institutions. Furthermore, finding junior and senior bonds with the similar maturity is often not possible.

Therefore, in the next chapter we will present another financial instrument, the credit default swap, and the reduced-form model for its pricing. In order to estimate loss given default, we will introduce two theoretical methods. Firstly, the simultaneous estimation of *PD* and *LGD* based on market prices of risky and risk-free bonds together with market prices of credit default swaps. Secondly, due to better attributes for empirical usage, we will transform the *ARS* model discussed in this section and apply it to senior and junior credit default swaps.

## Chapter 4

# Reduced-form Modeling Based on Credit Default Swaps

Guo *et al.* (2009) provide explanation why it has lately become more important to model also recovery rate process, not only default intensity process as it has been done by most authors so far. Moreover, they claim that nowadays, it is more realistic to model recovery rates thanks to two changes in financial markets in last few years. These are:

- the expansion of markets for defaulted debt,
- the expansion of credit derivatives markets<sup>1</sup>.

Higher efficiency and liquidity of these markets is significantly transferred into more accurate pricing of credit derivatives on defaultable debt as well as defaultable debt itself. Thus, it makes it easier to estimate default probability of the debt and its recovery rate from market data. In this chapter, we will not consider loss given default rate to be exogenously given and we will focus on its calculation together with probability of default parameter. We continue to assume face value convention, which is also consistent with credit default swap method, where protection payment is equal to fraction of nominal value in case of default.

Market with credit default swaps is the most developed market with credit risk derivatives. Generally, trading with derivatives only started in late 1990's, and has rapidly increased since that time. According to data provided by International Swaps and Derivatives Association (ISDA), the sum of underlying

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<sup>1</sup>Introduction of recovery rate swaps is an important issue in pricing risky debt. In order to price these derivatives correctly, it is necessary for investors to estimate recovery rates based on other market information such as credit spread between risky and risk-free bonds.

notional amount of credit default swaps was \$0.92 billion in 2002, \$62.17 billion in 2007 and \$26.30 billion in 2010. Slowdown in 2010 is associated with the global financial crisis, but despite this fact, the expansion of the market compared to the situation at the beginning of this century is enormous. Motivation behind development of these derivatives was firstly to hedge against credit risk. However, lately it has been more often used for speculation purposes. Many investors are nowadays trading with credit derivatives without holding underlying securities. The liquidity significantly increased due to higher number of participants and thus higher number of closed trades on the market, which allows for better and more precise modeling of derivative prices.

Firstly, we will describe terms of the credit default swap contracts and their standardization under ISDA, then the model for pricing CDS will be presented and adjusted for further empirical use. Next, using this model, we will introduce the method for  $PD$  and  $LGD$  calculation using market prices of credit default swaps and defaultable bonds with different maturities. Finally, the reduced-form model for pricing credit default swaps will be extended using different  $LGD$  for CDS written on bonds with different seniorities. We will show that this approach is more practical for empirical usage, compared to the  $ARS$  model based on bond prices, as CDS contracts are standardized.

## 4.1 CDS Contractual Terms

Credit default swap (CDS) can be defined as a bilateral contract between two counterparties, one of which is buying the guarantee (buyer of CDS) and the other one is selling the guarantee for regular payments (seller of CDS). Risk is transferred from the buyer of CDS to the seller. Credit default swap is issued for underlying firm's debt with some nominal value - it is referred to as notional amount. The buyer pays regular fixed payments, known as CDS spread, most usually annually, semi-annually or quarterly to the seller until the specified end of the contract<sup>2</sup> or until default of the underlying firm occurs. The seller pays only in case of default and the payment depends on what is agreed in the contract.

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<sup>2</sup>In order to harmonize the CDS contracts, under ISDA Master Agreement, which is used for most of the CDS contracts, maturity date as well as dates of payments by the buyer can be only on March 20, June 20, September 20, December 20.

Usually the settlement in case of default is done in one of the three following possibilities:

- physical settlement: buyer of CDS delivers defaulted bonds with total underlying nominal value equal to notional amount of CDS and is paid the nominal value from seller<sup>3</sup>,
- cash settlement: seller of CDS pays the difference between nominal value and the corresponding market price of defaulted bonds of the underlying issuer,
- fixed settlement: fixed amount agreed in contract regardless of after-default market price of bonds.

Apart from different types of settlement, as discussed in Packer & Zhu (2005), there are two more issues in credit default swap contracts:

- definition of deliverable bonds within physical settlement or definition of reference bond whose market price is used to calculate the amount of cash settlement
- definition of credit default that triggers the payment from CDS seller

In the CDS contracts the underlying issuer is agreed upon, not specific bonds. It is possible that only some type of bonds of the issuer are considered as underlying. CDS can be defined for only senior or only subordinated bonds of the issuer<sup>4</sup>. Then CDS buyer can deliver any bond of the issuer, which has the specified seniority. However, it is not so straightforward within cash settlement. As market prices of different bonds of the issuer can differ (even for bonds having the same seniority), it is not clear how much CDS seller should pay to CDS buyer when default occurs. Therefore, it has been generally accepted to take the market price of the bond, which is referred to as *cheapest to deliver*. Based on this rule and the fact that investors will always try to deliver the cheapest bond under contract with physical settlement, value of CDS is not dependent on the settlement method.

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<sup>3</sup>Physical settlement is used mainly when credit default swap is actually bought for hedging credit risk. The buyer owns underlying bonds and these, if defaulted, are “sold” to CDS seller for nominal price, so that CDS buyer does not incur any loss. Physical settlement was mostly used until 2005 [Mengle (2007)].

<sup>4</sup>Apart from single entity CDS, there are also basket CDS, which underlie two to ten reference entities, and index CDS underlying all entities included in specified market index. See Mengle (2007) for more details on basket and index CDS. We will focus on credit default swaps underlying single entity.

Regarding the definition of default, it must be defined in CDS contract which credit events are considered as default. Based on Mengle (2007), credit events that can be included in CDS contracts fall into following categories:

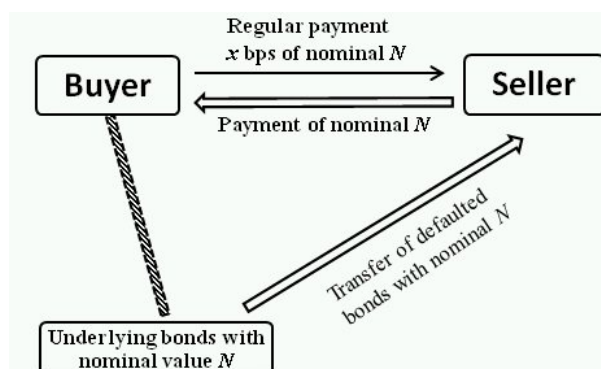
- *failure to pay*
- *bankruptcy* - refers to the bankruptcy of a corporate reference entity
- *restructuring* - refers to events such as coupon change or maturity extension due to threat of bankruptcy
- *repudiation, moratorium* - refers to specified actions of government reference entities, usually it is relevant only to emerging markets
- *obligation acceleration, obligation default* - refers to technical a default such as violation of bond covenants

According to Packer & Zhu (2005), in most of the CDS contracts, failure to pay and bankruptcy events are considered as default, while repudiation, moratorium, obligation acceleration or obligation default are not. It is more complicated with regards to restructuring, as in some cases bondholders do not incur any loss, rather they profit from restructuring of the underlying firm. A restructuring clause in CDS contract specifies which bonds can be delivered in case of default. Under *full restructuring* any bond is eligible, under *modified* (or *modified-modified*) restructuring only bonds with maturities until 30 months (or 60 months) after the maturity of CDS contract can be delivered. Also, there are CDS contracts with no restructuring clause, and so these events are not considered as default. See Packer & Zhu (2005) for more information on why different restructuring clauses have been developed and which of them are actually used in various regions of the world.

The transfers of money and securities between buyer and seller is depicted on the scheme below (Figure 4.1). The idea of credit default swap can be compared to insurance contracts. In case of no default during the time of CDS contract, buyer pays regular fixed payments to seller - like insurance premium payments. These are defined in basis points of the underlying nominal. The price of credit default swap is then quoted as the annual payment in bps of the underlying notional amount. Usually, contracts are issued with maturity of 1, 2, 3, 4, 5, 7 and 10 years.

When the underlying firm defaults, the buyer of CDS must pay corresponding accrued premium since the last premium payment to the default date. Then,

Figure 4.1: Transfer of payments and securities between buyer and seller of CDS contract



Source: Adopted and changed from O’Kane & Turnbull (2003).

depending on the settlement payment, the seller pays appropriate amount to the buyer of CDS. Credit default swaps can be considered as insurance against credit loss incurred in case of default of the underlying firm. Thus in order to price CDS, investors must estimate the future evolution of credit risk parameters ( $PD$ ,  $EAD$  and  $LGD$ ), therefore, these are included in the market prices of credit default swaps. We will not use the fixed settlement method in the pricing model of CDS because it does not directly disclose the credit risk parameters.

Risk is transferred in credit default swap trades. However, as discussed in Mengle (2007), this risk is not symmetrical as it is in other derivatives (e.g. interest rate swap). The buyer of CDS transfers credit risk of the reference entity to the seller. Apart from this, the buyer takes on the risk that the seller and the reference entity will simultaneously default. Furthermore, the buyer bears a liquidity risk which arises due to a lower liquidity of some CDS contracts, which imply that he cannot buy CDS with precise maturity that he would need for hedging. Besides credit risk of reference entity, the seller of CDS takes on the risk that the buyer will default and will not pay all promised premiums. In our model, we would assume no counterparty credit risk (risk of default of buyer or seller) and no liquidity credit risk. Therefore CDS spread is assumed to disclose information about credit risk of the reference entity. This assumption is based on Hull & White (2000).

The above described derivative is the plain vanilla credit default swap. There are other, more complex, derivatives traded on financial markets, such as total return swap, constant maturity CDS, first to default CDS, portfolio CDS, secured loan CDS, CDS on asset-backed securities, credit default swap-

tion, recovery lock transaction, credit spread option and different CDS index products. Moreover, there are variations of plain vanilla CDS, underlying only specific debt type, e.g. binary CDS, basket CDS. These derivatives have either more complex structure or, more usually, are not traded in such high volumes as plain vanilla CDS, thus liquidity risk would play a significant role in their pricing. Therefore, we will focus on plain vanilla contracts.

## 4.2 Reduced-form Approach to Pricing CDS

Credit default swap contract follows the basic insurance rule: expected value of premium payments gained by the insurer (CDS seller) must be equal to expected loss that can be incurred by the seller of CDS contract<sup>5</sup>. According to this rule, for calculation of CDS spread it is enough to evaluate what the possible cash flows are and assign to them the probability with which they might occur. Figure 4.1 shows these possible payments for physical recovery, while the cash settlement method can substitute the transfer of defaulted bonds from the buyer and the payment of nominal value from the seller by seller's payment of the difference between nominal value and market value of reference bonds.

According to the reduced-form approach, market value of a defaulted bond is equal to the fraction recovered from the exposure at default. We continue to use the face value convention, as stated previously, therefore in case a default occurs at time  $\tau$ , market value of defaulted bonds with notional principle  $N$  is  $(N \times R(\tau))$ . Amount to be payed within cash settlement is then  $N - N \times R(\tau) = N \times LGD(\tau)$ . This payment must be properly discounted in order to calculate the present value. Because timing of the default is not known when CDS is issued, value of cash settlement is equal to the present value of losses that can occur in case of possible future defaults during the life of the CDS contract. It is referred to as the *protection leg* of the CDS contract.

$$PV(\textit{protection}) = \int_0^T LGD(t) \times PD(t) e^{-tr_t} dt \quad (4.1)$$

where  $T$  is the maturity of CDS. To simplify, based on Hull & White (2000),

<sup>5</sup>This general idea is widely used in literature of reduced-form modeling for pricing credit default swaps. Models basically differ only in notation and assumptions about credit risk parameters. Examples of valuation of CDS can be found in Hull & White (2000), Schlaefler & Uhrig-Homburg (2010) or Doshi (2011).

we assume probabilities of default and loss given default to be constant over some intervals of time, as we did in the previous chapter when estimating  $PD$  from the credit spread. If life of CDS contract is divided into  $k$  intervals so that  $PD$  and  $LGD$  are constant on intervals  $(0, t_1), (t_1, t_2), \dots, (t_{k-1}, t_k = T)$ , then  $LGD(t_i)$  denotes percentage of loss incurred when default occurs within interval  $(t_{i-1}, t_i)$  and  $PD(t_i)$  denotes the probability that the default happens in the interval  $(t_{i-1}, t_i)$ . Also, we denote  $r_i$  as risk free rate relevant for interval  $(t_{i-1}, t_i)$ . Adjusting Equation 4.1, present value of possible protection payments from the CDS seller to the CDS buyer is following

$$PV(\text{protection}) = \sum_{i=1}^N LGD(t_i) \times PD(t_i) e^{-t_i r_i} \quad (4.2)$$

Secondly, it is necessary to calculate present value of regular fees paid by the buyer of CDS. Following Hull & White (2000), let's denote  $u(t)$  the present value of the sum of fixed payments from the beginning of the CDS contract to time  $t$  if CDS spread is 1 bps. Apart from this, in case of default the buyer needs to pay accrued fee since the last payment, let's denote its present value by  $e(t)$ . How to calculate these variables will be more precisely defined in the empirical part of this work, because it depends on the combination of frequency of fee payments and intervals  $(t_{i-1}, t_i)$  and it is not necessary to go into such details and complicate structure of the model at this point. Just for illustration, if buyer's fee of 1 bps of notional amount (in annual terms) is paid semiannually, then

$$u(t = 3.3) = \sum_{i=1}^{3 \times 2} \frac{N \times 10^{-4}}{2} \times e^{-\frac{i}{2} r_i} \quad (4.3)$$

and

$$e(t = 3.3) = 0.3 \times N \times 10^{-4} \times e^{-3.3 r_6}$$

Taking into account possibilities of default at any time up to the CDS maturity and the possibility that no default occurs until the maturity so that all premiums are paid, expected value of the buyer's fixed premium payments is following

$$PV(\text{premiums}) = s \times \left[ \int_0^T PD(t) \times (u(t) + e(t)) dt + (1 - PD(T)) u(T) \right] \quad (4.4)$$

Credit default swaps are contracts with two legs of payment, the buyer's premium leg and the seller's protection leg. Assuming no arbitrage, present



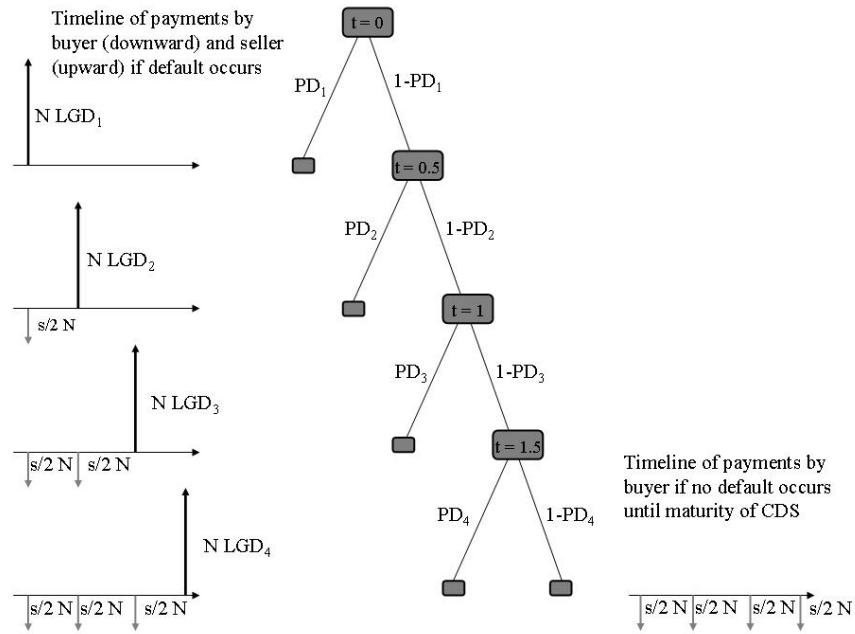
values of these two legs must be the same, thus present value of protection payment must be equal to the present value of premiums. Therefore, the price of credit default swap defined as the annual buyer's fee in bps of notional amount, the CDS spread  $s$ , can be expressed as:

$$s = \frac{\int_0^T LGD(t) \times PD(t) e^{-tr_t} dt}{\int_0^T PD(t) \times (u(t) + e(t)) dt + (1 - PD(T))u(T)} \quad (4.5)$$

Similarly, but a bit more illustrative, payments on the CDS contract can be specified using the so called "probability model". This is based on the binomial tree diagram described in Jarrow & Turnbull (1995) as a general mechanism for pricing derivatives or bonds. At each node of the tree there are two possibilities, either the underlying instrument will default in the next period of time or not. Probability of occurrence of the default needs to be specified for each leg, as well as payments triggered within each leg. This is depicted in Figure 4.2 for credit default swap with maturity of two years and semiannual premium payments. As already defined,  $N$  is the notional amount of CDS,  $s$  is annual spread in bps of  $N$ , thus  $\frac{s}{2}N$  is the premium paid every half-year.  $PD(i)$  denotes the probability that default will occur during the  $i^{th}$  half-year interval and  $LGD(i)$  is the loss given default rate at this interval. Because the reimbursement by CDS seller in case of default is equal to the difference between face value of bonds and its recovered value, payment triggered if default occurs in interval  $i$  is thus  $N \times LGD(i)$ . All these cash flows are assumed to be paid at the end of the interval, therefore discount factor can be considered to be  $e^{-\frac{i}{2}r_i}$ .

After assigning probabilities and cash flows to edges, in order to calculate the present value of the CDS contract it is sufficient to sum up the expected cash flows from all final edges. For example, the probability that the default will occur in the third interval is  $(1 - PD_1) \times (1 - PD_2) \times PD_3$  and the present value of cash flow paid and received by the buyer until termination of the contract (after the third interval) is  $N \times LGD_3 \times e^{-\frac{3}{2}r_3} - N \times \frac{s}{2}(1 \times e^{-\frac{1}{2}r_1} + 1 \times e^{-\frac{2}{2}r_2})$ . Expected value is then calculated by multiplying with the probability that

Figure 4.2: Binary Tree Mechanism of CDS contract



Source: Adopted and changed from O'Kane & Turnbull (2003) and Jarrow & Turnbull (1995).

default occurs in the third period. Under the no arbitrage condition, total expected value of the CDS contract should be equal to zero, so that the buyer pays on premiums as much as is expected to receive from the seller in case of default. In our example from Figure 4.2 this would imply that the following equation must hold

$$\begin{aligned}
0 = & \left( N \times LGD_1 \times e^{-\frac{1}{2}r_1} \right) PD_1 + \\
& + \left( N \times LGD_2 \times e^{-\frac{2}{2}r_2} - N \frac{s}{2} (e^{-\frac{1}{2}r_1}) \right) (1 - PD_1) PD_2 + \\
& + \left( N \times LGD_3 \times e^{-\frac{3}{2}r_3} - N \frac{s}{2} (e^{-\frac{1}{2}r_1} + e^{-\frac{2}{2}r_2}) \right) (1 - PD_1)(1 - PD_2) PD_3 + \\
& + \left( N \times LGD_4 \times e^{-\frac{4}{2}r_4} - N \frac{s}{2} (e^{-\frac{1}{2}r_1} + e^{-\frac{2}{2}r_2} + e^{-\frac{3}{2}r_3}) \right) \\
& \times (1 - PD_1)(1 - PD_2)(1 - PD_3) PD_4 + \\
& - \left( N \frac{s}{2} (e^{-\frac{1}{2}r_1} + e^{-\frac{2}{2}r_2} + e^{-\frac{3}{2}r_3} + e^{-\frac{4}{2}r_4}) \right) \\
& \times (1 - PD_1)(1 - PD_2)(1 - PD_3)(1 - PD_4)
\end{aligned} \tag{4.6}$$

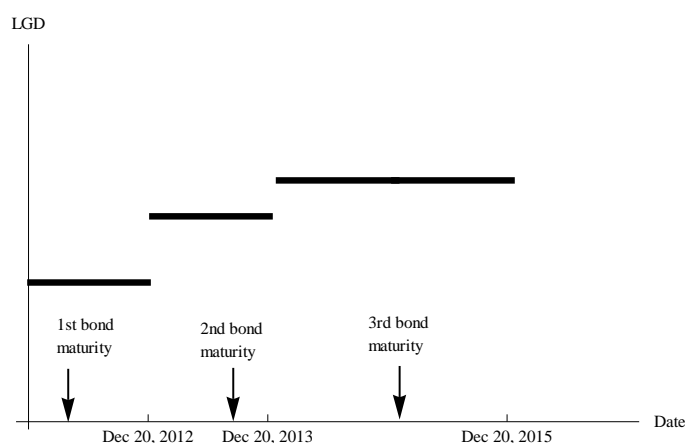
### 4.3 PD and LGD Simultaneous Estimation

In the previous chapter we explained the reduced-form model for pricing a risky bond. The same approach is used in this chapter to model credit default swaps. Both market prices of bond and CDS disclose information about credit risk expectations. Based on this, we will describe an analytical approach to extract the credit risk parameters  $PD$  and  $LGD$ .

From the purely mathematical point of view, if we assume that parameters  $PD_i$  and  $LGD_i$  are independent, the number of parameters that are included in the equations must be no more than the number of explanatory equations. If probability of default and loss given default are assumed to be constant, two explanatory equations are needed to calculate these two variables. Model for pricing the credit spread of bonds must be used together with the model for pricing credit default swaps, so that information about one bond and one CDS derivative is used.

More generally, based on the principle used in Hull & White (2000), parameters are assumed to be constant on some intervals of time. To calculate the probabilities of default and the loss given default of the reference firm, we suggest to use market prices of  $k$  CDS contracts, written on senior bonds of the reference entity with maturities  $t_1, t_2, \dots, t_k$ , together with market prices of  $k$  defaultable senior bonds issued by the firm, with maturity of the  $i^{th}$  bond in the interval  $(t_{i-1}, t_i)$ , assuming  $PD_i$  and  $LGD_i$  to be constant between CDS maturities. Example of possible outcome of the evolution of  $LGD$  is illustrated in Figure 4.3 for three bonds and credit default swaps that mature in approximately 1, 2 and 4 years.

Figure 4.3: Example of time-varying  $LGD$  evolution in time



Source: Author's construction

The assumptions in Equation 4.6 are too simplifying due to long intervals and protection payments are assumed to be paid only at the end of these intervals which implies imprecise discounting of  $N \times LGD_i$ . Furthermore, accrued premium payment since the last payment up to the time of the default that should be paid by the buyer at default was neglected. In order to incorporate the accrued premiums and enhance the discount factor of protection payment, we can design a binary tree with one day intervals, so that it specifies the day when the default occurs. Then the equation can be designed as follows

$$\begin{aligned}
0 = & \sum_{i=1}^{360 \times T} N \times LGD_i \times e^{-\frac{i}{360}r_i} \times PD_i \prod_{k=1}^{i-1} (1 - PD_k) \\
& - \left( \sum_{i=1}^{360 \times T} N \times \sum_{j=1}^{Div[i,90]} \frac{s}{4} e^{-\frac{j}{4}r_{90j}} + \frac{Mod[i,90]}{360} \times s \times e^{-\frac{i}{360}r_i} \right) \times PD_i \prod_{k=1}^{i-1} (1 - PD_k) \\
& - N \times \sum_{n=1}^{4 \times T} \frac{s}{4} e^{-\frac{n}{4}r_{90n}} \times \prod_{m=1}^{360 \times T} (1 - PD_m)
\end{aligned} \tag{4.7}$$

where  $T$  denotes time to maturity of credit default swap in years, first row calculates the protection payment received by the buyer if default occurs at the  $i^{th}$  day, second row calculates the sum of quarterly paid premium payments and the accrued premium payment since the last premium payment if default occurs at the  $i^{th}$  day<sup>6</sup>. This sum is multiplied by the probability that the default occurs at  $i^{th}$  day and not before. The last row calculates the present value of all premiums paid in case that no default occurs before the CDS contract matures.

In the previous chapter we presented the reduced-form model for the credit spread between prices of risk-free and risky bonds (see Equation 3.16 and Equation 3.17), which must be a bit modified so that it is compatible with Equation 4.7.  $LGD$  is assumed to be a time-varying variable, constant only during the specified intervals, and we follow face value convention for  $EAD$  estimation. Similarly to Equation 4.7, we try to model credit spreads as precisely as possible. Although probability of default is constant on the intervals between CDS maturities, we model that a default can occur on any day. For example, probability that the default occurs on the  $k^{th}$  day is  $PD_k$ , which is the same

<sup>6</sup> $Mod[x, y]$  stands for the modulo of  $x$  divided by  $y$  and  $Div[x, y]$  stands for the whole part of such division.

for any day in the interval. Note that the probability that a default occurs on the  $k^{\text{th}}$  day and not before is  $PD_k \times \prod_{i=1}^{k-1} (1 - PD_i)$ .

$$g^t - b^t = \sum_{i=1}^{360 \times t} N \times LGD_i \times e^{-\frac{i}{360} r_i} \times PD_i \prod_{k=1}^{i-1} (1 - PD_k) \quad (4.8)$$

Based on the idea of Hull & White (2000), credit risk parameters can be then iteratively calculated from the spread between defaultable and default-free bonds with maturity at  $t$  (Equation 4.8) and the CDS with maturity at  $T$  (Equation 4.7). This means that for the first bonds and first CDS we extract one day  $PD$  and  $LGD$  that are considered to be the same until  $T$ . These values are then substituted into the equations for second bonds and second CDS contract to extract credit parameters in the second interval, and so we continue until the last pair of equations.

It is important to bear in mind that this approach is based on the following simplifying assumptions:

- spread between prices of risky and risk-free bond is assumed to be incurred only by the credit risk, therefore differences in liquidity or market risks are assumed to be insignificant,
- market prices of risky and risk-free bonds must be bootstrapped and interpolated to achieve their comparability (the same coupon structure and same maturity) for calculation of the spread,
- German government bonds will be considered as reference risk-free bonds, although they are not completely risk-free,
- all senior bonds of one issuer are assumed to bear the same credit risk regardless the maturity and coupon structure,
- pricing model for credit default swaps neglects counterparties credit risks and liquidity risk of CDS contracts,
- probability of default and loss given default are constant on the intervals between CDS maturities.

We will apply this approach in Chapter 5 on the market data and calculate what credit risk parameters are expected by bonds and CDS investors. The following section contains the last theoretical part of this work - the adjusted relative spread based on credit default swaps.

## 4.4 Extracting LGD from CDS with Different Seniority

In this section we show that the concept of adjusted relative spread introduced by Unal *et al.* (2003) on bonds with different seniorities can be applied on market prices of credit default swaps written on the debt with different seniority. As far as we are informed, at the time of writing this work there was no theoretical or empirical study combining ARS and credit default swaps.

The approach is very similar to the already explained *ARS* method. The main advantage lies in the better empirical application. Note that to calculate *ARS* it is necessary to group junior, senior and risk-free bonds with the same maturity and zero coupons. However, in practice, this will not be possible for most of the bonds. In theory, after few adjustments to market data<sup>7</sup>, *ARS* can be calculated, but the precision is questionable. On the contrary, credit default swap market prices are free from any consideration of coupon payments and maturities are easily matched due to standardization of the CDS contracts by ISDA.

To overcome these shortcomings, we transform the model of adjusted relative spread based on bonds with different seniority into the model of relative spread based on credit default swaps with different seniority. Firstly, we define the relative spreads (for swap) variable (*RSS*) as

$$RSS = \frac{s_J^T - s_S^T}{s_J^T} \quad (4.9)$$

where  $s_J$  and  $s_S$  denote prices of junior and senior CDS in the form of their spreads that are quoted on the market. Based on the reduced-form model explained in this chapter, junior and senior CDS spread for the contract with maturity at  $T$  can be expressed as

$$s_i^T = \frac{\sum_{t=1}^T LGD_i \times \prod_{k=1}^{t-1} (1 - PD_k) PD_t e^{-tr_t} \times N}{\int_0^T PD(t) \times (u(t) + e(t)) dt + (1 - PD(T))u(T)} \quad (4.10)$$

for  $i = J$  or  $i = S$ , assuming different *LGD* and the same probability of default

<sup>7</sup>Adjustments to bond prices at this point consist of two steps. Both will be explained in more detail in the next, empirical, chapter. It is needed, firstly, to bootstrap the value of coupon bond and calculate the equivalent zero bond value. Secondly, to interpolate zero senior bonds and risk-free bonds to obtain theoretical value of senior and risk-free bond with same maturity as junior bond.

for senior and junior CDS. After substitution into Equation 4.9 we get pure recovery model that is not dependent on default probabilities or timing.

$$RSS = \frac{LGD_J - LGD_S}{LGD_J} = 1 - \frac{LGD_S}{LGD_J} = \frac{R_S - R_J}{1 - R_J} \quad (4.11)$$

$RSS$  is independent from any maturity or default time considerations. Therefore when calculating it from market spreads of credit default swaps with maturity  $T_1$ , then with maturity  $T_2$ , etc.,  $RSS$  should be the same, regardless the maturity of chosen CDS. However, this holds only under the assumption that the expectation about the loss given default is constant, so that expected  $LGD$  in one year is the same as expected  $LGD$  in 5 or 10 years.

Expected values of junior and senior recoveries can be calculated as it was shown in Equation 3.28, using density function of aggregate recovery from Equation 3.29 and payoff functions described in Table 3.1. We assume the aggregate recovery to be logit transformation of normally distributed variable  $x$  with mean  $\mu$  and standard deviation  $\sigma$ . Then, in order to calculate the standard deviation of  $x$  and the mean of  $x$  based on the volatility of  $RSS$  and expected value of  $RSS$ , it is necessary to estimate parameters  $\psi$ ,  $\theta$  and  $p_S$ .

In this chapter, the reduced-form model for pricing plain vanilla CDS was described, followed by the theoretical method to extract  $LGD$  and  $PD$  using information from CDS market prices and credit spread between risk-free and risky bonds. Then we introduced the method of  $RSS$  which can be used for numerical estimation of  $LGD$  for both junior and senior bonds. Next, we conclude this work with empirical application of the both explained methods.

# Chapter 5

## Extracting Loss Given Default from Market Data

This chapter deals with the extraction of risk parameters from market prices of selected risky instruments. At the beginning we briefly describe employed data and how it needs to be modified in order to be applicable for the models. Then we present results of the two described reduced-form approaches for *LGD* calculation. Firstly, we discuss the empirical implementation and the calculated results of the method for extracting loss given default and probability of default simultaneously from the market prices of defaultable bonds and the market spreads of credit default swaps. We analyze the sensitivity of these data on implied credit risk parameters and model shortcomings. We conclude this part with modification, which proves itself to enable better implementation of the method. Secondly, we apply the model of relative spread for credit default swaps written on bonds with different seniorities and discuss the results.

### 5.1 Data

Due to the fact that most empirical studies which estimate credit risk parameter(s) have been done for USA<sup>1</sup>, we decided to apply the model to European bonds and their CDS. However, one reason why the U.S. bonds have been more popular in empirical studies is their higher liquidity, hence they are believed to be priced more precisely (see e.g. Biais & Declerck (2006)). Moreover, for European companies it is still more common to increase capital by issuing new

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<sup>1</sup>Even though few theoretical models for LGD estimation have been developed in the last decade, empirical literature concerning loss given default estimation is still not very broad.



loan rather than by issuing bonds and junior or subordinated bonds are rarely used<sup>2</sup>. Also, the market with credit default swaps is more liquid for the U.S. reference entities. Therefore, we tried to choose firms whose bonds as well as credit default swaps are traded on the market. In order to overcome the problem with differences in average *LGD* for firms in different industries, we decided to choose all issuers from the same industry. Among firms that have issued at least one subordinated bond and have market data available for both senior and subordinated CDS were mostly financial institutions. Based on this, our bond and CDS data set contains nine European banks, each from a different country:

- Zurich Finance
- Swedbank AB
- Allied Irish Banks
- HSH Nordbank AG
- Bayerische Landesbank
- Banco Financiero y de Ahorros S.A
- Banco Popolare Societa Cooperativa
- Lloyds TSB Bank
- Royal Bank of Scotland

All data were downloaded from the Bloomberg database. The first method can be divided into two parts. In the first one, we use 3 bonds denominated in EUR<sup>3</sup> with time to maturities around 1, 2 and 4 years for Zurich Finance, Swedbank AB, Allied Irish Banks, HSH Nordbank AG and Bayerische Landesbank. For each bond, we use daily market closing prices from December 1, 2010 to May 9, 2011. See Table B.1 in Appendix B for summary about chosen bonds, their maturities and coupons, together with basic characteristics of the

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<sup>2</sup>James (2010) provides figures which support the fact that for most companies in Europe loans were considered as the only way of financing. For example, in 2006, the sum of issued corporate loans was €945.3 billion, while the sum of issued corporate bonds was €454.5 billion. However, James (2010) points out the increasing trend in issuing corporate bonds in 2009 and 2010.

<sup>3</sup>It is preferred to use only bonds denominated in the same currency, in order to eliminate the exchange rate risk from the model.

quoted market price (average, minimum, maximum and volatility) during the interval from December 1, 2010 to May 9, 2011. In the second part, we modify the methodology slightly and use weekly average market yields to maturity of senior bonds. This is applied to two bonds of Bayerische Landesbank, Lloyds TSB Bank and Royal Bank of Scotland with time to maturities around 2 and 4 years, again denominated in EUR. See Table B.2 in Appendix B for summary about these bonds' yields to maturity.

CDS data set contains quoted market spreads for derivatives with maturities on June 20 for years 2012, 2013, 2014, 2015, 2016, 2018 and 2021. Note that traded credit default swaps follow the standardized contractual terms. Necessary senior CDS data for the first model cover daily closing prices from December 1, 2010 to May 9, 2011, while the second model uses senior and subordinated weekly spreads from January 1, 2011 to December 31, 2011. These are then used in the *RSS* approach. See Table B.3 in Appendix B for basic characteristics (average, minimum and maximum) of market quoted credit default swap spreads for different maturities and chosen issuers. Based on the comparison of the average CDS spreads across issuers we can claim that the bonds of Allied Irish Banks are considered to be significantly the most risky ones. Investors evaluate credit default swaps spreads with reference entities Swedbank AB and Zurich Finance to be between 50 and 200 bps, which imply that they are exposed to a very small credit risk. Other banks are in the middle in terms of credit risk.

The relationship between average credit default swap spread and the maturity of the contract is increasing for all reference entities except the Allied Irish Banks. Moreover, CDS spreads in time follow similar evolution in time for different maturities. See Figure B.1 to Figure B.8 in Appendix B. A possible explanation based on the reduced-form approach is that issuers are considered to be of a very low risk and stable, thus expectations about credit risk parameters are the same for this year as for any other year in the future. CDS spreads are then increasing with maturity due to a lower probability that no default will occur until the end of the contract. On the contrary, for Allied Irish Banks the average senior CDS spread decreases with time to maturity and the average junior CDS spread does not follow any pattern. This is in line with empirical findings in the Trueck *et al.* (2004). They showed that there is ambiguous relationship between CDS spreads and maturity for speculative

grade debt, while for investment grade debt the relationship is positive<sup>4</sup>.

Furthemore, it is important to notice, that there are constant spreads as of May 2011 for Lloyds TSB Bank (Figure B.4), Roayal Bank of Scotland (??) and Banco Popolare (Figure B.7), which imply that credit default swaps of these banks were not traded since then. Another exception of positive relationship between maturities and CDS spreads can be seen in Figure B.2. This is however only due to significant rise of first and second-year CDS of Swedbank and no trading of the other CDS. In order to take into account that certain CDS were not traded, we will shorten the analyzed time accordingly.

Apart from defaultable bond prices and CDS spreads, the yields or prices of a default-free bonds are necessary for our calculations. Due to the fact that we deal with European bonds, we decided to consider the prices of German government zero coupon bonds denominated in EUR with different maturities as the benchmark for calculation of forward risk-free rates.

Before we implement the market data to the first model, few modifications are necessary to meet the model assumptions. First of all, bonds should have zero coupon, which is true for the risk-free German government bonds but not for the risky bonds issued by Zurich Finance, Swedbank AB, Allied Irish Banks, HSH Nordbank AG, nor Bayerische Landesbank. *Bootstrapping* method is performed to get equivalent zero coupon prices of these bonds. Bootstrapping is the analytical approach to calculate prices from zero coupon bonds based on the prices of coupon bearing bonds. Based on Dedek (2010), we use the bond stripping technique, so that coupons and the principal of a bond are considered as separate securities with maturity equal to its payment date. Moreover, it is necessary to adjust the technique to precise maturities and coupon payment dates.

Secondly, default-free and defaultable bonds must have the same maturity in order to have comparable prices or yields. Therefore, we need to calculate the price of the equivalent risk-free bond for each risky bond. To solve this problem, it is necessary to assume how prices of default-free bonds would evolve on a specific day in relation to the remaining time to maturity. It is usual to assume this relationship to be linear between two adjacent pairs of time to maturity and price which are known. This method is referred to as *linear interpolation*. The

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<sup>4</sup>The division between investment grade and speculative grade is usually done based on the credit ratings. Debt with the rating above BBB- by Standard & Poor's or Baa by Moody's is considered as investment grade, anything below as speculative grade. However, we can assume, based on the average CDS spreads, that Zurich Finance, Swedbank AB, Nordbank AG and Bayerische Landesbank debt is of investment grade.

calculation of the interpolated risk-free bond at time  $t$  consists of the following three steps:

1. Find one risk-free bond with the closest maturity before  $t$  and one after  $t$ . Denote these maturities as  $t_1$  and  $t_2$ .
2. Find prices of risk-free bonds with maturities  $t_1$  and  $t_2$  and denote them  $g_1$  and  $g_2$ .
3. Calculate the interpolated price of risk-free bond with maturity at time  $t$  as  $\frac{t_2-t}{t_2-t_1} \times (g_1 - g_2) + g_2$ .

See the example of calculated prices of risky and risk-free bonds after their bootstrapping and linear interpolation for all banks, as of May 6, 2011, in Table 5.1. The calculated figures satisfy our expectations that prices of bootstrapped defaultable zero bonds are decreasing with time to maturity. Furthermore, prices of defaultable zero bonds are always higher than prices of the equivalent default-free bonds, where this difference is the credit spread that we assume to be equal to the expected credit loss of a defaultable bond.

**Table 5.1:** Prices of Bootstrapped Defaultable Zero Bonds and Interpolated Default-free Bonds with the Same Maturities, May 6, 2011

Issuer	Maturity	Zero risky bond	Risk-free eq.
Zurich Finance	14-Apr-2012	97.740	99.046
	17-Sep-2014	87.122	92.965
	14-Oct-2015	82.770	89.738
Swedbank AB	4-Oct-2011	97.583	99.594
	4-Mar-2013	94.624	96.898
	19-Aug-2014	85.748	93.204
Irish Allied Banks	30-Sep-2011	93.978	99.613
	1-Oct-2012	97.791	81.650
	12-Nov-2014	64.510	92.503
HSH Nordbank AG	10-Oct-2011	97.071	99.565
	19-Oct-2012	94.309	97.692
	13-Feb-2015	83.232	91.743
Bayerische Landebank	11-Nov-2011	96.501	99.421
	11-Dec-2012	94.007	97.400
	12-Dec-2014	86.907	92.255

*Source:* Bloomberg, author's calculations.

## 5.2 Methodology and Results

This section is divided into two parts based on the two models explained in Chapter 4. Firstly, following the method of *PD* and *LGD* simultaneous estimation based on the credit default swaps and on the credit spreads between defaultable and default-free bonds, we find out that the model is not applicable on daily market prices of bonds. We provide a detailed discussion on the shortcomings of the model as well as possible shortcomings in the market data. This methodology is then improved with modified data sample as we use weekly average market yields to maturity and the results are discussed.

Secondly, following the method of relative spread for CDS, we calculate the implied market loss given default rates for senior and subordinated bonds issued by Zurich Finance, Swedbank AB, Allied Irish Banks, Bayerische Landesbank, Banco Financiero, Banco Popolare Societa Cooperativa, Lloyds TSB Bank and Royal Bank of Scotland. The software *Mathematica 8.0 for Students* was used for calculations in both models.

### 5.2.1 PD and LGD Simultaneous Estimation

We present the process of implementation of the suggested reduced-form method for extracting loss given default and probability of default from the data set described above. This is then followed by a sensitivity analysis of credit risk parameters and market data on bonds and derivatives. Most of the analysis is done for daily market data<sup>5</sup>.

Maturities of bonds and credit default swaps satisfy the relationship suggested in the previous chapter, namely that the maturity of the first bond is before June 20, 2012, of the second bond between June 20, 2012 and June 20, 2013<sup>6</sup> and of the last bond between June 20, 2013 and June 20, 2015. Credit risk parameters are assumed to be constant on the intervals  $(0, t_1)$ ,  $(t_1, t_1 + 360)$  and  $(t_1 + 360, t_1 + 720)$ , where  $t_1$  denotes the number of days from the reference date until June 20, 2012. We can think of these parameters as being the short-term, the medium-term and the long-term estimates. Therefore we denote them by  $LGD_S$ ,  $PD_S$ ,  $LGD_M$ ,  $PD_M$ ,  $LGD_L$  and  $PD_L$ .

The process of the calculation should be iterative. In the first step, we

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<sup>5</sup>We used the 30/360 convention for the discounting of future cash flows.

<sup>6</sup>There is no bond denominated in EUR issued by Zurich Finance with the maturity between June 20, 2012 and June 20, 2013, therefore choice of CDS derivatives was modified according to the available bonds.

substitute the first bonds spread and the first credit default swap spread into Equation 4.7 and Equation 4.8 to express the implied  $LGD_S$  and  $PD_S$  parameters. In the second step, we repeat the process for the following bond spread and CDS spread using the implied  $LGD_S$  and  $PD_S$  to calculate  $LGD_M$ ,  $PD_M$ . The same is done in the third step.

The main advantages of the model compared to other studies are:

- default can occur at any time, not only at maturities
- present value of premium payments is calculated according to real CDS contractual terms, so that it is paid quarterly on specified days and in case of default, accrued interest since the last premium is paid

However, despite the precise calculation, the implied market credit risk parameters were often negative or greater than 1. To explain such results we provide the analysis of Equation 4.7 and Equation 4.8 that can be divided into the following two steps:

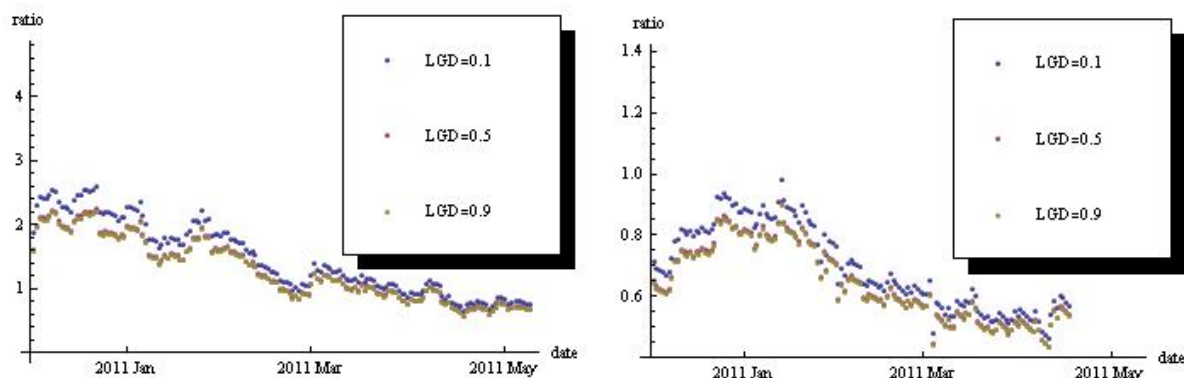
**1. Calculation of market implied CDS spreads based on bond spreads and assumption about LGD:** for the first bonds of each issuer, based on the Equation 4.8, we calculated the implied probability of default given different values of loss given default ( $LGD = 0.1, 0.2, \dots, 0.9$ ). Then we substituted these values into Equation 4.7 to express implied market CDS spreads. Firstly, we would like to point out that implied CDS spreads are very insensitive to change in  $LGD$  for all reference banks except for Allied Irish Banks. For illustration, we provide an example of the implied and the market CDS spreads as of May 6, 2011 in Table 5.2. There are only 5 bps differences in implied spreads in case of the lowest and the highest  $LGD$  assumption.

Table 5.2: Example of Implied Market CDS and Market CDS Spreads, 6 May 2011

	Implied	Market CDS		Market CDS
	$LGD = 0.1$	$LGD = 0.5$	$LGD = 0.9$	
Zurich Finance	115.274	110.347	109.834	64.017
Swedbank AB	166.618	162.072	161.587	57.228
Allied Irish Banks	1331.252	1225.970	1198.885	1662.497
Nordbank AG	167.530	162.753	162.245	113.030
Bayerisch Lndbk	146.157	141.780	141.315	89.428

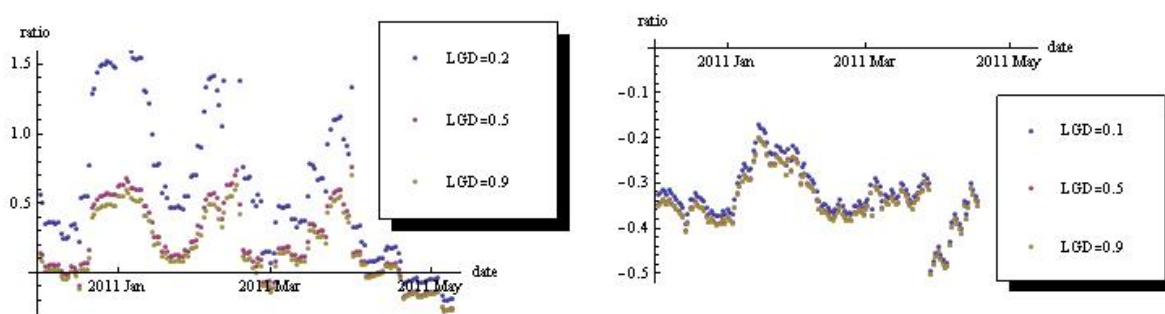
Source: Bloomberg, author's calculations.

Figure 5.1: Difference between Implied CDS Spread and Market CDS Spread relative to the Market CDS Spread for Zurich Finance (on the left) and Swedbank AB (on the right), December 1, 2010 - May 9, 2011



Source: Bloomberg, author's calculation based on Equation 4.7 and Equation 4.8

Figure 5.2: Difference between Implied CDS Spread and Market CDS Spread relative to the Market CDS Spread for Allied Irish Banks (on the left) and Nordbank AG (on the right), December 1, 2010 - May 9, 2011

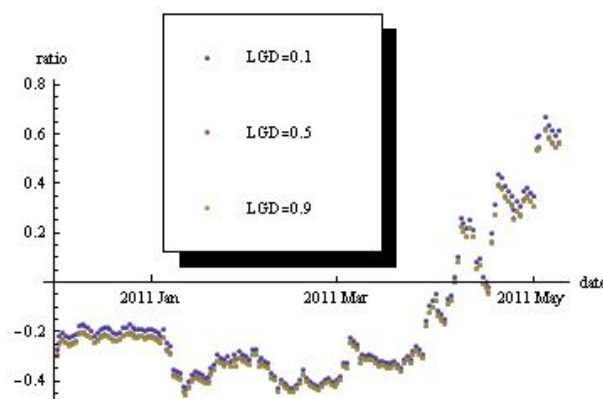


Source: Bloomberg, author's calculation based on Equation 4.7 and Equation 4.8

From another point of view, if we calculate the implied  $LGD$ , the estimates will be very sensitive to small differences in CDS spread. Thus only a short interval of CDS spreads would result in an implied  $LGD$  which is more than 0 and less than 1.

Secondly, it is important to notice that the market CDS spreads do not fit into the intervals that can be built based on the bounding points of  $LGD = 0.1$  and  $LGD = 0.9$ . To check if there is an exception on May 6, 2011, we compare the implied CDS spreads and quoted CDS spreads in time. As can be seen on Figure 5.1, the implied market CDS spreads of Zurich Finance and Swedbank AB are always higher than the quoted spread.

Figure 5.3: Difference between Implied CDS Spread and Market CDS Spread relative to the Market CDS Spread for Bayerische Landesbank, December 1, 2010 - May 9, 2011



Source: Bloomberg, author's calculation based on Equation 4.7 and Equation 4.8

However, the implied CDS spreads for Nordbank are significantly lower only until April 18, 2011, and then their relative difference jumps to positive values so high that it cannot be depicted on Figure 5.2. Implied CDS spreads for Bayerische Landesbank are below their market value until April 6, 2011 and then follow an increasing trend relatively to the market CDS (Figure 5.3). The behavior of the implied spreads for Allied Irish Banks seems to be very volatile and without any trend. As can be seen on Figure 5.2, there are few dates for which the implied spread is equal to the market spread. However, in some cases, more than 20% dispersion of  $LGD$  can satisfy that the difference is lower than 1%. Therefore, the estimated implied  $LGD$  based on the described reduced-form models will be very sensitive to the market data.

The differences between the implied market CDS spread and the quoted market CDS spread can be caused by:

- investors' wrong expectations about the credit risk, which in case of Zurich Finance and Swedbank AB would mean that investors on the CDS market constantly underestimate the credit risk,
- strict assumptions of reduced-form approach to credit default swaps pricing, especially the assumption that the counterparty credit risk can be neglected. As sellers of a CDS contract are often low risk financial institutions similar to e.g. Swedbank AB or Zurich Finance, the counterparty risk and reference risk is similar. However, we can neither calculate the counterparty credit risk based on the market available data, nor estimate



it precisely based on the evolution of differences between the implied CDS and the market CDS that were presented previously.

**2. Calculation of market implied defaultable bond prices based on CDS spreads and assumption about  $LGD$ :** for each reference entity and their first CDS, we calculated the implied probability of default based on Equation 4.7, given different values of loss given default ( $LGD = 0.1, 0.2, \dots, 0.9$ ). Then we substituted these values and prices of interpolated risk-free bonds into Equation 4.8 to express the implied market price of a risky bond. The process seems to be only the reversion of the previous step, but it is actually a completely different analysis. While in the previous step we assumed the market bond spread to be the base of the calculation and CDS spreads to be mispriced, here we reverse this assumption and extract the information about credit risk from CDS spreads and assume that these are priced correctly. This leads to different estimates of the implied market probability of default.

Table 5.3: Example of Implied Market Bond Prices and Quoted Market Bond Prices, 6 May 2011

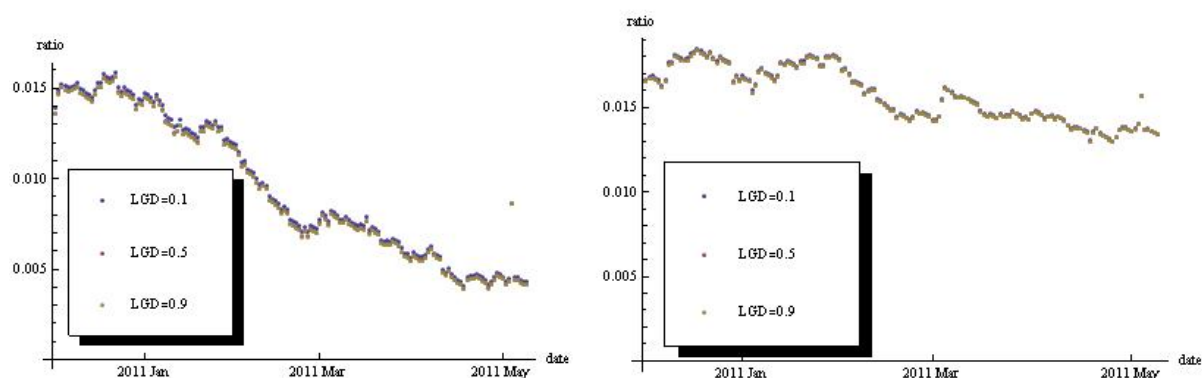
	Implied Price			Market Price
	$LGD = 0.1$	$LGD = 0.5$	$LGD = 0.9$	
Zurich Finance	98.459	98.445	98.443	98.014
Swedbank AB	99.358	99.356	99.356	98.923
Allied Irish Banks	94.558	93.156	92.968	94.733
Nordbank AG	99.087	99.078	99.077	98.865
Bayerisch Lndbk	98.966	98.957	98.956	98.688

Source: Bloomberg, author's calculations.

Similarly to the previous step, the sensitivity of implied bond prices to a change in  $LGD$  is very low. For example, on May 6, 2011, the differences between the implied prices under the assumption of  $LGD$  equal to 0.1 and 0.9 are 0.016, 0.002, 0.01 and 0.01 for Zurich Finance, Swedbank AB, Nordbank and Bayerische Landesbank (Table 5.3).

Next, we examine the evolution of differences between the implied market prices and market prices. Results, which are graphically depicted in Figure 5.4, Figure 5.5 and Figure 5.6, seem to be more consistent than those in the previous step of our analysis. For all low risk issuers, the implied bond prices are always higher regardless the assumed  $LGD$ . For Allied Irish Banks the opposite is true. The average relative difference which is calculated as  $E\left[\frac{b_i(\text{implied}) - b_i(\text{market})}{b_i(\text{market})}\right]$  is

Figure 5.4: Difference between Implied Price of Risky Bond and Market Price of Risky Bond relative to the Market Price for Zurich Finance (on the left) and Swedbank AB (on the right), December 1, 2010 - May 9, 2011



Source: Bloomberg, author's calculation based on Equation 4.7 and Equation 4.8

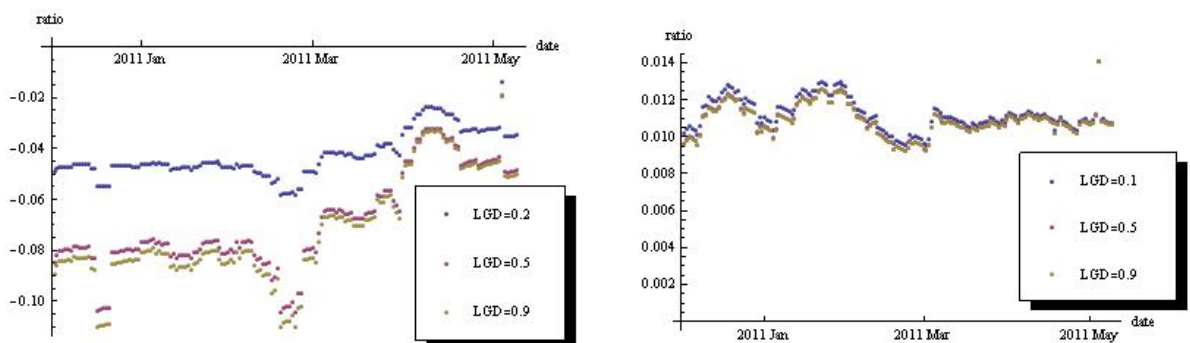
equal to 0.0095 for Zurich Finance, 0.0156 for Swedbank AB, 0.0108 for Nordbank AG and 0.0090 for Bayerische Landesbank.

These differences between the implied market prices and the quoted prices can be caused by:

- investors' wrong expectations about the credit risk, which in case of low-risk issuers would mean that investors on the bonds market overestimate the credit risk,
- wrong choice of the reference risk-free bonds, which is not a reasonable explanation in case of low-risk issuers, because price of a risk-free bond would need to be lower than price of then German government bonds to decrease the bond spread,
- strict assumptions of reduced-form approach to defaultable bonds pricing, especially the assumption that the bond spread can be explained only by the credit risk, but there are other risks, such as liquidity risk which might comprise significant part of the bond spread, especially for low-risk banks,
- market data transformation, during bootstrapping, interpolation or even rounding, as credit risk parameters are very sensitive to small differences of market prices.

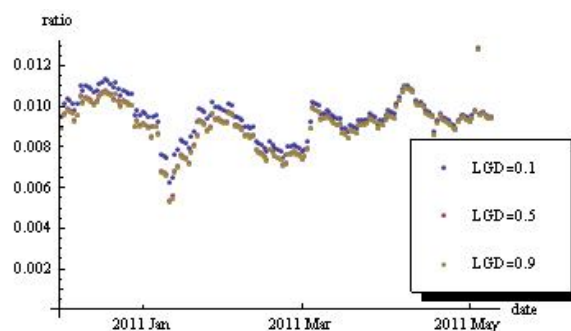
To sum up, the above described empirical implementation of the method of simultaneous  $PD$  and  $LGD$  estimation shows that loss given default rate

Figure 5.5: Difference between Implied Price of Risky Bond and Market Price of Risky Bond relative to the Market Price for Allied Irish Banks (on the left) and Nordbank AG (on the right), December 1, 2010 - May 9, 2011



Source: Bloomberg, author's calculation based on Equation 4.7 and Equation 4.8

Figure 5.6: Difference between Implied Price of Risky Bond and Market Price of Risky Bond relative to the Market Price for Bayerische Landesbank, December 1, 2010 - May 9, 2011



Source: Bloomberg, author's calculation based on Equation 4.7 and Equation 4.8

is a very sensitive to small differences in market CDS and bond prices. If we assume that investors on both markets have correct expectations about credit risk, it is necessary to modify the model.

- First possibility is to extend the model for the liquidity consideration in both instruments and for the counterparty credit risk in the CDS. However, this is not possible to estimate liquidity and counterparty risk from market available data and thus not feasible to include into provided model.
- Second possibility is to use quoted bonds yields to maturity, which are already free of coupon consideration, instead of market prices that need to be bootstrapped. This should increase the precision of data we use in the model.
- Third possibility is to slightly modify the assumption of the same credit risk expectation by investors on bond and CDS market. We may assume, that investors have access to the same information regarding the reference firm, however at these are not revealed in market data at the same day. Therefore, use of weekly average data instead of daily data would smooth these timing differences.

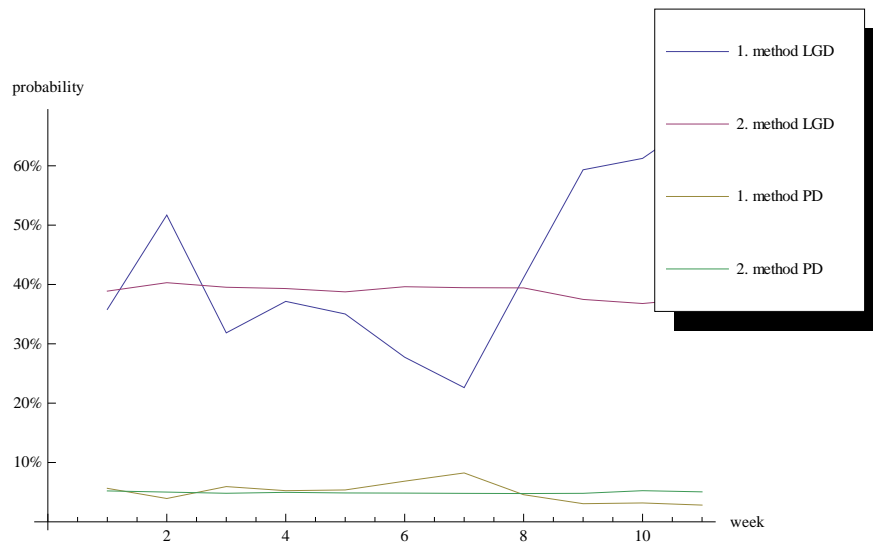
As the first improvement is not possible using available market data, we continue the analysis of the model with second and third modification. This is applied to market data of two bonds for Bayerische Landesbank, Lloyds TSB Banks and Royal Bank of Scotland. See the summary information about these bonds in Table B.2. Similarly, the corresponding interpolated risk-free yields must be calculated for these bonds. Again, we rather use quoted yield to maturity instead of quoted closing price of German government bonds. Thereafter, we follow the model of simultaneous  $PD$  and  $LGD$  calculation, using weekly average risky and risk-free yields and weekly averages of credit default swaps.

Although, as can be seen in Table 5.4, results for some weeks are apparently not corresponding to reality (e.g. first month for Royal Bank of Scotland), in general, the weekly smoothing and use of yields have improved the applicability of the model. Implied  $PD$  and  $LGD$  are compared to credit risk parameters calculated based on RSS method using weekly data in Figure 5.7, Figure 5.8 and Figure 5.9.<sup>7</sup> The time evolutions of loss given default parameter do not

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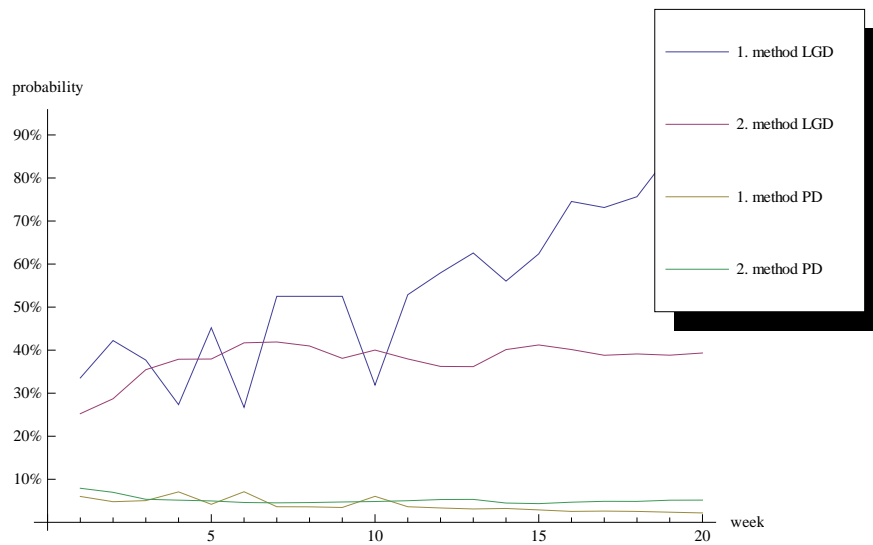
<sup>7</sup>In order to preserve visibility of the rest of the data, we do not include values that exceed 100%

Figure 5.7: Weekly *PD* and *LGD* Comparison across Two Models for Bayerische Landesbank



Source: Bloomberg, author's calculation

Figure 5.8: Weekly *PD* and *LGD* Comparison across Two Models for Lloyds TSB Bank



Source: Bloomberg, author's calculation

follow the same pattern. Furthermore, first method implies more volatile parameters, compared to almost stable results of second method, which could be explained by market inconsistencies. In order to generalize the results, we exclude the extraordinary high and low data and calculate overall average value. We conclude the average loss given defaults for Bayerische Landesbank, Lloyds TSB Bank and Royal Bank of Scotland are respectively 43.63\$, 46.27\$, 39.25\$ and one-year probabilities of default are 4.98%, 3.96% and 2.91%. In the following section, we illustrate further application of the second method and its results.

Table 5.4: Implied Market *LGD* at Weekly Basis from January 2011

	1	2	3	4	5	6	7
B.L.	35.80%	51.70%	31.85%	37.15%	35.03%	27.74%	22.62%
RBS	125.74%	169.62%	287.06%	136.35%	36.19%	32.70%	40.00%
Lloyds	33.55%	42.23%	37.70%	27.34%	45.20%	26.72%	52.51%
	8	9	10	11	12	13	14
B.L.	41.15%	59.33%	61.25%	68.14%			
RBS	40.00%	39.17%	34.17%	41.30%	41.33%	43.36%	53.44%
Lloyds	52.51%	52.51%	31.89%	52.88%	57.96%	62.58%	56.04%
	15	16	17	18	19	20	
RBS	35.20%	40.96%	34.15%	34.60%	40.64%	40.78%	
Llyods	62.38%	74.54%	73.14%	75.65%	85.61%	94.04%	

Source: Bloomberg, author's calculations.

## 5.2.2 Relative Spread for CDS

As discussed above, assumptions of the reduced-form models might be too strict for the method of simultaneous estimation of credit risk parameters based on the market data of defaultable bonds and the appropriate credit default swaps. Credit risk seems to be priced differently on these two markets. Therefore, it is necessary to decide what market data we take as the base for *PD* and *LGD* estimation. Doshi (2011) claims that credit default swap spreads better disclose expectations of investors about credit risk of the reference issuers. Moreover, the availability of market data on subordinated CDS compared to market data on subordinated bonds is another advantage<sup>8</sup>.

<sup>8</sup>Subordinated CDS with different maturities (1 year to 10 years) are traded for all chosen banks, except of HSH Nordbank AG. Their prices are directly comparable to senior CDS spreads with the same maturities. On the contrary, only very few subordinated bonds were issued by the chosen banks with maturities less than 10 years.

Figure 5.9: Weekly  $PD$  and  $LGD$  Comparison across Two Models for Royal Bank of Scotland



Source: Bloomberg, author's calculation

In order to follow the method of relative spread based on Equation 4.11 and Table 3.1, it is necessary to estimate parameter  $p_S$ , the amount of issued senior to all issued debt and parameters  $\psi$  and  $\theta$ , that describe at what level and to what extent is APR violated. To estimate  $p_S$ , it is sufficient to look up the data in the financial statements of a company. Based on annual reports for 2010 and half-year reports for 2011, where available, we calculated that  $p_S$  is equal to 70.5% for Zurich Finance, 96.2% for Swedbank AB, 78.3% for Allied Irish Banks, 91.2% for Bayerische Landesbank, 81.9% for Banco Financiero y de Ahorros S.A, 71% for Banco Popolare Societa Cooperativa, 88.7% for Lloyds TSB Bank and 83.5% for Royal Bank of Scotland.

Due to the lack of empirical evidence about APR violation in Europe (except for Germany), it is difficult to precisely estimate parameters  $\psi$  and  $\theta$ . In Germany, based on Brouwer (2006), we can estimate that the absolute priority rule is always strictly kept, thus for Bayerische Landesbank is  $\psi$  equal to 1 and  $\theta$  is not applicable. For other 7 banks that are based on the Swiss, Swedish, Irish, English, Scottish, Italianish and Spanish law, we provide a sensitivity analysis of  $LGD_J$  and  $LGD_S$  on parameters  $\psi$  and  $\theta$ . Based on the bounding condition about  $\theta$  defined in Equation 3.34, we created the following scenarios to investigate possible  $LGD_J$  and  $LGD_S$  estimates.

Table 5.5: Scenarios for  $LGD_J$  and  $LGD_S$  estimation in  $RSS$  model

<b>Zurich Fin.</b>	$p_S$	$\psi$	$\theta$		$p_S$	$\psi$	$\theta$
Scenario 1	0.705	0.10	0.70	Scenario 7	0.705	0.70	0.50
Scenario 2	0.705	0.10	0.90	Scenario 8	0.705	0.70	0.90
Scenario 3	0.705	0.30	0.70	Scenario 9	0.705	0.90	0.30
Scenario 4	0.705	0.30	0.90	Scenario 10	0.705	0.90	0.90
Scenario 5	0.705	0.50	0.60	Scenario 11	0.705	1.00	N/A
Scenario 6	0.705	0.50	0.90				
<b>Swedbank</b>	$p_S$	$\psi$	$\theta$		$p_S$	$\psi$	$\theta$
Scenario 1	0.962	0.10	0.98	Scenario 6	0.962	0.60	0.95
Scenario 2	0.962	0.20	0.98	Scenario 7	0.962	0.70	0.90
Scenario 3	0.962	0.30	0.98	Scenario 8	0.962	0.80	0.90
Scenario 4	0.962	0.40	0.98	Scenario 9	0.962	0.90	0.80
Scenario 5	0.962	0.50	0.95	Scenario 10	0.962	1.00	N/A
<b>Allied Irish B.</b>	$p_S$	$\psi$	$\theta$		$p_S$	$\psi$	$\theta$
Scenario 1	0.783	0.10	0.80	Scenario 7	0.783	0.70	0.60
Scenario 2	0.783	0.10	0.90	Scenario 8	0.783	0.70	0.90
Scenario 3	0.783	0.30	0.80	Scenario 9	0.783	0.90	0.30
Scenario 4	0.783	0.30	0.90	Scenario 10	0.783	0.90	0.90
Scenario 5	0.783	0.50	0.70	Scenario 11	0.783	1.00	N/A
Scenario 6	0.783	0.50	0.90				
<b>Bayerische Lndbk</b>	$p_S$	$\psi$	$\theta$				
Scenario 1	0.911	1.00	N/A				

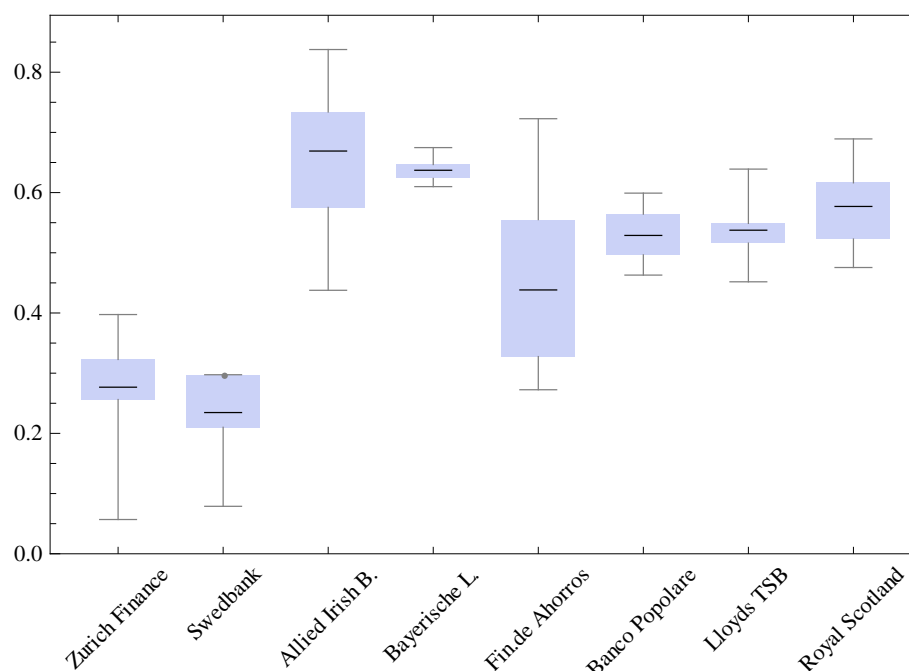


Table 5.6: Scenarios for  $LGD_J$  and  $LGD_S$  estimation in  $RSS$  model

<b>Fin. de Ahorros</b>	$p_S$	$\psi$	$\theta$		$p_S$	$\psi$	$\theta$
Scenario 1	0.819	0.10	0.90	Scenario 6	0.819	0.70	0.90
Scenario 2	0.819	0.30	0.90	Scenario 7	0.819	0.90	0.60
Scenario 3	0.819	0.50	0.70	Scenario 8	0.819	0.90	0.90
Scenario 4	0.819	0.50	0.90	Scenario 9	0.819	1.00	N/A
Scenario 5	0.819	0.70	0.60				
<b>B. Popolare</b>	$p_S$	$\psi$	$\theta$		$p_S$	$\psi$	$\theta$
Scenario 1	0.71	0.10	0.70	Scenario 7	0.221	0.70	0.50
Scenario 2	0.71	0.10	0.90	Scenario 8	0.221	0.70	0.90
Scenario 3	0.71	0.30	0.70	Scenario 9	0.221	0.90	0.20
Scenario 4	0.71	0.30	0.90	Scenario 10	0.221	0.90	0.90
Scenario 5	0.71	0.50	0.60	Scenario 11	0.221	1.00	N/A
Scenario 6	0.71	0.50	0.90				
<b>Lloyds TSB</b>	$p_S$	$\psi$	$\theta$		$p_S$	$\psi$	$\theta$
Scenario 1	0.887	0.10	0.90	Scenario 6	0.887	0.70	0.90
Scenario 2	0.887	0.30	0.90	Scenario 7	0.887	0.90	0.50
Scenario 3	0.887	0.50	0.80	Scenario 8	0.887	0.90	0.90
Scenario 4	0.887	0.50	0.90	Scenario 9	0.887	1	N/A
Scenario 5	0.887	0.70	0.75				
<b>Royal Scotland</b>	$p_S$	$\psi$	$\theta$		$p_S$	$\psi$	$\theta$
Scenario 1	0.835	0.10	0.90	Scenario 6	0.835	0.70	0.90
Scenario 2	0.835	0.30	0.90	Scenario 7	0.835	0.90	0.40
Scenario 3	0.835	0.50	0.80	Scenario 8	0.835	0.90	0.90
Scenario 4	0.835	0.50	0.90	Scenario 9	0.835	1.00	N/A
Scenario 5	0.835	0.70	0.65				

However, it is possible to set some estimates based on the discussion about the countries' law origins in Chapter 3. In Switzerland, Sweden, Spain and Italy, which can be included into common law countries, we would expect APR violation to be quite low. On the contrary, in Ireland, England and Scotland that can be included into civil law countries, we would expect the APR violation to occur more often. Occurrence of the APR violation in USA [Longhofer & Carlstrom (1995)] was empirically shown to be around 70%.

Next, we calculated the mean value, minimum and maximum value of  $RSS$  (Equation 4.9) for all maturities based on the daily market spreads of senior and subordinated CDS. The sample time was shorten for due no trading and thus constant CDS spreads for Swedbank, Allied Irish Banks, Lloyds TSB Bank, Rooyal Bank of Scotland and Banco Popolare. For illustration, see the distribution of one-year  $RSS$  for each bank in Figure 5.10. It represents lower

Figure 5.10: Date Distribution of 1-year  $RSS$ 

Source: Bloomberg, author's calculation based on Equation 4.9

and upper boundaries of calculated values, together with first and third quantile and the mean value.

The spread of daily  $RSS$  values is quite significant, especially for Zurich Finance, Allied Irish Banks and Banco Financiero y de Ahorros S.A, therefore using only average  $RSS$  will not provide us with robust information about expected credit risk. In order to avoid calculation of too many combinations of different scenarios, different dates, we use average  $RSS$  as a proxy to specify the best scenario. This is then used for further calculation. For Swedbank, Bayerische Landesbank, Banco Popolare Societa Cooperativa, Lloyds TSB Bank and Royal Bank of Scotland, we decided to use average  $RSS$ , as the first and third quantiles of  $RSS$  fall into relatively short intervals.

For all scenarios described in Table 5.5, given the average value of  $RSS$ , we numerically calculate the best  $\mu$  and  $\sigma$ , which define the distribution of the aggregate recovery rate  $R = 1 - LGD$ . Following the relationships explained in the previous chapters (Equation 3.28, Equation 3.31, Equation 3.29 and Table 3.1), we calculate the market implied values of  $E[R_S]$  and  $E[R_J]$ . Thereafter, according to the estimates of loss given default, we calculate the appropriate probability of default.

It is important to notice that, when calculating  $RSS$  for different maturities

we assumed that the loss given default is constant and 1-day probability of default is the same on any day until the maturity of CDS contract. Therefore, the calculated estimates e.g.  $PD_2$  and  $LGD_2$  from the  $RSS_2$  refer to the whole two-year interval. For the expected probability of default, which refers only to the second year (let's denote by  $PD_{12}$ ) the relationship in Equation 5.1 must hold, so that probability a default occurs until maturity of the second CDS ( $T_2$ ) is equal to sum of the probability that it occurs until maturity  $T_1$  and the probability that it occurs between  $T_1$  and  $T_2$ .

$$\sum_{i=1}^{T_2} (1 - PD_2)^{i-1} PD_2 = \sum_{i=1}^{T_1} (1 - PD_1)^{i-1} PD_1 + \sum_{i=1}^{T_2 - T_1} (1 - PD_{12})^{i-1} PD_{12} \quad (5.1)$$

Concerning the  $LGD$ , which refers only to the interval  $(T_1, T_2)$  (let's denote by  $LGD_{12}$ ), we assume that relationship in Equation 5.2 must hold, so that  $LGD_2$  is a weighted average of loss given default rates in intervals  $(0, T_1)$  and  $(T_1, T_2)$ . All other credit risk parameters regarding different maturities can be calculated accordingly.

$$LGD_2 = \frac{T_1}{T_2} LGD_1 + \frac{T_2 - T_1}{T_2} LGD_{12} \quad (5.2)$$

Below, we present and analyze results of the model. First of all, in Appendix B, we present results of different scenarios for parameters of APR violation based on the average  $RSS$  - after modification according to Equation 5.2. Figure B.9 up to Figure B.16 show that recovery rates remain the same since certain scenario. More specifically, if  $RSS$  is lower than  $\psi p_S$  then subordinated recovery  $R_J$  is implied to be zero (see Figure B.10, Figure B.12, Figure B.14 and Figure B.16) and senior recovery  $R_S$  is at the level of  $RRS$  regardless the  $\psi$  or  $\theta$ .

The estimated value of  $p_S$  is not changing over different scenarios, thus  $\psi$  seems to have the most significant impact on the estimates. As discussed above, we expected that for Swedbank and Banco Popolare the APR will be violated only to small extent, which imply that  $\psi$  should be high. According to the results in Figure B.9 and Figure B.11, implied recovery rate is the same for all scenarios since  $\psi = 0.4$  and  $\psi = 0.5$ , respectively, thus it is not necessary to make a decision about precise values of  $\psi$  or  $\theta$ . According to  $RSS$  method, the implied market recoveries of senior bonds are for different maturities between 23% and 35% for Swedbank AB and between 48% and 67% for Banco Popo-

lare. Based on this, the implied probabilities of default can be calculated from Equation 4.7 a modified according to Equation 5.1. It is irrelevant whether we calculate the probabilities of default from subordinated or senior CDS quotas.

The implied probabilities of default relevant in the intervals between CDS maturities are around 1% for Swedbank and 5% for Banco Popolare. See the complete results below in Table 5.7. For Swedbank, despite very low probability of default, the implied recoveries seem to be quite low for a low-risk bank. However, this would not change significantly if we use daily (or other frequency) CDS spreads instead of the average value. In comparison, implied senior recoveries of Banco Popolare are significantly higher, but also probability of default is much higher. We can conclude that investors consider Banco Popolare more risky, but if default occurs, more than 47% would be recovered.

Table 5.7: Implied  $R_S$ ,  $R_J$  and  $PD$  Relevant for the Interval between CDS Maturities, Estimated from  $RSS$  Model

Issuer	1Y	2Y	3Y	4Y	5Y	7Y	10Y
Swedbank	23.45%	26.76%	32.22%	34.53%	34.68%	34.84%	35.09%
	0.05%	0.02%	0.12%	0.14%	0.03%	0.02%	0.20%
	1.23%	1.45%	1.11%	1.19%	1.30%	1.38%	1.46%
Banco Pop.	66.88%	64.10%	56.45%	50.86%	49.97%	49.06%	47.90%
	4.59%	3.84%	1.87%	0.53%	0.87%	0.77%	0.28%
	5.02%	5.14%	4.62%	4.46%	4.67%	4.81%	4.91%

*Source:* Bloomberg, author's computation

On the contrary, we expected quite high APR violation for Lloyds and Royal Bank of Scotland. Therefore, we assume that the second scenario with  $\psi = 0.30$  and  $\theta = 0.90$  could correspond to reality. 70% frequency of APR violation with only very small extent of it is also in line with Longhofer & Carlstrom (1995). According to the second scenario, the implied market recoveries of senior bonds are between 51% and 64% for Lloyds and between 46% and 64% for Royal Bank of Scotland. Based on this, the implied probabilities of default, as well as, senior and subordinated recoveries are summarized in Table 5.8.

Table 5.8: Implied  $R_S$ ,  $R_J$  and  $PD$  Relevant for the Interval between CDS Maturities, Estimated from  $RSS$  Model

Issuer	1Y	2Y	3Y	4Y	5Y	7Y	10Y
Lloyds	63.93%	N/A	62.60%	N/A	57.62%	53.23%	51.14%
	29.60%	N/A	28.43%	N/A	24.09%	20.26%	18.43%
	3.02%	N/A	3.83%	N/A	4.36%	4.22%	4.29%
Scotland	64.01%	60.51%	55.73%	52.94%	50.32%	48.39%	46.87%
	19.12%	17.15%	14.47%	12.90%	11.43%	10.34%	9.49%
	2.49%	3.05%	3.29%	3.55%	3.75%	3.87%	3.99%

*Source:* Bloomberg, author's computation

For Bayerische Landesbank, we expected no APR violation to occur, therefore  $\psi = 1.00$ . See below in Table 5.9 complete results of implied recovery rates for senior and subordinated debt, together with implied probability of default. For Bayerische Landesbank, Lloyds and Royal Bank of Scotland, the expectations about overall credit risk are increasing throughout the year 2011, as the probability of default is increasing and implied recovery is decreasing.

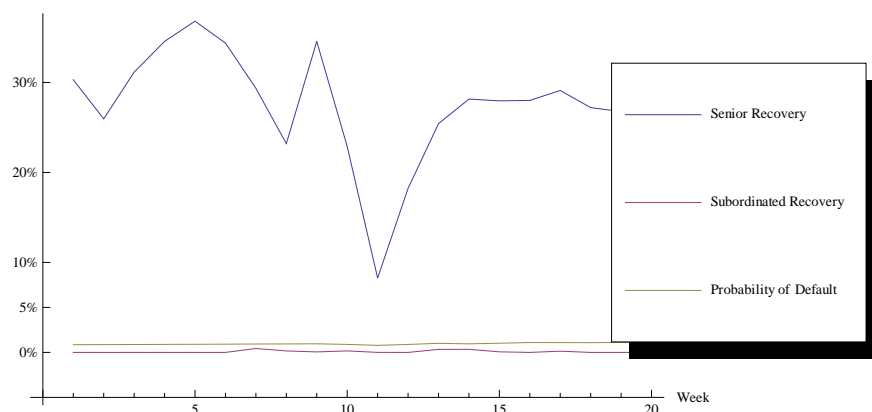
Table 5.9: Implied  $R_S$ ,  $R_J$  and  $PD$  Relevant for the Interval between CDS Maturities, Estimated from  $RSS$  Model

Issuer	1Y	2Y	3Y	4Y	5Y	7Y	10Y
Bayer. L.	63.73%	63.15%	61.93%	60.34%	59.23%	58.81%	58.31%
	0.00%	0.00%	0.19%	0.24%	0.05%	0.00%	0.02%
	3.96%	4.28%	4.51%	4.67%	4.83%	4.98%	5.13%

*Source:* Bloomberg, author's computation

We continue with the rest of the banks - Zurich Finance, Allied Irish Banks and Banco Financiero y de Ahorros S.A. We firstly follow the same process. In Appendix B, we present results of different scenarios for parameters of APR violation based on the average  $RSS$ . Similarly, Figure B.17, Figure B.19 and Figure B.21 show that recovery rates remain the same since certain scenario. Thanks to that, and the expectation of low APR violation for Zurich Finance and Banco Financiero, we can choose the proxy scenarios for further calculation. For Zurich Finance we use scenario 7 and for Banco Financiero we use scenario 4, both with APR parameters  $\psi = 0.50$  and  $\theta = 0.90$ . On the contrary, we expected quite high frequency of APR violation with low extent for Allied Irish Banks, thus we chosen the scenario 4 with  $\psi = 0.30$  and  $\theta = 0.90$ . As the implied recoveries are only slightly different from scenario 2 with  $\psi = 0.10$  and  $\theta = 0.90$ , we believe that this scenario is a good proxy for further calculation.

Figure 5.11: First-year Implied Recoveries and Probabilities of Default for Zurich Finance



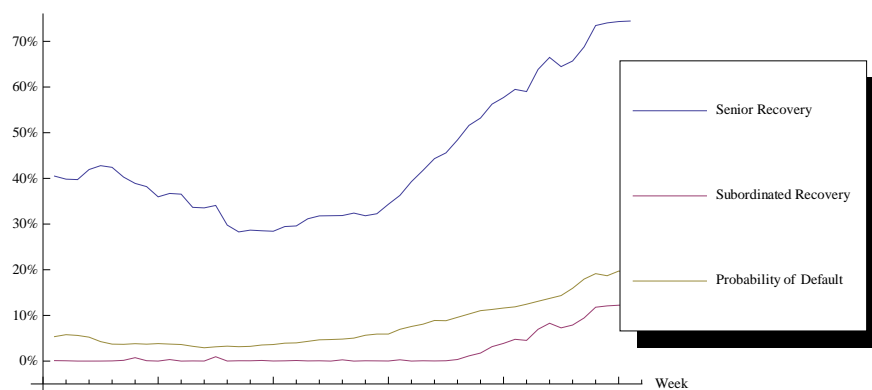
Source: Bloomberg, author's calculation based on weekly averages of *RSS*

Thereafter, we calculate implied recovery rates and probabilities of default for weekly averages of *RSS* for the chosen scenario. The whole analysis is done for 20, 24 and 51 weeks, respectively, for Zurich Finance, Allied Irish Banks and Banco Financiero. This limitation stands from availability of market data, as can be seen in Figure B.1 and Figure B.3. In Figure 5.11, Figure 5.12, Figure 5.13 see the weekly timeline of first year implied recoveries for senior and subordinated debt, as well as first year probabilities of default as they are expected by investors.

We can conclude that weekly results for Zurich Finance do not follow any trend or huge differences. The expected recoveries are quite low for the whole analyzed period and these are compensated with very low probability of default of less than 1%, which is similar to the case of Swedbank AB. On the contrary, expected credit risk parameters of Banco Financiero differ significantly throughout the year. For this Spanish bank, the probability of default has increased from 5.3% to 19.4%. However, also expected senior debt recoveries are increasing rapidly from the 21st week which is May 2011 (29.4%) until end of the year (74.4%).

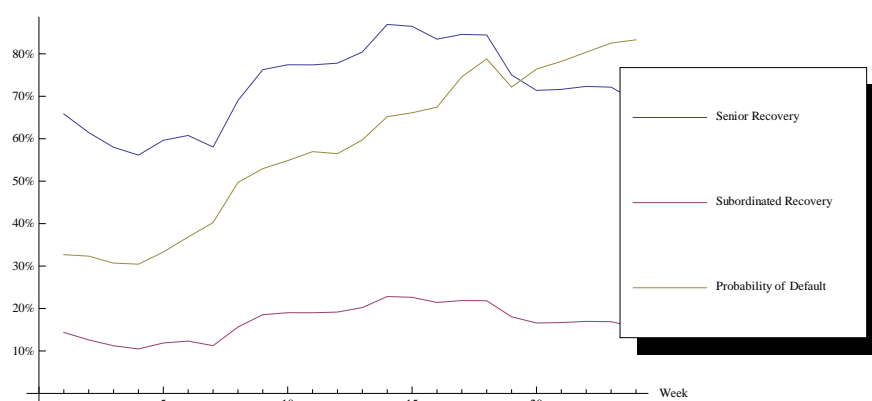
Lastly, according to Figure 5.13, credit risk of Allied Irish Banks was significantly increasing between week 15 and 24, as the recoveries have downgrade trend and expected probability of default rises to values of 80%. Results can be supported with the actual default in mid June 2011. Moreover, accord-

Figure 5.12: First-year Implied Recoveries and Probabilities of Default for Banco Financiero y de Ahorros S.A.



Source: Bloomberg, author's calculation based on weekly averages of  $RSS$

Figure 5.13: First-year Implied Recoveries and Probabilities of Default for Allied Irish Banks



Source: Bloomberg, author's calculation based on weekly averages of  $RSS$

ing to Creditex:<sup>9</sup> ”‘According to initial data submitted to ISDA (for more on the mechanics of CDS auctions read here) on behalf of buyers and sellers of CDS into the auction, the AIB senior bonds will see a final recovery value of about 71.375%, while the sub will barely recover 10%’”

In general, differences between implied market credit risk parameters and real market credit risk parameters can be possibly explained either by investors' mispricing the credit risk or by shortcomings of the model. For the following discussion it is important firstly to note that in the model  $RSS$  is an increasing function of junior CDS spread and decreasing function of senior CDS spread.

<sup>9</sup>Online article from 30th June 2011, <http://www.zerohedge.com/article/initial-results-allied-irish-cds-settlement-auction-senior-bonds-71375-subs-12>

Furthermore, if both spreads are increased by the same percentage then  $RSS$  decreases, while if both spreads are decreased by the same percentage then  $RSS$  increases. Secondly, note that implied recoveries have positive relationship to  $RSS$ .

Finally, we can divide results into three parts:

1. The low implied recoveries in case of Zurich Finance and Swedbank AB that are considered as low-risk can be caused by the following:
  - investors underestimate the risk of subordinated CDS of low-risk firms,
  - investors overestimate the risk of senior CDS of low-risk firms,
  - the model implies lower recovery rate, as liquidity risk and counterparties risk, that are neglected in  $RSS$  method, play a significant role when pricing CDS,
  - the expected recovery would be so low also in reality, however the default is highly improbable.
2. Credit risk of Allied Irish Banks as the reference entity is significantly higher than the counterparties credit risk, and liquidity risk is also probably relatively insignificant. Therefore, neglecting them should not impact the implied credit risk parameters. Such high recoveries and as well as probabilities of default are supported with the empirical findings.
3. In Table 5.9, Figure B.11, Figure B.13, Figure B.15 and Figure 5.12 we provide the results for Bayerische Landesbank, Banco Popolare, Lloyds TSB Bank, Royal Bank of Scotland and Banco Financiero. Implied first year recoveries around 64%, 70%, 50%, 63% and 30 up to 70%, respectively, seem to be reasonable for these firm. Compared to Zurich Finance and Swedbank AB, these banks are, based on the average CDS spreads, a bit more risky, which might explain why the neglected liquidity risk and counterparties risks do not have such a significant effect on the recovery rate underestimation.

To sum up, the results according to the  $RSS$  method can be seen in two ways. Firstly, investors actually expect very low probability of default for low-risk banks, but if a default occurs, the loss is expected to be high. On the contrary, they expect high probability of default for a high-risk bank, but the recovery in default is relatively high. Secondly, the results can be considered to



imprecisely disclose the expected risk by investors on the senior CDS market. For extremely low-risk banks the risk of senior bonds is overestimated, while for extremely high-risk banks it is underestimated.

# Chapter 6

## Conclusion

The main aim of this master thesis is to investigate theoretical concepts of the loss given default modeling (*LGD*). We focus mainly on the reduced-form approach which we employ for theoretical derivation and further for empirical estimation of implied market *LGD*. The reduced-form modeling is based on the assumption that market prices of defaultable financial instruments disclose the investors' expectations about credit risk parameters. On the contrary to structural models, timing of a default in reduced-form model is assumed to be unpredictable and it can occur at any time until maturity of risky instrument. The problem of the reduced-form approach is the separation of credit risk parameters – the probability of default and the loss given default – from the credit spread of risky instrument compared to its risk-free equivalent. In this work, we describe the reduced-form approach for pricing of two financial instruments: defaultable bonds and credit default swaps. These are then used to introduce the two following theoretical methods for the separation of the credit risk parameters:

1. Simultaneous *PD* and *LGD* estimation based on the market spread of the credit default swap and the bond spread between defaultable and default-free bonds: the model is based on the assumption that the market prices of credit default swaps and bonds spreads disclose the same investors' expectations about credit risk parameters. Equations for pricing the bonds with maturities in between the credit default swap maturities are then iteratively used to extract *PD* and *LGD* for each pair of CDS and bond spread.
2. Calculation of *LGD* referring to the senior and junior bonds based on the market CDS spreads with different seniorities: the main idea of the model

is the assumption that a default event is common for bonds with different seniorities, while only loss given default differs. Market implied senior and junior *LGD* can be then calculated based on the relative spread of junior and senior CDS spreads and on the assumption about the absolute priority rule violation (APR) which defines the relationship between *LGD* referring to a different seniority of the underlying bonds.

In the last chapter we provide an empirical application of the two described methods on the market data of nine European banks. Following the first method, we conclude that it is not possible to simultaneously extract *PD* and *LGD* for daily prices, however, smoothing of data to weekly average yields improves the applicability. This is thanks to two factors. Firstly, we believe that expectation about credit risk changes are not revealed on bond and CDS markets at the same time. Secondly, due to very high sensitivity of credit risk parameters to market data, we removed the step of bootstrapping the prices of risky coupon bonds and therefore increased the precision of data, which we use in the model. On weekly basis, we calculated that expected loss given default for Bayerische Landesbank, Lloyds TSB Bank and Royal Bank of Scotland is on average, respectively, 43.63%, 46.27% and 39.25%.

Following the second method, we calculated the time-varying expected senior and junior *LGD* for the assumed APR violation for eight banks. The results are much more appealing, which could be thanks to the use of data only from CDS market, not combining with bond market, as it was done previously. Regarding the low implied recoveries of 27.6% and 23.45% in case of Zurich Finance and Swedbank AB that are considered as extremely low-risk, we conclude that there are two probable explanations. Firstly, similarly as expected with previous model, it can be caused by neglected counterparty and liquidity risk when pricing CDS. Secondly, the expected recovery might be so low in reality, however the default with probability around 1% is highly improbable. For Bayerische Landesbank, Banco Popolare, Lloyds TSB Bank, Royal Bank of Scotland and Banco Financiero are the implied first year recoveries on average 63.73%, 66.88%, 63.93%, 64.01% and 44.19%. Another significant finding are the first year implied recoveries of Allied Irish Banks, which increased in May 2011 up to 71% for senior bonds and up 12% for subordinated bonds. This results are in line with actual default in June 2011.

Finally, we compare results from both of the introduced models. In the first model we calculated the expected loss given default based on the bonds

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with maturity up to 3 years and CDS with maturity on June 2014. These are on average 43.63%, 46.27% and 39.25%, respectively for Bayerische Landesbank, Lloyds TSB Bank and Royal Bank of Scotland. The average *LGD* of third-year CDS, that are calculated as the complement to recovery rates, are 39.07%, 38.40% and 45.27%. We conclude, that the results are very similar and both methods can serve as very good credit risk approximation, however the sensitivity of credit risk parameters to market data is too high to calculate the exact predictions.

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# Appendix A

## Definitions

### Homogenous Poisson process

Process  $T_i$  is a homogenous Poisson process with intensity  $\lambda$  if time intervals between subsequent arrivals are independent and exponentially distributed with parameter  $\lambda$ . Density and distribution function of the process  $T_i$  are following:

$$f(T, \lambda) = \lambda e^{-\lambda T}$$

$$F(T) = P[t \leq T] = 1 - e^{-\lambda T}$$

(See Hsu (1997).)

### Discrete time homogenous Poisson process

$N_t$  is discrete time homogenous Poisson process with intensity  $\lambda$  if increments  $N_t - N_s$  are independent and have Poisson distribution with parameter  $\lambda(t - s)$ . Probability distribution of process  $N_t$  is following:

$$P(N_t - N_s = k) = \frac{1}{k!} \lambda^k (t - s)^k \exp^{-\lambda(t-s)}$$

(See Hsu (1997).)

**Martingale** In reduced-form models, processes are specified under the martingale measure  $Q$ . Martingale is a stochastic process where the conditional expected value of next observation, conditional on all the past observations, is equal to the last observation. (See Hurt *et al.* (2003).)

### Stochastic Modeling with Time-varying Intensity $\lambda$

Deterministic intensity function  $\lambda(t)$  can be any discrete or continuous function known for  $t \geq 0$ . E.g. it is possible to use piece by piece constant intensities  $\lambda_1, \lambda_2, \dots$  changing throughout intervals. This can be limitly generalized into continuous intensity function. Then  $(N_t)$  is homogenous Poisson process where

increments  $N_t - N_s$  are independent and have Poisson distribution.

$$P[N_t - N_s = k] = \frac{1}{k!} \left( \int_s^t \lambda(u) du \right)^k e^{-\int_s^t \lambda(u) du}$$

Probability of survival until time  $T$  is calculated as follows:

$$P[\tau > T] = P[N_T = 0] = e^{-\int_0^T \lambda(u) du}$$

Valuation of bonds would be calculated similarly as with constant intensity. (See Giesecke (2004) and references therein.)

### Cox process

Cox process was used in reduced-form credit risk model by Jarrow & Turnbull (1995) to generate default timing. It is also called doubly stochastic Poisson process, because of two stages of uncertainty: default intensity process  $\lambda$  and Poisson arrival of default process conditional on  $\lambda$ . More mathematically said, intensity parameter ( $\lambda_t$ ) follows some stochastic process (e.g. mean-reverting process with jumps, CIR process, affine processes, see Duffie & Singleton (2003) for detailed descriptions) and  $N$  is conditionally on realization of  $\lambda$  an inhomogenous Poisson process with time-varying intensity  $\lambda$  :

$$P[\tau \leq T | (\lambda_t)_{0 \leq t \leq T}] = 1 - P[N_T = 0 | (\lambda_t)_{0 \leq t \leq T}] = 1 - e^{-\int_0^T \lambda(u) du}$$

Due to law of iterated expectations, unconditional probability of default can then be calculated as expected value of conditional probability.

$$P[\tau \leq T] = 1 - E[e^{-\int_0^T \lambda(u) du}]$$

(See Hsu (1997) and references therein.)

### Gauss-Wiener process

Standard Gauss-Wiener stochastic process is the continuous random variable  $W_t$  on  $t \in (0, T)$  for which following is true:

- (i)  $W_0 = 0$
- (ii) for any  $0 \leq s \leq t \leq T$

$$W_t - W_s \sim \sqrt{t - s} N(0, 1)$$

where  $N(0, 1)$  denotes normal distribution with zero mean and unit variance

(iii) for any  $0 \leq s \leq t \leq u \leq v \leq T$   $W_t - W_s$  and  $W_v - W_u$  are independent

(See Hurt *et al.* (2003).)

### Itô's lemma

Itô's lemma is used to find the differential of a function of a particular stochastic process. Let's assume  $X_t$  follows Itô's process (diffusion type stochastic process with standard Gauss-Wiener process) in the form:

$$dX_t = \mu_t dt + \sigma_t dW_t$$

where  $\mu_t$  is the drift term and  $\sigma_t$  is the volatility function and  $W_t$  is Gauss-Wiener process. Then the differential of process  $f(t, X_t)$  is following:

$$df(t, X_t) = \left( \mu_t \frac{\partial f}{\partial X_t} + \frac{\sigma_t^2}{2} \frac{\partial^2 f}{\partial X_t^2} + \frac{\partial f}{\partial t} \right) dt + \sigma_t \frac{\partial f}{\partial X_t} dW_t$$

(See Hurt *et al.* (2003).)

# Appendix B

## Empirical Data

Table B.1: Summary of Bonds Data, 1 December 2010 - 9 May 2011

Issuer	Zurich Finance		
<b>Maturity</b>	14 Apr 2012	17 Sep 2014	14 Oct 2015
<b>ISIN code</b>	XS0423888824	XS0201168894	XS0423888667
<b>Annual Coupon</b>	4.875	4.500	6.500
<b>Average Price</b>	103.065	103.957	111.993
<b>Minimum Price</b>	102.416	102.965	110.823
<b>Maximum Price</b>	103.826	105.988	114.393
<b>Volatility</b>	0.146	0.640	0.838
Issuer	Swedbank AB		
<b>Maturity</b>	04 Oct 2011	04 Mar 2013	19 Aug 2014
<b>ISIN code</b>	XS0455512888	XS0491438429	XS0445995896
<b>Annual Coupon</b>	2.340	3.125	5.000
<b>Average Price</b>	100.246	100.744	105.195
<b>Minimum Price</b>	99.816	100.121	103.505
<b>Maximum Price</b>	100.538	101.730	107.249
<b>Volatility</b>	0.036	0.184	0.912
Issuer	Allied Irish Bank		
<b>Maturity</b>	30 Sep 2011	01 Oct 2012	12 Nov 2014
<b>ISIN code</b>	XS0264548917	XS0455308923	XS0465876349
<b>Annual Coupon</b>	1.486	4.500	5.625
<b>Average Price</b>	89.750	82.911	75.987
<b>Minimum Price</b>	82.563	70.343	68.000
<b>Maximum Price</b>	95.375	90.707	83.900
<b>Volatility</b>	12.113	23.119	24.084

<b>Issuer</b>	<b>HSH Nordbank AG</b>		
<b>Maturity</b>	10 Oct 2011	19 Oct 2012	13 Feb 2015
<b>ISIN code</b>	DE000HSH29N9	DE000HSH3B28	DE000HSH2BK5
<b>Annual Coupon</b>	3.250	3.250	3.250
<b>Average Price</b>	100.931	101.639	100.222
<b>Minimum Price</b>	100.196	100.347	97.581
<b>Maximum Price</b>	101.447	102.777	103.282
<b>Volatility</b>	0.123	0.429	2.045

<b>Issuer</b>	<b>Bayerische Lndbk</b>		
<b>Maturity</b>	11 Nov 2011	11 Dec 2012	12 Dec 2014
<b>ISIN code</b>	DE000BLB5NA3	DE000BLB5M99	DE000BLB5HW9
<b>Annual Coupon</b>	4.750	4.750	4.750
<b>Average Price</b>	102.088	104.201	105.591
<b>Minimum Price</b>	101.068	102.831	103.685
<b>Maximum Price</b>	102.930	105.591	107.967
<b>Volatility</b>	0.279	0.571	1.220

*Source:* Bloomberg

Table B.2: Summary of Bonds Yields, 1 January 2011 - 31 December 2011

<b>Issuer</b>	Bayerische Landesbank	
<b>Maturity</b>	11 Dec 2012	12 Dec 2014
<b>ISIN code</b>	DE000BLB5M99	DE000BLB5HW9
<b>Average Yield</b>	2.396	3.209
<b>Minimum Yield</b>	1.909	2.802
<b>Maximum Yield</b>	2.864	3.560
<b>Volatility</b>	0.207	0.179

<b>Issuer</b>	Lloyds TSB Banks	
<b>Maturity</b>	20 Apr 2013	18 Jan 2015
<b>ISIN code</b>	XS0579627984	XS0550541691
<b>Average Yield</b>	2.908	4.612
<b>Minimum Yield</b>	2.244	3.869
<b>Maximum Yield</b>	4.082	5.815
<b>Volatility</b>	0.535	0.406

<b>Issuer</b>	Royal Bank of Scotland	
<b>Maturity</b>	15 May 2013	15 Jul 2015
<b>ISIN code</b>	XS0363669408	XS0526338693
<b>Average Yield</b>	4.184	4.925
<b>Minimum Yield</b>	3.234	4.186
<b>Maximum Yield</b>	6.098	6.634
<b>Volatility</b>	0.799	0.589

Table B.3: Summary of CDS Spread Data (bps), 1 January, 2011 - 31 December, 2011

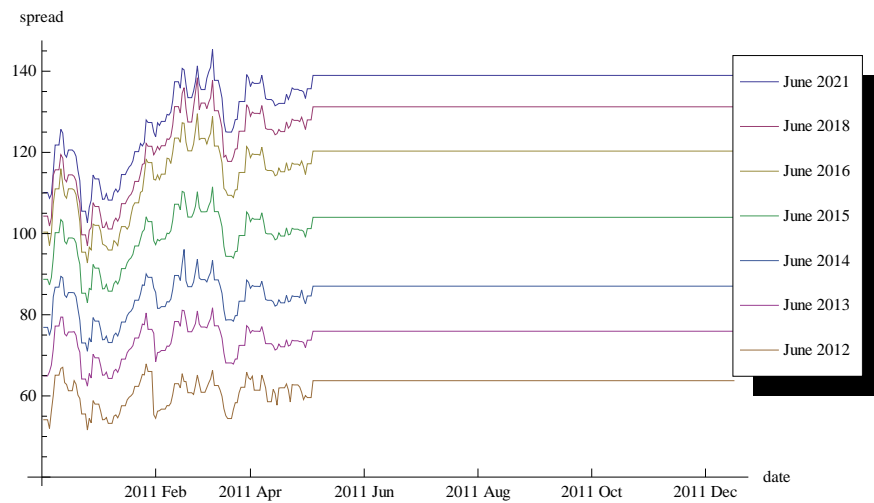
Maturity	Average	Senior		Average	Junior	
		Min	Max		Min	Max
<b>Zurich Fin.</b>						
20 June 2012	62.333	51.596	67.898	86.835	64.457	97.830
20 June 2013	74.722	62.405	81.711	109.560	109.560	109.560
20 June 2014	85.600	71.005	96.095	126.071	98.927	139.214
20 June 2015	101.814	82.914	111.508	151.298	151.298	151.298
20 June 2016	117.252	92.750	129.535	168.530	130.825	192.533
20 June 2018	126.749	96.999	138.382	178.546	155.753	196.755
20 June 2021	134.018	102.679	145.383	181.807	138.298	203.900
<b>Swedbank</b>						
20 June 2012	95.104	54.520	188.213	77.600	77.600	77.600
20 June 2013	107.057	60.250	197.199	92.561	92.561	92.561
20 June 2014	75.935	60.476	82.475	106.995	106.995	106.995
20 June 2015	78.225	69.326	87.283	119.980	119.980	119.980
20 June 2016	85.586	76.633	94.983	131.285	131.285	131.285
20 June 2018	89.020	80.205	97.778	137.216	137.216	137.216
20 June 2021	92.328	83.526	100.638	141.802	141.802	141.802
<b>Allied Irish B.</b>						
20 June 2012	1726.243	954.566	4334.058	5500.795	3032.469	7707.754
20 June 2013	1443.007	885.548	3000.289	4377.156	3010.669	6331.243
20 June 2014	1312.903	862.001	1635.491	4291.554	3118.777	5497.896
20 June 2015	1222.365	868.230	2209.385	4558.232	3347.279	5768.916
20 June 2016	1195.673	838.923	1560.871	4479.430	3542.605	5007.604
20 June 2018	1127.987	794.506	1910.709	4132.383	3241.435	4941.143
20 June 2021	1076.902	737.773	1403.633	4108.365	3047.725	4746.405
<b>Lloyds TSB Bank</b>						
20 June 2012	110.972	77.326	154.613	218.303	167.742	400.691
20 June 2014	146.416	120.186	194.198	276.360	226.210	452.036
20 June 2016	189.227	162.697	219.079	326.499	271.296	482.945
20 June 2018	198.480	171.020	231.105	337.031	278.224	485.990
20 June 2021	207.157	176.600	228.341	342.516	285.211	481.347

Maturity	Average	Senior		Average	Junior	
		Min	Max		Min	Max
<b>Bayerische L.</b>						
20 June 2012	146.985	75.877	274.280	330.080	321.367	444.385
20 June 2013	161.297	90.548	287.670	360.595	352.541	474.110
20 June 2014	176.149	105.094	302.760	387.119	379.680	499.630
20 June 2015	190.060	120.140	313.193	412.900	405.429	526.347
20 June 2016	202.180	132.695	322.635	446.266	427.917	548.161
20 June 2018	206.995	137.186	326.911	449.398	431.152	549.680
20 June 2021	212.718	141.933	333.759	450.400	435.147	551.195
<b>Royal B. Scotland</b>						
20 June 2012	91.077	59.978	171.386	205.107	160.013	444.836
20 June 2013	122.647	94.833	182.774	242.778	204.089	420.216
20 June 2014	148.528	121.180	203.993	282.709	243.000	475.125
20 June 2015	170.359	145.345	212.552	309.841	276.255	467.908
20 June 2016	190.170	169.833	236.680	335.440	302.053	499.592
20 June 2018	200.281	177.623	240.758	345.556	311.248	507.811
20 June 2021	209.265	187.552	257.356	354.864	322.424	511.745
<b>Banco Popolare</b>						
20 June 2012	170.856	114.874	287.342	530.634	286.541	661.469
20 June 2013	189.640	133.538	303.188	481.617	298.815	656.187
20 June 2014	206.211	150.473	315.473	423.311	327.091	678.928
20 June 2015	224.383	168.549	329.910	448.337	353.865	699.970
20 June 2016	239.647	187.978	344.368	470.158	386.847	701.595
20 June 2018	247.385	191.478	344.253	478.062	385.450	721.821
20 June 2021	253.895	197.825	343.654	482.707	399.631	705.725
<b>Banco Financiero</b>						
20 June 2012	231.354	155.170	362.303	441.396	237.227	738.728
20 June 2013	253.723	178.331	397.509	481.396	271.483	793.442
20 June 2014	268.572	196.210	419.134	522.123	306.425	845.124
20 June 2015	277.330	197.512	430.172	540.176	330.364	856.213
20 June 2016	285.812	196.921	440.717	553.194	348.807	862.622
20 June 2018	288.055	199.577	435.075	552.036	348.426	853.711
20 June 2021	291.221	203.464	431.627	551.484	349.074	845.316

*Source:* Bloomberg

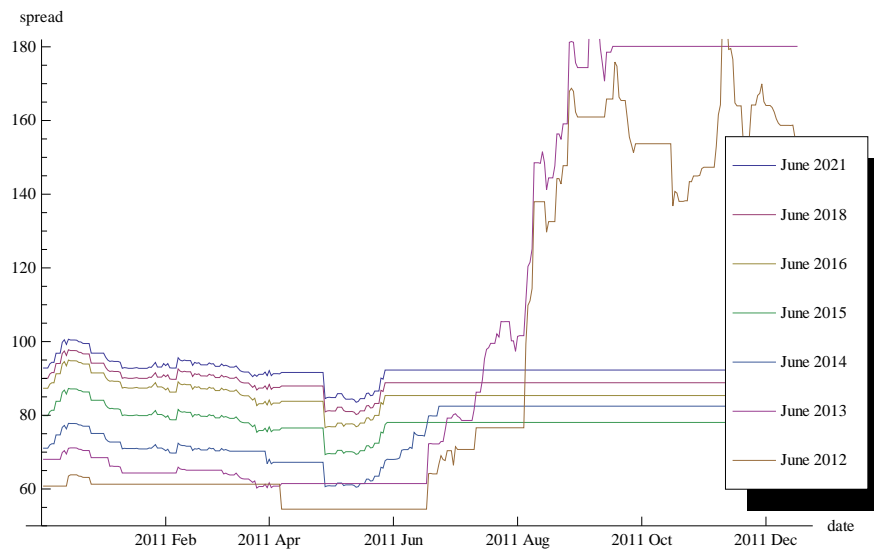


Figure B.1: Zurich Finance - Senior CDS spread for Different Maturities, Jan 1, 2011 - Dec 31, 2011



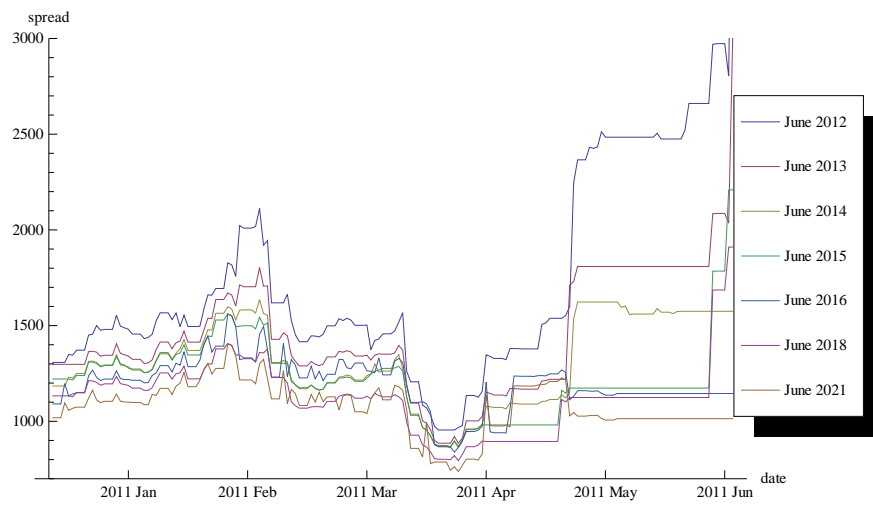
Source: Bloomberg

Figure B.2: Swedbank AB - Senior CDS spread for Different Maturities, Jan 1, 2011 - Dec 31, 2011



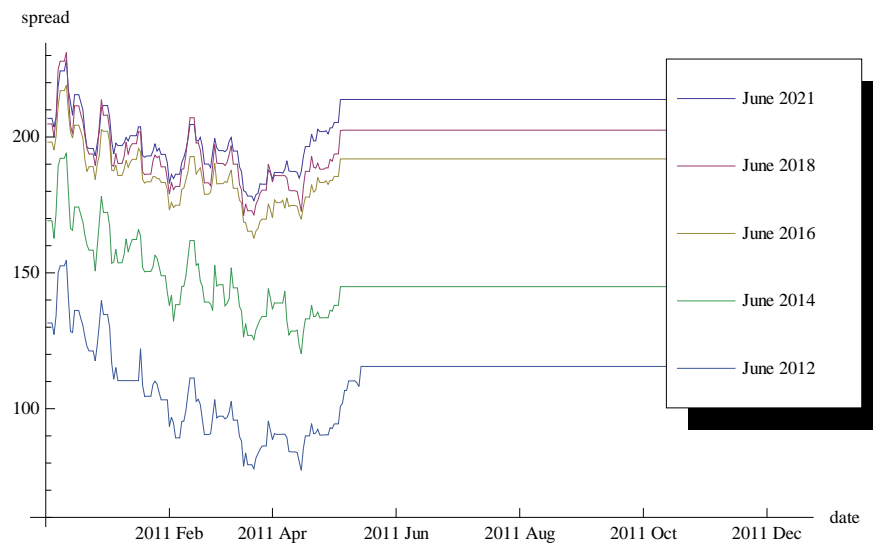
Source: Bloomberg

Figure B.3: Allied Irish Banks - Senior CDS spread for Different Maturities, Jan 1, 2011 - Dec 31, 2011



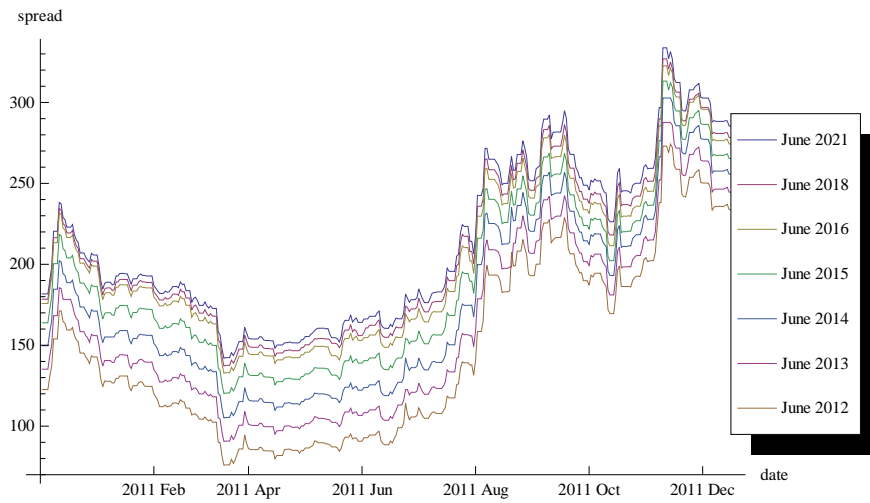
Source: Bloomberg

Figure B.4: Lloyds TSB Bank - Senior CDS spread for Different Maturities, Jan 1, 2011 - Dec 31, 2011



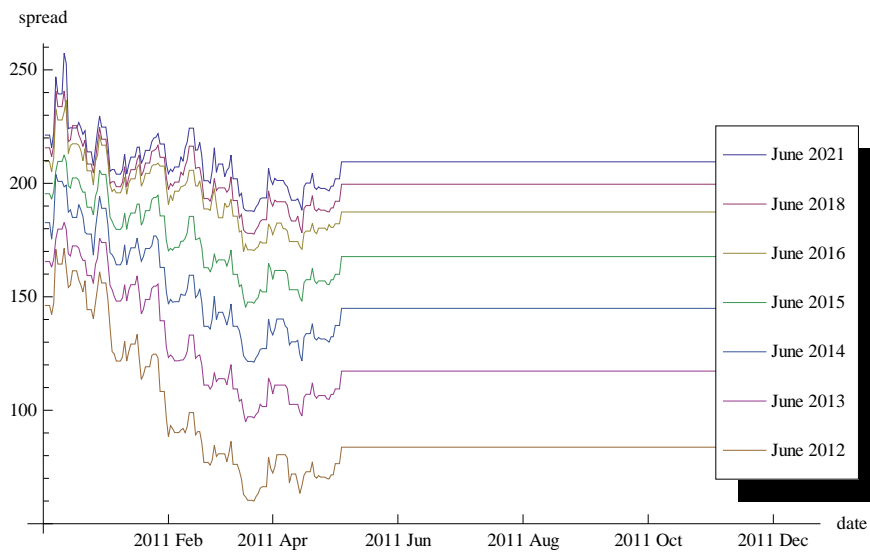
Source: Bloomberg

Figure B.5: Bayerische Landesbank - Senior CDS spread for Different Maturities, Jan 1, 2011 - Dec 31, 2011



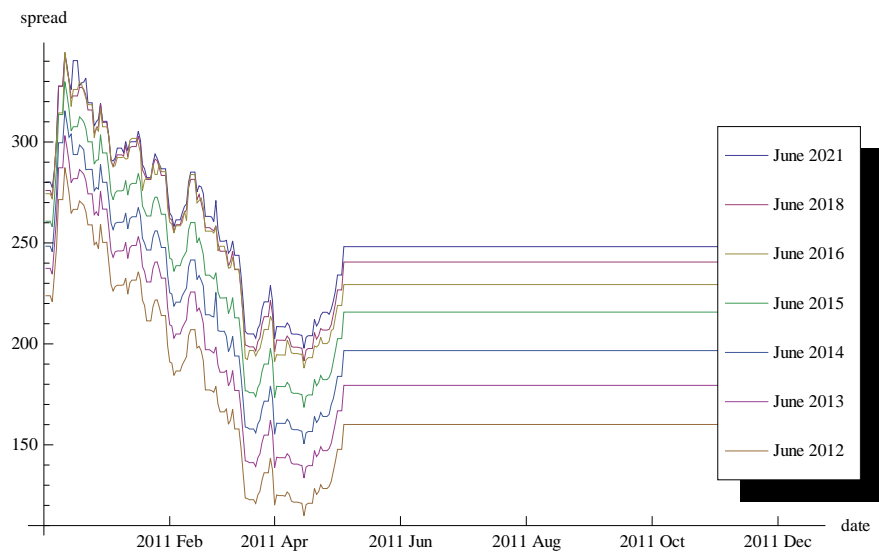
Source: Bloomberg

Figure B.6: Royal Bank of Scotland - Senior CDS spread for Different Maturities, Jan 1, 2011 - Dec 31, 2011



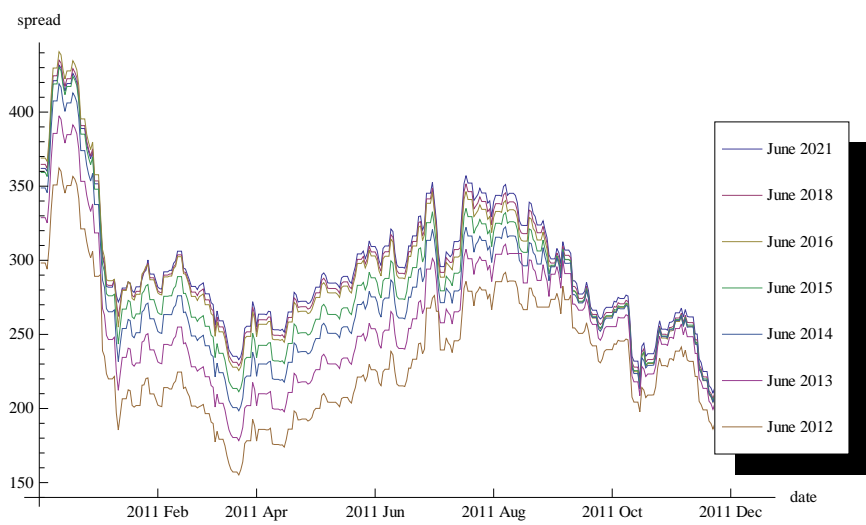
Source: Bloomberg

Figure B.7: Banco Popolare Societa Cooperativa - Senior CDS spread for Different Maturities, Jan 1, 2011 - Dec 31, 2011



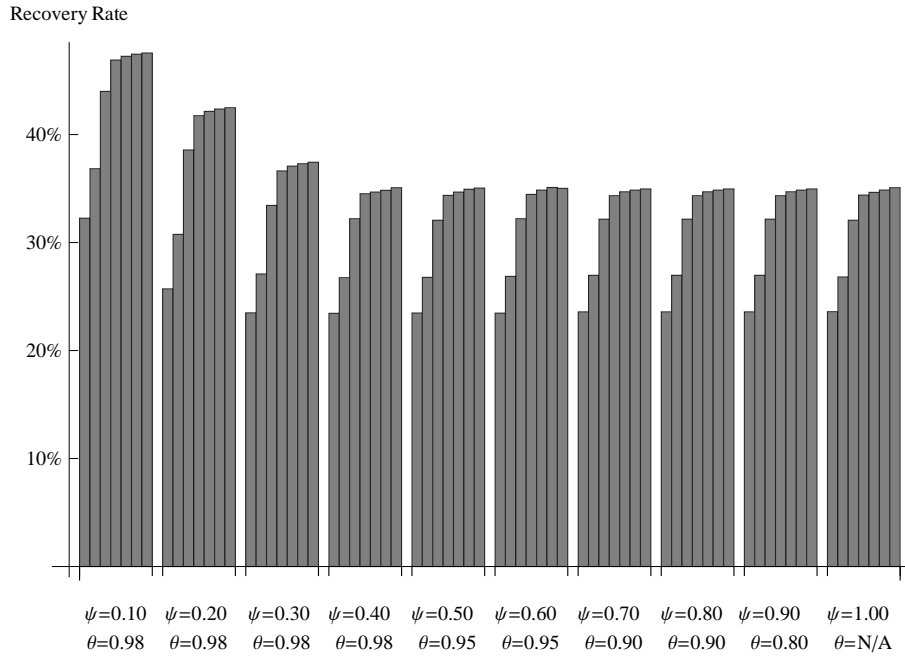
Source: Bloomberg

Figure B.8: Banco Financiero y de Ahorros S.A - Senior CDS spread for Different Maturities, Jan 1, 2011 - Dec 31, 2011



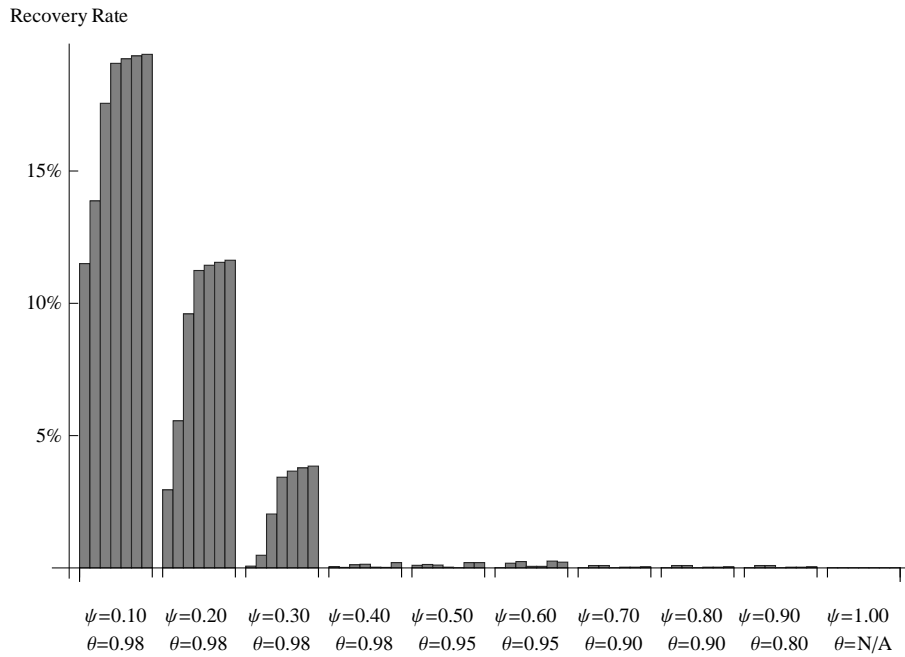
Source: Bloomberg

Figure B.9: Swedbank AB - Implied Market Recovery Rates For Senior Bonds Relevant at Intervals between CDS Maturities



Source: Bloomberg, author's computation

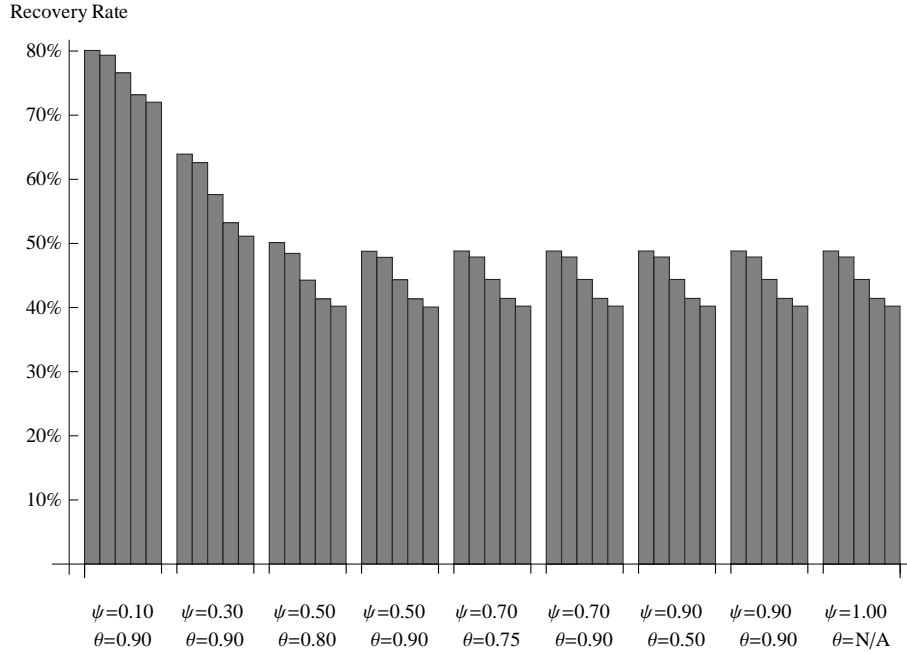
Figure B.10: Swedbank AB - Implied Market Recovery Rates For Subordinated Bonds Relevant at Intervals between CDS Maturities



Source: Bloomberg, author's computation

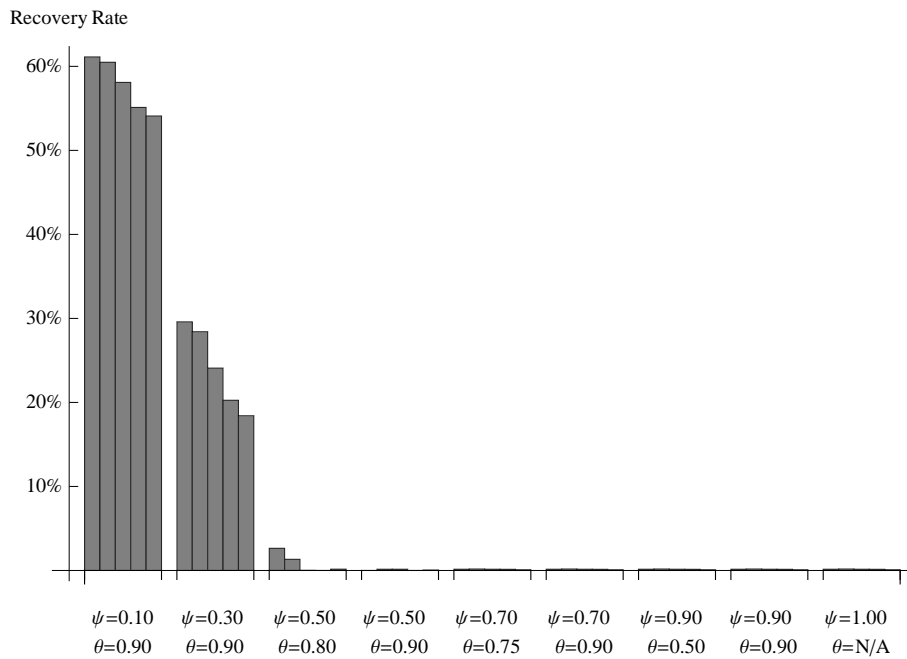


Figure B.13: Lloyds TSB Bank - Implied Market Recovery Rates For Senior Bonds Relevant at Intervals between CDS Maturities



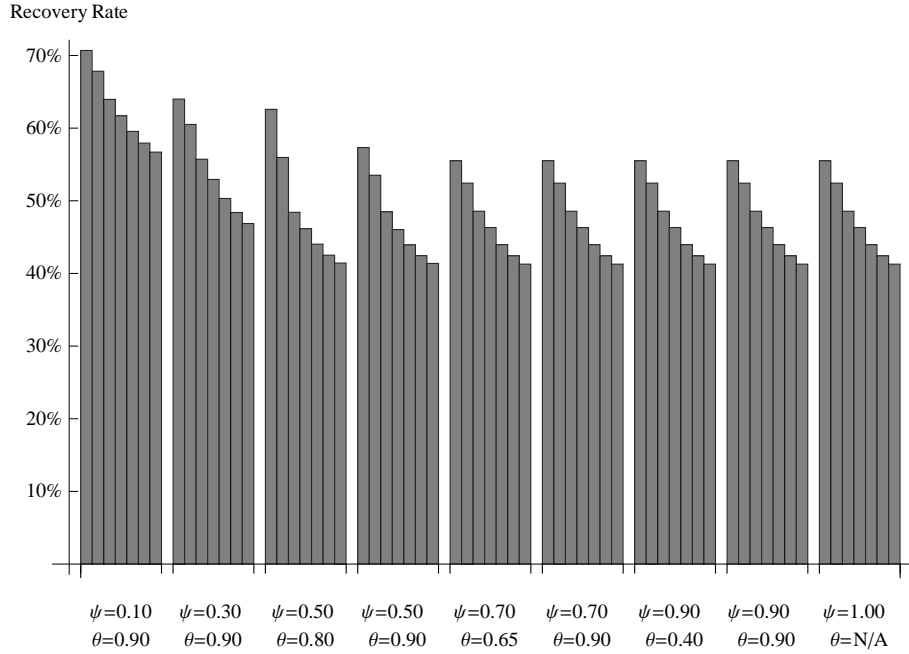
Source: Bloomberg, author's computation

Figure B.14: Lloyds TSB Bank - Implied Market Recovery Rates For Subordinated Bonds Relevant at Intervals between CDS Maturities



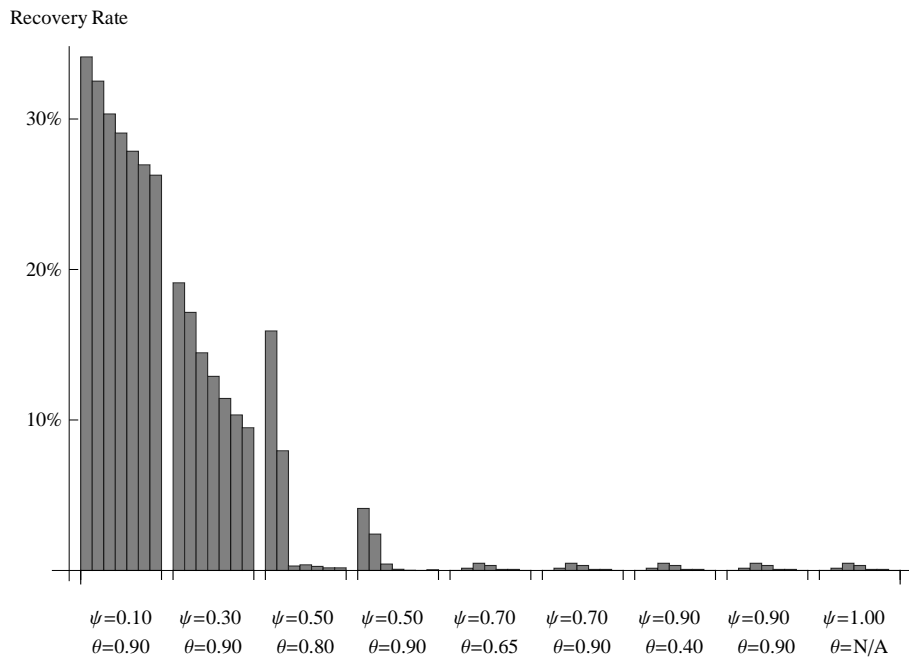
Source: Bloomberg, author's computation

Figure B.15: Royal Bank of Scotland - Implied Market Recovery Rates For Senior Bonds Relevant at Intervals between CDS Maturities



Source: Bloomberg, author's computation

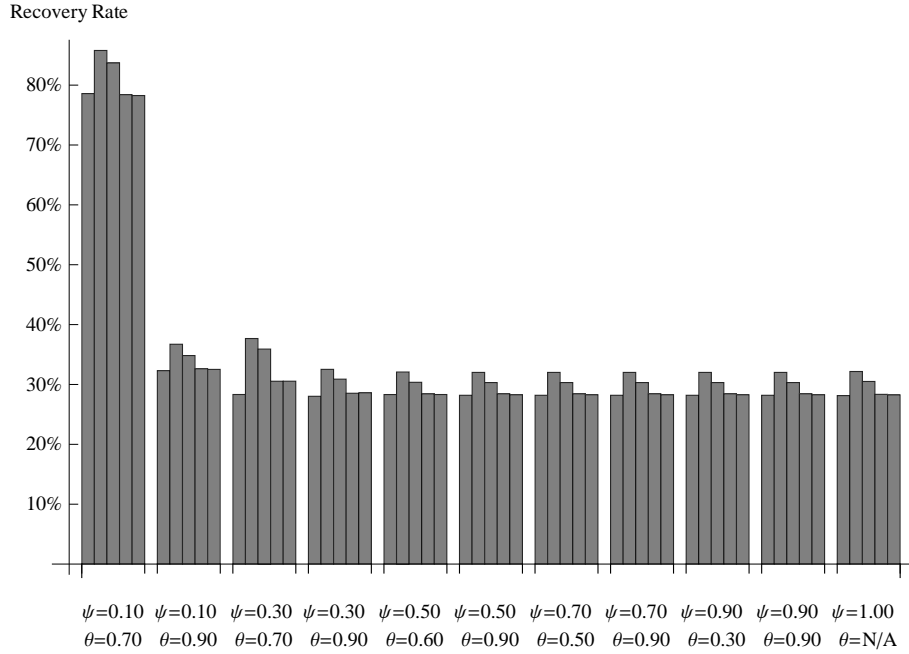
Figure B.16: Royal Bank of Scotland - Implied Market Recovery Rates For Subordinated Bonds Relevant at Intervals between CDS Maturities



Source: Bloomberg, author's computation

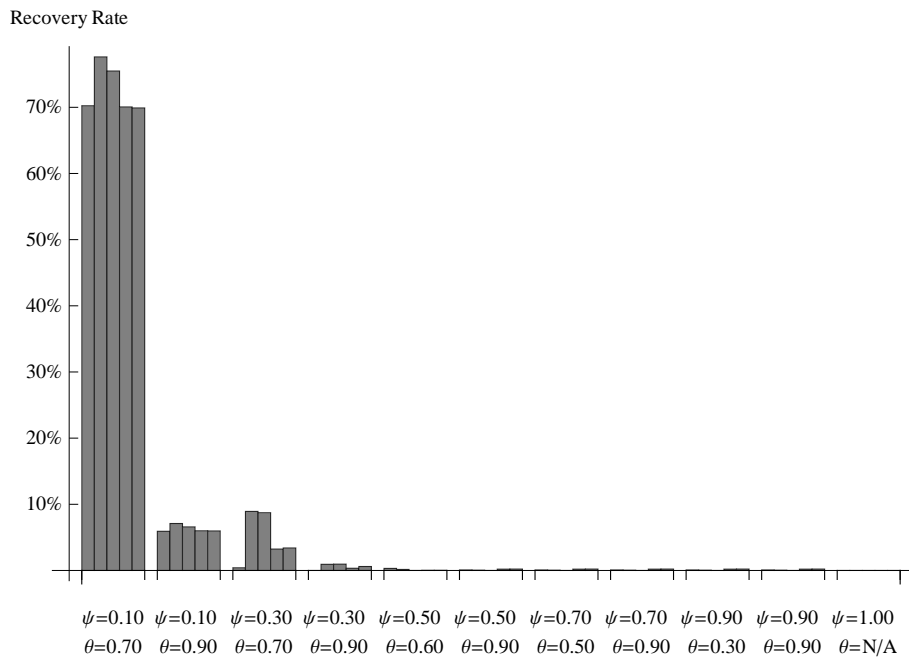


Figure B.17: Zurich Finance - Implied Market Recovery Rates For Senior Bonds Relevant at Intervals between CDS Maturities



Source: Bloomberg, author's computation

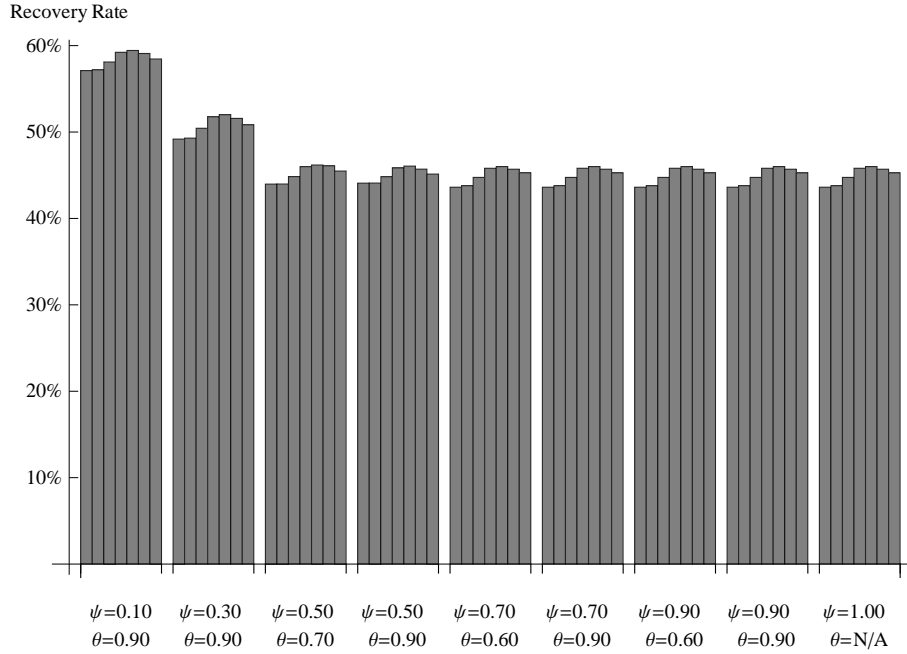
Figure B.18: Zurich Finance - Implied Market Recovery Rates For Subordinated Bonds Relevant at Intervals between CDS Maturities



Source: Bloomberg, author's computation

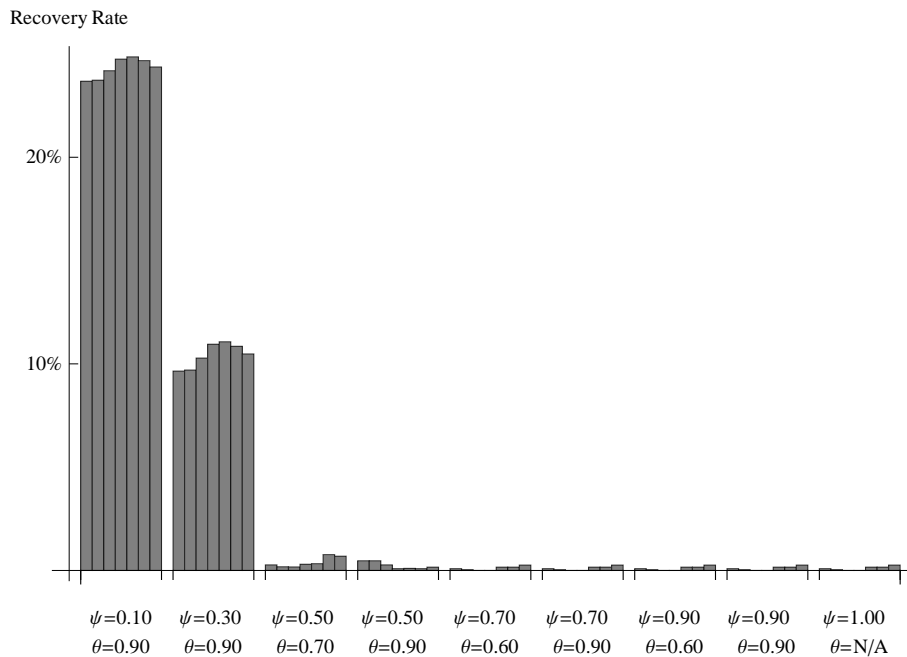


Figure B.21: Banco Financiero y de Ahorros S.A - Implied Market Recovery Rates For Senior Bonds Relevant at Intervals between CDS Maturities



Source: Bloomberg, author's computation

Figure B.22: Banco Financiero y de Ahorros S.A - Implied Market Recovery Rates For Subordinated Bonds Relevant at Intervals between CDS Maturities



Source: Bloomberg, author's computation

# Rigorous Thesis Proposal

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<b>Author</b>	Mgr. Ivana Hlavatá
<b>Supervisor</b>	PhDr. Jakub Seidler
<b>Proposed topic</b>	Reduced-form Approach to LGD Modeling

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**Topic characteristics** The author composed Master Thesis on the topic Reduced-form Approach to LGD Modeling, which was defended on 13.9.2011. In the Rigorous Thesis she will incorporate the remarks of supervisor and opponent.

The thesis deals with the advanced methods for estimating credit risk parameters from market prices: probability of default (PD) and loss given default (LGD). Precise evaluation of these parameters is important not only for banks to calculate their regulatory capital but also for investors to price risky bonds and credit derivatives. It provides forward looking reduced-form analytical method for calculation of PD and LGD of corporate defaultable bonds based on their quoted market prices, prices of equivalent risk-free bonds and quoted credit default swap spreads of the issuer of these bonds. The methodology is reversed to most of the studies on credit risk modeling, as aim is not to price instruments based on estimated credit risk parameters, but to calculate these parameters based on the available market prices. Furthermore, compared to other studies, the LGD parameter is assumed to be endogenous and in the thesis is provided the method for its simultaneous calculation with the probability of default. Finally, we use developed methods to estimate implied PD and LGD for five European banks, assuming that the risk is priced correctly by other investors and the markets are efficient.

**Aim of the Thesis** The aim of the rigorous thesis is to extend the analysis performed in the master thesis by enlarging the sample of bonds. Furthermore, we will incorporate liquidity risk and counterparty default risk into our model of pricing credit default swaps. This would lead to generalization of the original

methodology. We believe that this introduction of less restrictive assumptions will enable us to achieve more precise results of credit risk parameters.

**Methodology** In the thesis we will use market data of corporate and government bonds from various countries together with credit default swaps spreads in 2011.

## Outline

1. Introduction
2. Loss Given Default in Credit Risk
  - i. Credit Risk Management
  - ii. *LGD* Measurement
  - iii. Main characteristics of *LGD*
3. Implied Market Modeling Based on Bonds
  - i. Structural Models
  - ii. Standard Reduced-Form Models
  - iii. Extracting *LGD* from Bonds with Different Seniority
4. Reduced-form Modeling Based on Credit Default Swaps
  - i. CDS contractual terms
  - ii. Reduced-form Approach to Pricing CDS
  - iii. *PD* and *LGD* Simultaneous Estimation
  - iv. Extracting *LGD* from CDS with Different Seniority
5. Extracting Loss Given Default from Market Data
  - i. Data
  - ii. Liquidity Risk and Counterparty Default Risk Modeling
  - iii. Methodology and Results
6. Conclusion

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Supervisor