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Referee's report to doctoral thesis

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Optimization Problems under (max, min)-Linear Constraints and Some Related Topics

In the submitted thesis, Mr. Mahmoud Gad considers systems of (max, min)-linear of equations and inequalities with variables occurring on one side of the equations and inequalities, and similar systems with variables on both sides. Optimization problems with (max, min)-separable objective function and (max, min)-linear constraints are also studied. The results are extended to solving incorrectly posed (max, min)-linear systems, which have no exact solution. For these problems the optimal approximation of the input values is computed, for which the approximated system is solvable. The (max, min)-linear problems have many practical applications in different fields of human technical and research activities, such as manufacturing systems, control systems, information systems, graph theory, scheduling theory, or computer networks theory.

The problems investigated in the doctoral thesis belong to so-called extremal algebra, which is one of the principal objects of study in the research group Optimization and Decision Making (ODEMA) at the Faculty of Informatics and Management of the University in Hradec Králové. During his doctoral study at Charles University in Prague, Mr. Mahmoud Gad cooperated with this research group, presented his results on international conferences, and he is an author or co-author of several research papers which have been published or submitted for publication in scientific journals. The results presented in the papers are systematically summarized in the submitted thesis. The selected research area is topical and the content of the thesis corresponds to contemporary research trends. The achieved results are also promising for further continuation of the research in the indicated area.

The first chapter of the thesis describes the historical background and the state-of-art of the (max, min)-linear system solving and optimization problems. The rest of the thesis is based on author`s original results (as independent author or coauthor). In the second chapter, an algorithm is presented for computing the optimal solution of a two-sided (max, plus)-linear optimization problem. The computational complexity of the described algorithm depends on the objective function. The third chapter studies the optimization problem under one-sided (max, min)-linear inequality constraints. Polynomial algorithms are presented for finding the maximal feasible solution and the optimal solution, or recognizing when there is no feasible solution. The optimization problem under one-sided (max, min)-linear equality constraints is investigated independently in chapter four. Chapter five and six are devoted to two-sided systems of (max, min)-linear equations and inequalities. Algorithms for computing the optimal solutions are

presented, and the solvability of the system with inequalities is completely described. The incorrectly posed one-sided (max, min)-linear equations system are considered in chapter seven, where several approaches to finding an optimal solution are described. Finally, the eighth chapter contains a generalization of the optimization problems with one-sided max-separable equality and inequality constraints.

Besides the theoretical results and presented algorithms, the thesis contains a number of numerical examples which illustrate the proved theorems and the work of the algorithms. The theoretical results and algorithms are well-applicable in various practical optimization problems.

The elaboration of the selected method shows the research competency of the author. The introductory part and the state-of-the-art description are written with sufficient level of knowledge. The text contains adequate formulations, and the expression style is systematic, exact and clear. All items in the list of references are cited on proper places in the text. Summarizing, it can be said that the thesis is prepared on very good technical and scientific level. The thesis contains new results and new ideas, and indicates their possible applicability. The goals of the thesis have been achieved.

Conclusion

The thesis „Optimization Problems under (max, min)-Linear Constraints and Some Related Topics“ submitted by Mr. Mahmoud Gad at the Faculty of Mathematics and Physics, the Charles University in Prague and at the Faculty of Science, Sohag University, satisfies the requirements for a doctoral thesis, and therefore

I recommend

to award Mr. Mahmoud Attya Mohamed Gad the degree „Philosophiae Doctor (Ph.D.) - Doctor of Philosophy“ in the field Operations Research.

Hradec Králové, September 30, 2014 Prof. RNDr. Martin Gavalec, CSc.