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Review of the habilitation thesis of RNDR. Svatopluk Krysl, Ph.D.

"Symplectic spinors and Hodge theory"

The habilitation thesis consists of 10 papers, published in different journals between 2007 and 2016. There is an introduction with 32 pages and 82 references, in which the author describes very carefully his research during the last 10 years. It can mainly be subdivided in two large topics :

– symplectic spinors and symplectic Dirac operators (7 publications),

– de Rham and Hodge theory over C^*-algebras (3 publications).

The spinor bundle over a symplectic manifold is an infinite-dimensional vector bundle constructed via the Segal-Shale-Weil representation of the metaplectic group. More than 20 years ago, Katharina Habermann began to investigate symplectic spinors and symplectic Dirac operators from the viewpoint of differential geometry. In volume No. 1887 of the Lecture Notes in Mathematics, K. and L. Habermann summarized the results obtained until 2006, see [26] in the list of references. This has been the starting point of Krysl's work on the first topic. During the last 10 years, Svatopluk Krysl developed new direction in symplectic spin geometry and obtained a series of new and interesting results. Let me mention some of them.

1.) S. Krysl constructs higher symplectic spinor modules as well as higher symplectic spinor bundles by decomposing suitable tensor products of representations of the metaplectic group.

2.) He studies the behavior of the exterior symplectic spinor derivative depending on a Fedosov connection on these higher bundles. This is a family of torsion-free connections preserving the symplectic structure.
3.) S. Krysl studies the action of the Ricci tensor and Weyl tensor of a Fedosov connection on spinors.
4.) He constructs several (elliptic) complexes of higher spinor bundles, again depending on special Fedosov connections. The examples relate the first and second topic of Krysl's work, see below.
5.) He investigates the symplectic analogues of the Riemannian Dirac operator, the Rarita-Schwinger operator and of the Riemannian twistor operator.
6.) S. Krysl discusses symplectic Killing spinors and their integrability conditions. He computes the space of symplectic Killing spinors explicitely on the 2-dimensional symplectic vector space as well as on the 2-dimensional sphere. Moreover, he proves that a spinor field is a symplectic Killing spinor iff it is an eigenspinor of the symplectic Dirac operator and belongs to the kernel of the symplectic twistor operator.

The complexes over symplectic manifolds constructed in 4.) are defined in infinite-dimensional vector bundles. For these, there is no general index theory, no general de Rham or Hodge theory. The publications on the second topic (two papers in 2015 and 2016) are devoted to these questions in the context of functional analysis. In the first paper, S. Krysl considers so-called "self-adjoint parametrix possessing complexes" (in the obvious sense of a paramatrix in a complex) over C^*-algebras. The main result is a Hodge decomposition. In particular, this applies to A^*-elliptic complexes. In the second paper he discusses elliptic complexes of pseudodifferential operators acting in C^*-Hilbert-bundles over compact manifolds. Again, S. Krysl proves a Hodge decomposition.

By his substantial work on symplectic spin geometry of the past decade, Svatopluk Krysl was able to enlarge and develop this research area beyond the topics covered in the field before. He obtained a series of very original and interesting results, and showed up new directions for further research. He therefore proved his ability to carry out independent mathematic research. Hence, I stronlgy recommend to accept his habilitation thesis.

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