OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG MATH FAKULTÄT FÜR MATHEMATIK

Institut für Mathematische Optimierung
Geschäftsführender Leiter

**Prof. Dr. Volker Kaibel**

Otto-von-Guericke-Universität Magdeburg
Universitätsplatz 2
39106 Magdeburg

Telefon: +49 391 67-18759
Telefax: +49 391 67-11171

kaibel@ovgu.de
www.ovgu.de

Otto-von-Guericke-Universität Magdeburg, Postfach 4120, 39016 Magdeburg
Professor Jan Trlifaj
Research and International Affairs Department
CharlesUniversity
Ke Karlovu 3
**121 16 Praha 2**

28.10.2016

Dear Professor Trlifaj,

in response to your letter of July 7, I am happy to provide a review of the habilitation thesis *Linear and exact extended formulations* submitted by Dr. Hans R. Tiwary.

Extended formulations have been a very active research area in Combinatorial Optimization in the last few years. The basic idea here is to obtain a polytope representing the feasible solutions to the combinatorial problem under investigation as a linear projection of a higher dimensional polytope that admits a description by few linear inequalities. The submitted thesis is composed from several contributions Dr. Tiwary has made to this area.

In fact, Dr. Tiwary has been involved in some of the most exciting recent developments in that research area. Most notably, he is one of the authors (with Fiorini, de Wolff, Massar, and Pokutta) of the seminal paper *Exponential Lower Bounds for Polytopes in Combinatorial Optimization* (Journal of the ACM, 2015, included into the thesis as Appendix [A]) in which a 25 year old question posed by Yannakakis was answered: The traveling salesman polytope does not have an extended formulation of polynomial size. Due to complexity reasons this result did not come as a surprise, but rather the fact that a proof for it had eventually been found was extremely valuable. For good reasons this work has been distinguished with the Best Paper Award at the Symposium on Theory of Computing (STOC 2012). Prior to that result, Dr. Tiwary had already obtained another very interesting lower bound (together with Fiorini and Rothvoss): The extension complexity (i.e., the smallest size of an extended formulation) of an $n$-gon can in general not be bounded from above significantly better than by $\sqrt{n}$ (Appendix [C], Discrete & Comput. Geometry, 2012). In this early stage of his work on extended formulations, besides obtaining these remarkable concrete lower bound results, Dr. Tiwary was also involved in further developing the connections between extended formulations and Communication Complexity (Appendix [B], Math. Programming, 2015, jointly with Faenza, Fiorini, and Grappe) that had originally been observed by Yannakakis. He also, together with Avis, derived exponential lower bounds on the extension complexities of polytopes associated with several NP-hard problems by geometrically imitating Turing reductions (Appendix [D], Math. Programming, 2015). Dr. Tiwary moreover worked on the constructive side of the topic, coming up, e.g., with small extended formulations for polytopes associated with formal languages decidable via certain one-pass Turing machines (Appendix [E]). Other constructive results include extended formulations for polytopes associated with graph problems defined via monadic second order formulae; here, the sizes of the formulations are bounded in a fixed-parameter fashion by the product of the size of the graph and a function depending only on the formula and the treewidth of the graph (Appendix [G], Proceedings of SWAT, 2016, jointly with Kolman and Koutecký). Another one of Dr. Tiwary's papers (Appendix [F], jointly with Gajarský and Hlineňý) also contains quite interesting negative results on the existence of fixed-parameter small extended formulations of stable set polytopes. Together with Avis, Dr. Tiwary has also suggested and investigated interesting relaxations of the concept of extended formulations: $\mathcal{H}$-free extended formulations (Appendix [I], Infiormation Processing Letters, 2015) and weak extended formulations (Appendix [J], Math. Programming, 2015)

The thesis provides a thoughtfully structured overview over the theory of extended formulations and in particular over the contributions made by Dr. Tiwary. The individual concepts and results are put into context in a very suitable way. Thus, though compiled from ten different papers, the thesis nicely presents Dr. Tiwary's work as composed from well-fitting pieces. It is a bit of a pity that despite some typos and a few minor inconsistencies (e.g., the formula in Prop. 1.1.16 is not exactly correct with the definition of irredundancy given on p. 12) some of the useful notions (like a clan of polytopes) are not defined rigorously. This does not affect the correctness of any of the results presented, but would be desirable to be corrected in case the author plans to use the thesis as a basis for some lecture notes, as might be intended in view of several exercises that are already formulated in the text.

In summary, Dr. Tiwary has successfully worked in a quite active area of research over the last few years. His contributions have been very well recognized by the community. His research has been conducted in different groups of coworkers. I am sure that Dr. Tiwary has contributed significantly to all the papers the thesis is based upon. He is also able to set up research agendas in new working environments, as one can deduce, e.g., from the papers listed as Appendices [D,E,F,G,H,I,J].

Without any hesitation I recommend to accept the habilitation thesis submitted by Dr. Hans R. Tiwary and to appoint him as an associate professor.

Prof. Dr. Volker Kaibel