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February 14, 2017

Dear Professor Gáliková:

One of the fundamental problems is ring-theory is to determine to what extent ring-theoretic properties of a ring R are determined by properties of the category of right, respectively left R-modules. This investigation usually requires the identification of specific (full) subcategories of the module category, and then to investigate closure and other properties of these subcategories. Dr. Žemlička's Habiltation thesis focuses on questions related to this approach, and contains several nice results which have to be considered significant contributions to his chosen research area. In the following I want to comment on some of his results.

Finiteness conditions play a central role in ring theory, and many pathological examples of rings arise because of the absence of such conditions. Dr. Žemlička looks at a variety of finiteness conditions, and obtains characterizations of important classes of rings satisfying them. For instance, a ring R is called right steady if the classes of small and finitely generated right R-modules coincide. For instance, every right Noetherian ring is right steady, but there are many interesting classes of right steady rings which do not satisfy any of the standard finiteness conditions. Although the absence of such finiteness conditions makes any investigation of right steady rings very challenging, Dr. Žemlička's focuses on these difficult classes of rings. In particular, he obtains complete characterizations of right steady rings in several important cases. I particularly like his results for right chain rings and for rings with complete set of idempotents. These have to be considered important contributions to ring theory.

The discussion of right steady rings naturally leads to the investigation of the class of self-small right modules. While self-small Abelian groups play an important role in the investigation of mixed and torsion-free Abelian groups, only little is known about them in the non-commutative case. Therefore, it is natural to look for an extension of this concept to a categorical setting in order to get a better handle on how to deal with these modules in a more general setting. One such extension is the notion of C-compactness which Dr. Žemlička has investigated in a series of joint papers with Dr. Breaz. Their results offer interesting insights into the behavior of self-small modules over non-commutative rings.

Another part of Dr. Žemlička's thesis concentrates on semi-Artinian rings. Again, his investigation of these interesting rings is thorough, and enhances our understanding of this class of rings. A further interesting class of rings are the right tall rings which are precisely the rings R such that every non-Noetherian right R-module contains a proper submodule with the same property. Dr. Žemlička obtains a very nice description of the non-tall commutative rings in Theorem 3.6.

Finally, I want to discuss Dr. Žemlička's work on RM-rings. The restricted minimum condition arises in the discussion of Noetherian hereditary rings. Not much has been known about a RM-ring R and its modules except in the case that R is an integral domain. Dr. Žemlička work gives us some interesting insights in the non-commutative case. He describes the singular modules over such rings in Theorem 5.2. Theorem 5.3 determines when an RM-ring is Noetherian which is a very interesting result since it clarifies the relation between being Noetherian and having the restricted minimum condition.

In summary, I feel that Dr. Žemlička's research contains several important contributions to ring and module theory. It shows a deep understanding of the subject, and contains innovative ideas which will have a significant impact on the work of other researchers in these areas. His thesis is a systematic investigation of how categorical and finiteness conditions can interact to enhance our understanding of the structure of rings and modules. I find it is particular important that he uses his approach to study several different classes of rings, e.g. steady, tall, and RM-rings, since this nicely demonstrates the applicability of his approach to the investigation of interesting and important classes of rings which are very difficult to work with in general.

I strongly support that Dr. Žemlička's Habilitation thesis is accepted and that he is awarded the title of "Docent".

Sincerely,

U. Albrecht
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