Report on Habilitation Thesis
Classes of rings determined by a categorical property
Jan Žemlička

The main theme of this thesis is the relationship between the structure of a ring R and the categorical properties of classes of R–modules. On the one hand, given a ring R with a particular property, such as commutative or semi–simple, what are the categorical properties of a class of modules over R, for example all modules, or finitely generated modules? On the other hand, given a categorical property, what is the algebraic structure of rings R over which all modules or some class of modules have the required property? For example, a classical theorem of Bass states that R is left perfect if and only if every left R–module has a projective cover.

Questions such as these have been a major concern of 21st Century studies in algebra, and Žemlička has been a major contributor to answering them. The core of this thesis consists of nine of his publications between 2001 and 2015, all published in prestigious international journals. Five of these are joint publications and four solely authored by Žemlička.

The thesis begins with a 25–page extensive survey of published research into classes of rings determined by a categorical property. It establishes the notation and presents the history of the problem and the major known results, dating from the work of Bass in the 1960s up to the present day, 2016. It emphasises the major rôle that the Czech school has played in this important area of Algebra. This chapter, whose Bibliography contains 113 items, is itself a major contribution to the field.

The rest of the thesis is divided into four chapters, each containing one or more of the papers mentioned above.

Chapter 2: **Self–small modules and strongly steady rings** consists of

A. Simeon Breaz and Jan Žemlička *When every self–small module is finitely generated*, J. Algebra 315/2, (2007), 823–893.
B. Jan Žemlička *When products of self–small modules are self–small*, Comm. Algebra 36/7, (2008), 2570–2576.

Chapter 3: **Small modules and steady rings** consists of

C. Jan Žemlička *Steadiness is tested by single module*, Abelian groups, rings and modules, Proc. AGRAM 2000 Conference, Amer. Math. Soc. Cont. Math., 273, (2001), 301–308.
D. Jan Žemlička *Steadiness of regular semiartinian rings with primitive factors artinian*, J. Algebra 304/1, (2006), 500–509.

Chapter 4: **The defect functor of homomorphisms and direct unions** consists of a single important paper

E. Simeon Breaz and Jan Žemlička *The defect functor of homomorphisms and direct unions*, Algebr. Represent. Theory, 19/1, (2016), 181–208.

Chapter 5: **Reflection of categorical properties to a ring structure** consists of the following four papers:

F. Jan Žemlička *Socle chains of abelian regular semiartinian rings*, J. Pure Appl. Algebra, 217/6, (2013), 1018–1025.

G. Tomás Penk and Jan Žemlička *Commutative tall rings*, J. Algebra Appl., 13/4, (2014).

H. M. Tamer Kosan and Jan Žemlička *Mod-retractable rings*, Comm. Algebra 42/3, (2014), 998–1010.

I. M. Tamer Kosan and Jan Žemlička *On modules and rings with restricted minimum condition*, Colloq. Math. 140/1, (2015), 75–86

To summarise, Jan Žemlička has demonstrated by his scientific publications and this Thesis that he is a worthy candidate for Habilitation in the Faculty of Mathematics and Physics of Charles University.

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