

A parity path in a vertex colouring of a graph G is a path in which every colour is used even number of times. A parity vertex colouring is a vertex colouring having no parity path. Let $\chi_p(G)$ be the minimal number of colours in a parity vertex colouring of G . It is known that $\chi_p(B_n) \geq \sqrt{n}$ where B_n is the complete binary tree with n layers. We show that the sharp inequality holds. We use this result to obtain a new bound $\chi_p(T) > \sqrt[3]{\log n}$ where T is any binary tree with n vertices.

We study the complexity of computing the parity chromatic number $\chi_p(G)$. We show that checking whether a vertex colouring is a parity vertex colouring is coNP-complete and we design an exponential algorithm to compute it. Then we use Courcelle's theorem to prove the existence of a FPT algorithm checking whether $\chi_p(G) \leq k$ parametrized by k and the treewidth of G . Moreover, we design our own FPT algorithm solving the problem. This algorithm runs in polynomial time whenever k and the treewidth of G is bounded. Finally, we discuss the relation of this colouring to other types of colourings, specifically unique maximum, conflict free, and parity edge colourings.