Report on the doctoral thesis of Barbora Hudcová
“Complexity and Computational Capacity of Discrete Dynamical Systems”

As stated in the title, the main topics of the thesis are complexity and computational capacity of discrete dynamical systems, mainly of cellular automata. Beyond the introductory chapter, the manuscript consists of two quite standalone parts. Let me overview the matter of both of them.

Part I is divided into two parts, Chapter 2 and Chapter 3. In both of them, finite time-space approximations of cellular automata are treated. By numerical and statistical analysis of these finite dynamical systems, the limiting behavior of the original infinite-space system is estimated.

Chapter 2 focuses on the notion of transient length, i.e. the time before the attraction happens for a given starting configuration. The question is how this time averaged over all starting configurations depends on the space-width of the approximation of an infinite-space system. It is shown numerically, that for the main part of automata from some classical classes, the average transient length increases logarithmically or exponentially fast, when the space-width of approximation goes to infinity. Except these two rates, also bounded, linear and polynomial behavior is considered and the classification based on it is compared to the classical Wolfram's classification, Zenil's classification and Wuensche's Z-parameter. Let us recall that all of these classifications aim to distinguish between ordered, chaotic and complex behavior.

In Chapter 3, graphical cellular automata are considered, mainly those of them where the local rule strongly reflects the majority state among the neighbors. That majority is measured by the density of appearance of the state. The main result of this chapter is then the recognition of the phase-transition of the whole dynamical system when the density is changed. In order to detect such behavior, methods from statistical physics, namely the dynamical cavity method and its backtracking version, are involved.

Part II consists of Chapters 4 and 5. Complexity of the cellular automata is treated here via its computational capacity and ability to simulate each other. The simulation by a class of automata, means that one can reproduce another automata from a given class by standard categorical constructions such as the sub-automaton, the quotient automaton, the product automaton or the automaton's iterative power.

In Chapter 4, simulation capacity of permutive affine cellular automata is concerned. It is proved that the class of this automata has limited simulation power, namely, using automata from this class, one can simulate just another permutive affine automaton.

Chapter 5 dwell into more subtle question about the simulation capacity of one single affine cellular automata. Under the conditions of permutivity and double bijectivity, it is shown that the simulation capacity of a canonical affine automata is limited to the products of its iterative powers.

In other words, construction of sub-automata or quotient automata does not produce anything new. Based on these results, the simulation capacity of each single Wolfram elementary cellular automata is described.

Candidate's contribution

Chapter 2 is based on the candidate's own work ([67]), where her supervisor, Tomáš Mikolov, is added as co-author only for formal reasons. Chapter 3 is based on two candidate's joint works with Freya Behrens and Lenka Zdeborová ([15] and [16]). Her contribution was minor in the first paper, but distinctive in the second one. Part II presents the result of joint work with Jakub Krásenský ([66]). The contribution of Barbora Hudcová is major.

Evaluation and reservations

The thesis is well-structured and introduces new notions and relevant results for the theory of cellular automata and related research. The notions developed in this work help to put natural well-known phenomena, often described only intuitively and vaguely, on the solid ground. This is in my opinion a very important contribution of the candidate to the field, where the applications are developed much more rapidly than the fundamentals. I also appreciate the implementation of algebraic theory to the theory of simulation capacity that provides new insight.

My reservations are minor. As I stated in the beginning of the review, the thesis consists of two parts that are connected, in my opinion, only by the common topics, namely the complexity of the discrete systems. They differ by their approaches, used methods and notions they treated. Although the candidate had to demonstrate her ability to conduct research in both areas, I miss more efforts to connect both parts together. I also miss some theory on transient length, possibly elementary. General theoretical constraints on the behavior of transient length and some observations for well chosen automata would be good theoretical basis and framework for the experimental studies. This is a direction which I would suggest for further research.

Conclusions

I would like to clearly state that the manuscript presented by Barbora Hudcová meets, in quantity and quality, the standards of a doctoral thesis. The object of the thesis is a new contribution to rapidly developing multidisciplinary sciences on complex systems. Her research demonstrates her ability for independent creative work as well as teamwork. The novelty and relevance of her work have been accepted by the scientific community, as evidenced by two of her articles published in peer-reviewed journals. For all these reasons, I recommend the thesis for public defense.

Questions

At the end of the review, I would like to ask the following questions.

The way of construction of finite-space versions of a cellular automaton in Part I corresponds to the application of the global rule on infinite periodic configurations with fixed length of the period. What results do you expect if you consider other configurations, namely finite configurations of given length extended to the infinite space by the repetition of the same symbol?

In order to show elementary properties of the simulation ability of a class of cellular automata, some set inclusions are used. One of them is $SH(K) \subset HS(K)$. Do you have an example, when the inclusion is strict? Is there such an example of a class of affine automata?

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